Novi Sad J. Math. Vol. 49, No. 2, 2019, 49-57
https://doi.org/10.30755/NSJOM.07956

ALMOST CONTRA (I, J)-CONTINUOUS MULTIFUNCTIONS

Ennis Rosas[123], **Carlos Carpintero**[4] **and José Sanabria**[5]

Abstract. The purpose of the present paper is to introduce, study and characterize the upper and lower almost contra (I, J)-continuous multifunctions. Also, we investigate their relation with another well known class of continuous multifunctions.

AMS Mathematics Subject Classification (2010): 54C10; 54C08; 54C05; 54C60

Key words and phrases: weakly (I, J)-continuous multifunctions; upper almost contra (I, J)-continuous multifunctions; I-regular open set; I-regular closed set; contra (I, J)-continuous multifunctions

1. Introduction

It is well known today that the notion of multifunction is playing a very important role in general topology. Upper and lower continuity have been extensively studied on multifunctions $F : (X, \tau) \to (Y, \sigma)$. Currently using the notion of topological ideal, different types of upper and lower continuity in multifunction $F : (X, \tau, I) \to (Y, \sigma)$ have been studied and characterized [2], [6], [7], [14], [17]. The concept of ideal topological space has been introduced and studied by Kuratowski [9]. The local function of a subset A of a topological space (X, τ) was introduced by Vaidyanathaswamy [16] as follows. Let (X, τ) be a topological space with an ideal I on X. If $P(X)$ is the set of all subsets of X, the set operator $(\,)^* : P(X) \to P(X)$, called the local function of A with respect to τ and I, is defined as follows: for $A \subseteq X$, $A^*(\tau, I) = \{x \in X : U \cap A \notin I$ for every $U \in \tau_x\}$, where $\tau_x = \{U \in \tau : x \in U\}$. A Kuratowski closure operator $cl^*(\,)$ for the topology $\tau^*(\tau, I)$ is defined by $cl^*(A) = A \cup A^*(\tau, I)$. The topology $\tau^*(\tau, I)$ is called the *-topology and it is finer than τ. We will denote $A^*(\tau, I)$ by A^*. In 1990, Janković and Hamlett [9], introduced the notion of I-open set in a topological space (X, τ) with an ideal I on X. In 1992, Abd El-Monsef et al. [1] further investigated I-open sets and I-continuous functions. In 2007, Akdag

[1]Departamento de Ciencias Naturales y Exactas, Universidad de la Costa, Barranquilla, Colombia

[2]Departamento de Matemáticas, Universidad de Oriente, Cumaná, Venezuela.
e-mail: ennisrafael@gmail.com, erosas@cuc.edu.co

[3]Corresponding author

[4]Departamento de Matemáticas, Universidad de Oriente, Cumaná , Venezuela & Vicerrectoría de Investigación, Universidad Autónoma del Caribe, Barranquilla, Colombia.
e-mail: carpintero.carlos@gmail.com

[5]Facultad de Ciencias Básicas, Universidad del Atlántico, Barranquilla, Colombia.
e-mail: jesanabri@gmail.com; http://orcid.org/0000-0002-9749-4099

