Zbornik radova Prirodno-matematičkog fakulteta-Univerzitet u Novom Sadu, knjiga 9 (1979)
Review of Research Faculty of Science-University of Novi Sad, Volume 9 (1979)

Mileva Prvanović

HOLOMORPHICALLY SEMI-SYMMETRIC CONNEXIONS

§1. Object of the paper. — Let us consider a n-dimensional differentiable manifold M. The connexion of M is termed semi-symmetric if its torsion tensor S_{ij}^{k} has the form

$$S_{ij}^{k}=\delta_{i}^{k}S_{j}-\delta_{j}^{k}S_{i}, \tag{1.1}$$

where S_i are the components of a vector field. The geometrical meaning of the semi-symmetric connexion was given by E. Bartolotti [1] and it consists in the following. Let U^i and V^i be two vectors. The vectors $S_{ij}^{k}U^iV^j$, U^k and V^k are, in the general case, linearly independent. But if

$$S_{ij}^{k}U^iV^j=aU^k+bV^k \tag{1.2}$$

for every U and V, where a and b are scalars, then S_{ij}^{k} has the form (1.1) and conversely.

Let us now suppose that in the manifold M a tensor field $F_j^i\neq\delta_j^i$ satisfying

$$F_j^iF_k^j=\omega\delta_k^i, \quad \omega=-1 \quad \text{or} \quad \omega=+1 \tag{1.3}$$

is given. We suppose that the structure F_j^i is integrable. Then, if $\omega=-1$, the dimension of M is even, and M is a complex space [2]. If $\omega=+1$, M is a locally product space.

A semi-symmetric connexion is generalized on a complex space in [3] and on a locally product space in [4]. In [3], two connexions of a Kähler space are investigated, each of which can be considered a generalization of a semi-symmetric connexion. Furthermore, each of these connexions is a metric F-connexion.

The purpose of the present paper is the generalization of the semi-symmetric connexion, too. We generalize Bartolotti's geometrical property in the case that the manifold M has the integrable structure (1.3). Namely, we consider the skew-symmetric tensor S_{ij}^{k} satisfying condition

$$S_{ij}^{k}U^iF_a^jU^a=aU^k+bF_a^kU^a \tag{1.4}$$

instead of (1.2). We shall show, in § 2, that if S^k_{ij} satisfies (1.4) for every U, then

$$S^k_{ij}=\delta^k_j S_i-\delta^k_i S_j-\omega S_a F^a_i F^k_j+\omega S_a F^a_j F^k_i+ \\ +\frac{1}{2}(\delta^a_i\delta^b_j+\omega F^a_i F^b_j)\, W^k_{ab} \tag{1.5}$$

where S_i is a vector field and W^k_{ij} is an arbitrary skew-symmetric tensor. The conversely being obvious, we have:

Theorem 1. — *The skew-symmetric tensor S^k_{ij} satisfies condition (1.4) for every U^i if and only if it has the form (1.5).*

Definition. — We term an affine connexion having a torsion tensor of the form (1.5) a *holomorphically semi-symmetric connexion.*

The simplest connexion of the Riemannian space whose torsion tensor has the form (1.5), is

$$\Gamma^k_{ij}=\left\{ {k \atop ij} \right\}+S_i\delta^k_j-\omega S_a F^a_i F^k_j \tag{1.6}$$

This connexion is an F-connexion (*i. e.* $\nabla F=0$), but it is not a metric one.

Let R^i_{rkj} and K^i_{rkj} be the components of the curvature tensors of the connexions (1.6) and $\left\{ {k \atop ij} \right\}$ respectively and $R_{rk}=R^a_{rka}$, $K_{rk}=K^a_{rka}$. We shall prove, in §3,

Theorem 2. — *We have*

$$R^i_{rkj}+\frac{1}{2}(R_{kj}-R_{jk})\,\delta^i_r+\frac{1}{2}(R_{ka}F^a_j-R_{ja}F^a_k)\,F^i_r=K^i_{rkj}+K_{ka}F^a_j F^i_r \tag{1.7}$$

in the Kähler space, and

$$R^i_{rkj}+\frac{1}{2}(R_{kj}-R_{jk})\,\delta^i_r-\frac{1}{2}(R_{ka}F^a_j-R_{ja}F^a_k)\,F^i_r=K^i_{rkj} \tag{1.8}$$

in the locally decomposable Riemannian space.