[2], introduced the concept of I-continuous multifunctions in a topological space with an ideal on it. For two ideal topological spaces (X, τ, I) and (Y, σ, J)we consider the multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$. We want to study some type of upper and lower continuity of F as was done in Rosas et al. [12]. In this paper, we introduce, study and characterize a new class of multifunction called almost contra (I, J)-continuous multifunctions in topological spaces. We investigate its relation with another class of continuous multifunctions. Also its properties when the ideal $J = \{\emptyset\}$.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces in which no separation axioms are assumed, unless explicitly stated and if I is an ideal on X, (X, τ, I) means an ideal topological space. For a subset A of (X, τ), $\mathrm{cl}(A)$ and $\mathrm{int}(A)$ denote the closure of A with respect to τ and the interior of A with respect to τ, respectively. A subset A is said to be regular open [15] (resp. semiopen [10], preopen [11] , semi-preopen [3]) if $A = \mathrm{int}(\mathrm{cl}(A))$ (resp. $A \subseteq \mathrm{cl}(\mathrm{int}(A)), A \subseteq \mathrm{int}(\mathrm{cl}(A)), A \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)))$). The complement of a regular open (resp. semiopen, semi-preopen) set is called regular closed (resp. semiclosed, semi-preclosed) set. A subset S of (X, τ, I) is I-open [8], if $S \subseteq \mathrm{int}(S^*)$. The complement of an I-open set is called an I-closed set. The I-closure and the I-interior, can be defined in the same way as $\mathrm{cl}(A)$ and $\mathrm{int}(A)$, respectively. They will be denoted by $I\,\mathrm{cl}(A)$ and $I\mathrm{int}(A)$, respectively. A subset S of (X, τ, I) is I-regular open (resp. I-regular closed), if $S = I\mathrm{int}(I\,\mathrm{cl}(S))$ (resp. $S = I\,\mathrm{cl}(I\mathrm{int}(S))$). The family of all I-open (resp. I-closed, I-regular open, I-regular closed, semiopen, semi closed, preopen, semi-preclosed) subsets of a (X, τ, I), is denoted by $IO(X)$ (resp. $IC(X), IRO(X), IRC(X), SO(X), SC(X), PO(X), SPO(X), SPC(X)$). We set $IO(X, x) = \{A : A \in IO(X)$ and $x \in A\}$. It is well known that in a topological space (X, τ, I), $X^* \subseteq X$ but if the ideal is codense, that is $\tau \cap I = \emptyset$, then $X^* = X$.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, also we always assume that $F(x) \neq \varnothing$ for all $x \in X$. For a multifunction $F : X \to Y$, the upper and lower inverse of any subset A of Y denoted by $F^+(A)$ and $F^-(A)$, respectively, that is $F^+(A) = \{x \in X : F(x) \subseteq A\}$ and $F^-(A) = \{x \in X : F(x) \cap A \neq \varnothing\}$. In particular, $F^+(y) = \{x \in X : y \in F(x)\}$ for each point $y \in Y$.

Definition 2.1. [14] A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be

1. upper weakly continuous if for each $x \in X$ and each open set V of Y such that $x \in F^+(V)$, there exists an open set U containing x such that $U \subseteq F^+(Cl(V))$.

2. lower weakly continuous if for each $x \in X$ and each open set V of Y such that $x \in F^-(V)$, there exists an open set U containing x such that $u \in F^-(Cl(V))$ for every $u \in U$.

3. weakly continuous if it is both upper weakly continuous and lower weakly continuous.

Definition 2.2. [2] A multifunction $F : (X, \tau, I) \to (Y, \sigma)$ is said to be

1. upper I-continuous if for each $x \in X$ and each open set V of Y such that $x \in F^+(V)$, there exists an I-open set U containing x such that $U \subseteq F^+(V)$.

2. lower I-continuous if for each $x \in X$ and each open set V of Y such that $x \in F^-(V)$, there exists an I-open set U containing x such that $U \subseteq F^-(V)$.

3. I-continuous if it is both upper I-continuous and lower I-continuous.

Definition 2.3. [4] A multifunction $F : (X, \tau, I) \to (Y, \sigma)$ is said to be

1. upper weakly I-continuous if for each $x \in X$ and each open set V of Y such that $x \in F^+(V)$, there exists an I-open set U containing x such that $U \subseteq F^+(Cl(V))$.

2. lower weakly I-continuous if for each $x \in X$ and each open set V of Y such that $x \in F^-(V)$, there exists an I-open set U containing x such that $U \subseteq F^-(Cl(V))$

3. weakly I-continuous if it is both upper weakly I-continuous and lower I-weakly continuous.

Definition 2.4. [12] A multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ is said to be:

1. upper weakly (I, J)-continuous at a point $x \in X$ if for each J-open set V such that $x \in F^+(V)$, there exists an I-open set U containing x such that $U \subseteq F^+(JCl(V))$

2. lower weakly (I, J)-continuous at a point $x \in X$ if for each J-open set V of Y such that $x \in F^-(V)$, there exists an I-open set U of X containing x such that $U \subseteq F^-(JCl(V))$.

3. upper (resp. lower) (I, J)-continuous on X if it has this property at every point of X.