Finally, in the case of a locally decomposable Riemannian space, we consider the connexion

$$\Gamma^k_{ij}=\left\{ {k \atop ij} \right\}+S_i\delta^k_j+g_{ij}S^i-F^a_i F^k_j S_a+F_{ij}F^k_a S^a \tag{1.9}$$

where g_{ij} are the components of the metric tensor and $F_{rj}=F^a_r g_{aj}$. The torsion tensor of this connexion has the form (1.5), too. It is also an F-connexion. We shall prove, in §4, Theorem 3. — *The curvature tensor R^i_{rkj} of the connexion (1.9) satisfies the relation*

$$(1.10)\quad \begin{cases} R^i_{rkj}-\frac{1}{2}\delta^i_r(R_{ab}-R_{ba})O^{ab}_{rk}+\frac{1}{2}F^i_rR_{pq}(O^{pq}_{ak}F^a_j-O^{pq}_{aj}F^a_k) \\ +\frac{1}{2}R_{pq}(g_{kr}g^{ia}O^{pq}_{aj}-g_{jr}g^{ai}O^{pq}_{ak}-F_{kr}F^{ia}O^{pq}_{aj}+F_{jr}F^{ia}O^{pq}_{ak}) \\ +\frac{1}{2}(\alpha R+\beta R^*)(g_{jr}\delta^i_k-g_{kr}\delta^i_j+F_{jr}F^i_k-F_{kr}F^i_j) \\ +\frac{1}{2}(\beta R+\alpha R^*)(g_{jr}\delta^a_k-g_{kr}\delta^a_j+F_{jr}F^a_k-F_{kr}F^a_j)F^i_a= \\ =K^i_{rkj}-\frac{1}{2}(g_{jr}K^i_k-g_{kr}K^i_j+F_{jr}F^i_aK^a_k-F_{kr}F^i_aF^a_j) \\ +\frac{1}{2}(\alpha K+\beta K^*)(g_{jr}\delta^i_k-g_{kr}\delta^i_j+F_{jr}F^i_k-F_{kr}F^i_j) \\ +\frac{1}{2}(\beta K+\alpha K^*)(g_{jr}\delta^a_k-g_{kr}\delta^a_j+F_{jr}F^a_k-F_{kr}F^a_j)F^i_a, \end{cases}$$

where we use the notations (2.3), (4.3) and

$$\alpha=\frac{n-2}{(n-2)^2-\varphi^2},\qquad \beta=-\frac{\varphi}{(n-2)^2-\varphi^2},\qquad \varphi=F^a_a.$$

§2. Proof of Theorem 1. — To eliminate a and b from (1.4), we multiply (1.4) by $\delta^r_{[l}\delta^s_t\delta^m_{k]}U^lF^t_aU^a$. We get

$$\delta^r_{[l}\delta^s_t\delta^m_{k]}S^k_{ij}U^iU^lF^j_aF^t_bU^aU^b=0$$

This condition being satisfied for every U^i, we have

$$\delta^r_{[l}\delta^s_t\delta^m_{k]}S^k_{ij}F^j_aF^t_b+(l,i,a,b)=0,$$

where the above expression means that it should sum up all the terms obtained by the cyclic permutation of indices l, i, a, b.

Multiplying this equation by $F^a_pF^b_q$, and then contracting with respect to r and l and with respect to s and q, we obtain

$$\begin{aligned} &[n(n+2)\omega-\varphi^2](\omega S^m_{ip}+S^m_{ab}F^a_pF^b_i)+2\varphi(\omega S^r_{ip}+S^r_{ab}F^a_pF^b_i)F^m_r \\ &+\delta^m_p[\omega(n+2)\Theta_aF^a_i+\omega\varphi\Theta_i-\omega^2(n+2)\psi_i-\omega\varphi\psi_aF^a_i] \\ &-\delta^m_i[\omega(n+2)\Theta_aF^a_p+\omega\varphi\Theta_p-\omega^2(n+2)\psi_p-\omega\varphi\psi_aF^a_p] \\ &-F^m_p[\omega(n+2)\Theta_i+\varphi\Theta_aF^a_i-\omega(n+2)\psi_aF^a_i-\omega\varphi\psi_i] \\ &+F^m_i[\omega(n+2)\Theta_p+\varphi\Theta_aF^a_p-\omega(n+2)\psi_aF^a_p-\omega\varphi\psi_p]=0, \end{aligned}$$

where

$$\varphi = F^a_a, \qquad S^r_{ir} = \psi_i, \qquad S^a_{ib} F^b_a = \Theta_i.$$

Putting

$$S_i = \omega\,(n+2)\,\Theta_a F^a_i + \omega\,\varphi\,\Theta_i - (n+2)\,\psi_i - \omega\,\varphi\,\psi_a F^a_i,$$

we reduce the preceding equation to the form:

$$\text{(2.1)}\qquad \begin{aligned}[n\,(n+2)\,\omega - \varphi^2]\,(\omega S^m_{ip} + S^m_{ab} F^a_p F^b_i) + 2\varphi\,(\omega S^r_{ip} + S^r_{ab} F^a_p F^b_i) F^m_r = \\ = \delta^m_i S_p - \delta^m_p S_i - \omega S_a F^a_p F^m_i + \omega S_a F^a_i F^m_p.\end{aligned}$$

In complex spaces $\varphi = 0$. Also, in some locally product spaces, $\varphi = 0$. Namely, in every locally product space th. structure tensor F^i_j determines two globally complementary distributions. If the dimensions of the distributions are p and q, then $\varphi = p - q$. We mean here by the distribution of dimension p, p-dimensional plane in the n-dimensional tangent plane at each point of the manifold.