Theorem 2.5. *[13] For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are equivalent:*

1. *F is upper weakly (I, J)-continuous.*

2. *$F^+(V) \subseteq I\,int(F^+(J\,\mathrm{cl}(V)))$ for any J-open set V of Y.*

3. *$I\,\mathrm{cl}(F^-(J\,int(B))) \subset F^-(B)$ for any every J-closed subset B of Y.*

Theorem 2.6. *[13] For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are equivalent:*

1. *F is lower weakly (I, J)-continuous.*

2. *$F^-(V) \subseteq I\,int(F^-(J\,\mathrm{cl}(V)))$ for any J-open set V of Y.*

3. *$I\,\mathrm{cl}(F^+(J\,int(B))) \subset F^+(B)$ for any every J-closed subset B of Y.*

Definition 2.7. [12] A multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ is said to be:

1. upper (I, J)-continuous at a point $x \in X$ if for each J-open set V such that $x \in F^+(V)$, there exists an I-open set U containing x such that $F(U) \subset V$.

2. lower (I, J)-continuous at a point $x \in X$ if for each J-open set V of Y such that $x \in F^-(V)$, there exists an I-open set U of X containing x such that $u \in F^-(V)$ for each $u \in U$.

3. upper (resp. lower) (I, J)-continuous on X if it has this property at every point of X.

Theorem 2.8. *[13] For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are equivalent:*

1. *F is lower weakly (I, J)-continuous.*

2. *$F^-(V) \subseteq I\,int(F^-(J\,\mathrm{cl}(V)))$ for any J-open set V of Y.*

3. *$I\,\mathrm{cl}(F^+(J\,int(B))) \subset F^+(B)$ for any every J-closed subset B of Y.*

Definition 2.9. [13] A multifunction $f : (X, \tau, I) \to (Y, \sigma, J)$ is said to be:

1. upper contra (I, J)-continuous if for each $x \in X$ and for each J-open set V such that $x \in F^+(V)$, there exists an I-open set U containing x such that $F(U) \subset V$.

2. lower contra (I, J)-continuous if for each $x \in X$ and for each J-open set V of Y such that $x \in F^-(V)$, there exists an I-open set U of X containing x such that $U \subseteq F^-(V)$.

3. contra (I, J)-continuous if it is upper contra (I, J)-continuous and lower contra (I, J)-continuous.

Definition 2.10. [5] A multifunction $f : (X, \tau, I) \to (Y, \sigma)$ is said to be:

1. upper almost contra I-continuous if for each $x \in X$ and for each regular closed set V such that $x \in F^+(V)$, there exists an I-open set U containing x such that $F(U) \subset V$.

2. lower almost contra I-continuous if for each $x \in X$ and for each regular closed set V of Y such that $x \in F^-(V)$, there exists an I-open set U of X containing x such that $U \subseteq F^-(V)$.

3. almost contra I-continuous if it is upper almost contra I-continuous and lower almost contra I-continuous.

3. Upper and Lower almost contra (I, J)-continuous multifunctions

Definition 3.1. A multifunction $f : (X, \tau, I) \to (Y, \sigma, J)$ is said to be:

1. upper almost contra (I, J)-continuous if for each $x \in X$ and for each J-regular closed set V such that $x \in F^{+}(V)$, there exists an I-open set U containing x such that $F(U) \subset V$.

2. lower almost contra (I, J)-continuous if for each $x \in X$ and for each J-regular closed set V of Y such that $x \in F^{-}(V)$, there exists an I-open set U of X containing x such that $U \subseteq F^{-}(V)$.

3. almost contra (I, J)-continuous if it is upper almost contra (I, J)-continuous and lower almost contra (I, J)-continuous.

Example 3.2. Let X be the set of real numbers with the topology $\tau = \{\emptyset, \mathbb{R}, \mathbb{R} \setminus \mathbb{Q}\}$, $Y = \mathbb{R}$ with the topology $\sigma = \{\emptyset, \mathbb{R}, \mathbb{Q}\}$ and $I = \{\emptyset\}$=J. Define $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $F(x) = \mathbb{Q}$ if $x \in \mathbb{Q}$ and $F(x) = \mathbb{R} \setminus \mathbb{Q}$ if $x \in \mathbb{R} \setminus \mathbb{Q}$. Recall that in this case, the I-open sets are the preopen sets. It is easy to see that F is upper (resp. lower) almost contra (I, J)-continuous.

Example 3.3. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$, $\sigma = \{\emptyset, Y, \{a\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset, \{b\}\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $F(c) = \{b\}$, $F(b) = \{c\}$ and $F(a) = \{a\}$. It is easy to see that:
The set of all I-open sets is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open sets is $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$.
The set of all J-regular closed sets is $\{\emptyset, \{c\}, Y\}$.
It is easy to see that F is upper (resp. lower) almost contra (I, J)-continuous but is not upper (resp. lower) (I, J)-continuous on X.

Example 3.4. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$ $\sigma = \{\emptyset, Y, \{a\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset, \{b\}\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $F(a) = \{b\}$, $F(b) = \{c\}$ and $F(c) = \{a\}$. It is easy to see that:
The set of all I-open sets is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open sets is $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$.
The set of all J-regular closed sets is $\{\emptyset, \{c\}, \{a, b\}, Y\}$.
It is easy to see that F is upper (resp. lower) (I, J)-continuous but is not upper (resp. lower) almost contra (I, J)-continuous on X.

Example 3.5. The multifunction F defined in Example 3.2, is upper (resp. lower) almost contra (I, J)-continuous but is not upper (resp. lower) (I, J)-continuous on X and the multifunction F defined in Example 3.3, is upper (resp. lower) (I, J)-continuous but is not upper (resp. lower) almost contra (I, J)-continuous. In consequence, both concepts are independent of each other.

Theorem 3.6. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are equivalent:*

1. *F is upper almost contra (I, J)-continuous.*
2. *$F^+(V)$ is I-open for each J-regular closed set V of Y.*
3. *$F^-(K)$ is I-closed for every J-regular open subset K of Y.*
4. *$F^-(J\,int(J\,\mathrm{cl}(B)))$ is I-closed for every J-open subset B of Y.*
5. *$F^+(J\,\mathrm{cl}(J\,int((V))))$ is I-open for every J-closed subset V of Y.*

Proof. (1)⇔(2): Let $x \in F^+(V)$ and V be any J-regular closed set of Y. From (1), there exists an I-open set U_x containing x such that $U_x \subset F^+(V)$. It follows that $F^+(V) = \bigcup_{x \in F^+(V)} U_x$. Since any union of I-open sets is I-open, $F^+(V)$ is I-open in (X, τ). The converse is similar.

(2)⇔(3): Let K be any J- regular open set of Y. Then $Y \backslash K$ is a J-regular closed set of Y. By (2), $F^+(Y \backslash K) = X \backslash F^-(K)$ is an I-regular open set. Then it is obtained that $F^-(K)$ is an I-regular closed set. The converse is similar.
(3)⇔(4): Let A be an I-open set of Y. Since $J\mathrm{int}(J\,\mathrm{cl}(B))$ is a J-regular open subset of Y, then by (3), $F^-(J\mathrm{int}(J\,\mathrm{cl}(B)))$ is an I-closed subset of X. The converse is clear.
(5)⇔(2): It follows in the same form as (3)⇔(4), only it is necessary to see that $J\,\mathrm{cl}(J\mathrm{int}((V)))$ is a J-regular closed set. □

Theorem 3.7. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are equivalent:*

1. *F is lower almost contra (I, J)-continuous.*
2. *$F^-(V)$ is I-open for each J-regular closed set V of Y.*
3. *$F^+(K)$ is I-closed for every J-regular open subset K of Y.*
4. *$F^+(J\,int(J\,\mathrm{cl}(B)))$ is I-closed for every J-open subset B of Y.*
5. *$F^-(J\,\mathrm{cl}(J\,int((V))))$ is I-open for every J-closed subset V of Y.*

Proof. The proof is similar to the proof of Theorem 3.6. □

Remark 3.8. It is easy to see that if $J = \{\emptyset\}$ and $F : (X, \tau, I) \to (Y, \sigma, J)$ is upper (resp. lower) almost contra (I, J)-continuous then F is upper (resp. lower) almost contra I-continuous.

Remark 3.9. When the ideal $J = \{\emptyset\}$, the J-regular open sets are the regular open sets and then every almost contra I-continuous is upper (resp. lower) almost contra (I, J)-continuous.

Remark 3.10. When the ideal $J = \{\emptyset\}$, the notions of almost contra (I, J)-continuous and almost contra I-continuous are the same.

Example 3.11. Let $\mathbb{R}$ be the set of the real numbers with the usual topology, take $I = J = \{\emptyset\}$. Define the multifunction $F : \mathbb{R} \to \mathbb{R}$ as $F(x) = \{x\}$. Recall that the I-open sets are the preopen sets. Observe that F is not: almost contra (I, J)-continuous, almost contra I-continuous but is (I, J)-continuous, weakly I-continuous.