Let us consider firstly, the case $\varphi = 0$. The vector field S_i being arbitrary we can put S_i instead of $\bar{S}_i = -\frac{1}{2n\,(n+2)} S_i$. Taking into account the skew-symmetry of S^i_{jk}, we transcribe (2.1) in the following form:

$$\text{(2.2)}\qquad {}^*O^{ab}_{pi} S^m_{ab} = S_i \delta^m_p - S_p \delta^m_i + \omega S_a F^a_p F^m_i - \omega S_a F^a_i F^m_p,$$

where

$$\text{(2.3)}\qquad O^{ab}_{ij} = \frac{1}{2}\,(\delta^a_i \delta^b_j + \omega F^a_i F^b_j), \qquad {}^*O^{ab}_{ij} = \frac{1}{2}\,(\delta^a_i \delta^b_j - \omega F^a_i F^b_j).$$

The condition

$$O^{pi}_{qj}\,(S_i \delta^m_p - S_p \delta^m_i + \omega S_a F^a_p F^m_i - \omega S_a F^a_i F^m_p) = 0$$

being satisfied, we can apply the well known lemma ([2], p. 133). Thus, (2.2) permits a solution, and the general solution is given by (1.5).

Now suppose that $\varphi \neq 0$. This can be the case only if $\omega = 1$. Taking this into account, we put

$$\text{(2.4)}\qquad S^m_{ip} + S^m_{ab} F^a_p F^k_i = T^m_{ip}$$

$$\text{(2.5)}\qquad \delta^m_i S_p - \delta^m_p S_i - S_a F^a_p F^m_i + S_a F^a_i F^m_p = V^m_{ip}.$$

Then (2.1) obtains the form:

$$\text{(2.6)}\qquad [n\,(n+2) - \varphi^2]\; T^m_{ip} + 2\varphi T^r_{ip} F^m_r = V^m_{ip}.$$

Multiplying (2.6) by F^s_m, we get

$$\text{(2.7)}\qquad 2\varphi T^m_{ip} + [n\,(n+2) - \varphi^2]\; T^r_{ip} F^m_r = V^r_{ip} F^m_r.$$

Multiplying (2.6) by $[n(n+2)-\varphi^2]$ and (2.7) by 2φ, and subtracting the second from the first, we obtain

$$\{[n(n+2)-\varphi^2]^2-4\varphi^2\}\, T_{ip}^{m}=[n(n+2)-\varphi^2]\, V_{ip}^{m}-2\varphi\, V_{ip}^{a}F_{a}^{m}.$$

This can be written in the form:

$$T_{ip}^{m}=AV_{ip}^{m}+BV_{ip}^{a}F_{a}^{m}, \tag{2.8}$$

where

$$A=\frac{n(n+2)-\varphi^2}{[n(n+2)-\varphi^2]^2-4\varphi^2}, \qquad B=-\frac{2\varphi}{[n(n+2)-\varphi^2]^2-4\varphi^2}.$$

It can be easily verified that

$$[n(n+2)-\varphi^2]^2-4\varphi^2\neq 0.$$

The tensor V_{ip}^{m} having the form (2.5), we find

$$AV_{ip}^{m}+BV_{ip}^{a}F_{a}^{m}=(AS_p-BS_aF_p^a)\,\delta_i^m-(AS_i-BS_aF_i^a)\,\delta_p{}^m-$$

$$-(AS_aF_p^a-BS_p)\,F_i^m+(AS_aF_i^a-BS_i)\,F_p^m.$$

Consequently, putting

$$V_i=AS_i-BS_aF_i^a,$$

and taking into account (2.4) and the skew-symmetry of S_{ij}^{k}, we express (2.8) in the form:

$$*O_{ip}^{ba}S_{ba}^{m}=\frac{1}{2}\,(V_p\delta_i^m-V_i\delta_p^m-V_aF_p^aF_i^m+V_aF_i^aF_p^m),$$

i. e. in the form (2.2). This completes the proof of Theorem 1.