Example 3.12. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$, $\sigma = \{\emptyset, Y, \{a\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset, \{b\}\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $f(a) = \{a\}$, $f(b) = \{c\}$ and $f(c) = \{b\}$. It is easy to see that:
The set of all I-open sets is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open sets is $\{\emptyset, \{a\}, \{c\}\{a, b\}, \{a, c\}, \{b, c\}, Y\}$.
the set of all J-regular open sets is $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{b, c\}Y\}$.
In consequence, F is not: upper (resp. lower) weakly (I, J)-continuous, upper (resp. lower) almost contra (I, J)-continuous, upper (resp. lower) (I, J)-continuous, upper (resp. lower) contra (I, J)-continuous but F is upper (resp. lower) contra I-continuous.

Example 3.13. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{b\}\}$ and two ideals $I = \{\emptyset, \{b\}\}$, $J = \{\emptyset, \{b\}\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $f(a) = \{a\}$, $f(b) = \{c\}$ and $f(c) = \{b\}$. It is easy to see that:
The set of all I-open is $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open is $\{\emptyset, Y, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
The set of all J-regular closed is $\{\emptyset, Y, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$.
The set of all preopen sets in Y is $\{\emptyset, Y, \{b\}, \{a, b\}, \{b, c\}\}$.
Observe that F is almost contra (I, J)-continuous and almost contra $(I, \{\emptyset\})$-continuous but is not (I, J)-continuous, weakly I-continuous.

Remark 3.14. Observe that if the ideal $J \neq \emptyset$, the notions of almost contra (I, J)-continuous multifunctions and the almost contra I-continuous multifunctions are independent.

Theorem 3.15. *If $F : (X, \tau, I) \to (Y, \sigma, J)$ is upper (resp. lower) almost contra (I, J)-continuous multifunction then it is upper (resp. lower) weakly (I, J)-continuous multifunction.*

Proof. Let $x \in X$ and V a J-open set containing $F(x)$. Follows that $J\,\mathrm{cl}(V)$ is a J-regular closed set of Y and $F(x) \subseteq J\,\mathrm{cl}(V)$. Using the hypothesis, there exists an I-open set U containing x such that $F(U) \subset J\,\mathrm{cl}(V)$. In consequence, F is upper weakly (I, J)-continuous. The proof for the case when F is lower almost contra (I, J)-continuous is similar. □

The following example shows that the converse of the Theorem 3.15 is not necessarily true.

Example 3.16. In Example 3.11, the multifunction F is not almost contra (I, J)-continuous but is weakly (I, J)-continuous multifunction.

Theorem 3.17. *If $F : (X, \tau, I) \to (Y, \sigma, J)$ is upper (resp. lower) contra (I, J)-continuous multifunction then it is upper (resp. lower) almost contra (I, J)-continuous multifunction.*

Proof. Since every J-regular closed set is a J-closed set the result is clear. □

The following example shows that the converse of the Theorem 3.17 is not necessarily true.

Example 3.18. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{b\}\}$ and two ideals $I = \{\emptyset, \{b\}\}$, $J = \{\emptyset, \{b\}\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $f(a) = \{b\}$, $f(b) = \{c\}$ and $f(c) = \{a\}$. It is easy to see that:
The set of all I-open is $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open is $\{\emptyset, Y, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. The set of all J-regular open is $\{\emptyset, Y\}\}$.
Observe that F is is almost contra (I, J)-continuous multifunction but is not contra (I, J)-continuous.

Example 3.19. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{b\}\}$ and two ideals $I = \{\emptyset, \{b\}\}$, $J = \{\emptyset\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $f(a) = \{b\}$, $f(b) = \{c\}$ and $f(c) = \{a\}$. It is easy to see that:
The set of all I-open sets is $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open sets are the set of preopen sets $\{\emptyset, Y, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
The set of all J-regular open sets is $\{\emptyset, Y, \{a, c\}, \{b\}\}$.

Observe that F is almost contra $(I, \{\emptyset\})$-continuous multifunction but is not contra $(I, \{\emptyset\})$-continuous multifunction.

Example 3.20. Let $\mathbb{R}$ be the set of the real numbers with the usual topology, take $I = J = \{\emptyset\}$. Define the multifunction $F : \mathbb{R} \to \mathbb{R}$ as $F(x) = \{x\}$. Recall that the I-open sets are the preopen sets. Observe that F is not almost contra $(I, \{\emptyset\})$-continuous but is contra I-continuous multifunction.

Remark 3.21. The notions of almost contra $(I, \{\emptyset\})$-continuous multifunctions and contra I-continuous multifunctions are independent.

Example 3.22. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$, $\sigma = \{\emptyset, Y, \{a\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $f(a) = \{a\}$, $f(b) = \{c\}$ and $f(c) = \{b\}$. It is easy to see that:
The set of all I-open sets is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open sets is $\{\emptyset, Y, \{a\}, \{a, b\}, \{a, c\}\}$.
the set of all J-regular open sets is $\{\emptyset, Y\}$.
In consequence, F is upper (resp. lower) almost contra (I, J)-continuous on X but is not upper (resp. lower) (I, J)-continuous

Remark 3.23. It is easy to see that if $F : (X, \tau, I) \to (Y, \sigma, J)$ is a multifunction and $JO(Y) \subset \sigma$. If F is upper (lower) almost contra I-continuous, then F is upper (lower) almost contra (I, J)-continuous. Even more, if $F : (X, \tau, I) \to (Y, \sigma, J)$ is a multifunction and $JO(Y) \nsubseteq \sigma$, we can find upper (resp. lower) almost contra (I, J)-continuous multifunctions that are not upper (lower) almost contra I-continuous multifunctions.

References

[1] Abd El-Monsef, M. E., Lashien, E. F., and Nasef, A. A. On I-open sets and I-continuos functions. *Kyungpook Math. J. 32*, 1 (1992), 21–30.

[2] Akdag, M. On upper and lower I-continuos multifunctions. *Far East J. Math. Sci. 25*, 1 (2007), 49–57.

[3] Andrijević, D. Semipreopen sets. *Mat. Vesnik 38*, 1 (1986), 24–32.

[4] Arivazhagi, C., and Rajesh, N. On upper and lower weakly I-continuous multifunctions. *Ital. J. Pure Appl. Math. 36* (2016), 899–912.

[5] Arivazhagi, C., Rajesh, N., and Shanthi, S. On upper and lower almost contra I-continuous multifunctions. *Int. J. Pure Appl. Math. 115*, 4 (2017), 787–799.

[6] Ekici, E. Nearly continuous multifunctions. *Acta Math. Univ. Comen. 72* (2003), 229–235.

[7] Ekici, E. Almost nearly continuous multifunctions. *Acta Math. Univ. Comen. 73* (2004), 175–186.

[8] Janković, D., and Hamlett, T. R. Compatible extensions of ideals. *Boll. Un. Mat. Ital. B (7) 6*, 3 (1992), 453–465.

[9] Kuratowski, K. *Topology.* Academic Press, 1966.

[10] Levine, N. Semi-open sets and semi-continuity in topological spaces. *Amer. Math. Montly 70*, 1 (1963), 36–41.

[11] Mashhour, A. S., Abd El-Monsef, M. E., and El-Deeb, S. N. On precontinuous and weak precontinuous mappings. *Proc. Math. Phys. Soc. Egypt 53* (1982), 47–53.

[12] Rosas, E., Carpintero, C., and Moreno, J. Upper and lower (I, J)-continuous multifunctions. *Int J. Pure Appl. Math. 117*, 1 (2017), 87–97.

[13] Rosas, E., Carpintero, C., and Sanabria, J. Weakly (I, J)-continuous multifunctions and contra (I, J)-continuous multifunctions. *Ital. J. Pure Appl. Math. 41* (2918), 547–556.

[14] Simithson, R. E. Almost and weak continuity for multifunctions. *Bull. Calcutta Math. Soc. 70* (1978), 383–390.

[15] Stone, M. H. Applications of the theory of boolean rings to general topology. *Trans. Amer. Math. Soc. 41*, 3 (1937), 375–481.

[16] Vaidyanathaswamy, R. The localisation theory in set topology. *Proc. Indian Acad. Sci. 20*, 1 (1944), 51–61.

[17] Zorlutuna, I. ω-continuous multifunctions. *Filomat 27*, 1 (2013), 165–172.

Received by the editors May 9, 2018
First published online March 28, 2019