§ 3. Proof of Theorem 2. — Suppose that the Riemannian metric g_{ij} satisfying

$$g_{ts}F_j^tF_i^s=g_{ij}, \qquad \overset{\circ}{\nabla}_kF_j^i=0,$$

where $\overset{\circ}{\nabla}$ denotes the covariant differentiation with respect to the Christoffel symbols $\{{}^{i}_{jk}\}$, is given. Then M is, in the case $\omega=-1$, a Kähler space and, in the case $\omega=+1$, a locally decomposable Riemannian space.

Let us consider the connexion

$$\Gamma_{jk}^{i}=\{{}^{i}_{jk}\}+S_j\delta_i^k+P_jF_k^i. \tag{3.1}$$

If $P_i=-\omega S_aF_i^a$, the connexion (3.1) reduces to (1.6), i. e. to the holomorphically semi-symmetric connexion.

Let ∇ be the operator of the covariant derivative with respect to Γ_{jk}^{i}. Then $\nabla_kF_j^i=0$, i. e. (3.1), is an F-connexion.

The curvature tensor R^i_{rkj} of the connexion (3.1) can be expressed in the form:

$$\text{(3.2)}\quad R^i_{rkj}=K^i_{rkj}+(\overset{\circ}{\nabla}_k S_j-\overset{\circ}{\nabla}_j S_k)\,\delta^i_r+(\overset{\circ}{\nabla}_k P_j-\overset{\circ}{\nabla}_j P_k)\,F^i_r,$$

where K^i_{rkj} is the curvature tensor of the Riemannian connexion $\{^i_{jk}\}$.

On the other hand, from the Ricci indentity

$$\nabla_s\nabla_r F^i_j-\nabla_r\nabla_s F^i_j=F^a_j R^i_{asr}-F^i_a R^a_{jsr}-\nabla_a F^i_j\,(\Gamma^a_{sr}-\Gamma^a_{rs})$$

and the condition $\nabla_k F^i_j=0$, we get

$$F^a_j R^i_{asr}-F^i_a R^a_{jsr}=0$$

from which

$$R^i_{ksr}-\omega R^a_{bsr}F^i_a F^b_k=0.$$

Putting $R^a_{ksa}=R_{ks}$, we find

$$R_{ks}=\omega R^a_{bst}F^t_a F^b_k,\qquad R_{pq}F^p_k F^q_s=R^a_{kqt}F^t_a F^q_s.$$

and

$$\text{(3.3)}\qquad R_{ks}+\omega R_{pq}F^p_k F^q_s=\omega\,(R^a_{bst}F^t_a F^b_k+R^a_{kqt}F^t_a F^q_s).$$

Similarly, if $K^a_{ksa}=K_{ks}$,

$$\text{(3.4)}\qquad K_{ks}+\omega K_{pq}F^p_k F^q_s=\omega\,(K^a_{bst}F^t_a F^b_k+K^a_{kqr}F^r_a F^q_s).$$

On the other hand, we have from (3.2)

$$R^a_{bst}F^t_a F^b_k=K^a_{bst}F^t_a F^b_k+\omega\,(\overset{\circ}{\nabla}_s S_k-\overset{\circ}{\nabla}_k S_s)+\omega\,(\overset{\circ}{\nabla}_s P_a-\overset{\circ}{\nabla}_a P_s)\,F^a_k$$

and

$$R^a_{kqt}F^t_a F^q_s=K^a_{kqt}F^t_a F^q_s+(\overset{\circ}{\nabla}_q S_a-\overset{\circ}{\nabla}_a S_q)\,F^a_k F^q_s+\omega\,(\overset{\circ}{\nabla}_q P_k-\overset{\circ}{\nabla}_k P_q)\,F^q_s.$$

Adding these two equations and taking into account that $\overset{\circ}{\nabla}_k F^i_j=0$, both for the Kähler and for the locally decomposable Riemannian space, we get

$$\begin{aligned}R^a_{bst}F^t_a F^b_k+R^a_{kqt}F^q_s F^t_a&=K^a_{bst}F^t_a F^b_k+K^a_{kqt}F^t_a F^q_s\\&+\omega\,\overset{\circ}{\nabla}_s S_k-\omega\,\overset{\circ}{\nabla}_k S_s+\omega\,\overset{\circ}{\nabla}_s(P_a F^a_k)-\omega\,(\overset{\circ}{\nabla}_a P_s)\,F^a_k\\&+\overset{\circ}{\nabla}_q(S_a F^a_k)\,F^q_s-\overset{\circ}{\nabla}_a(S_q F^q_s)\,F^a_k+\omega\,(\overset{\circ}{\nabla}_a P_k)\,F^a_s-\omega\,\overset{\circ}{\nabla}_k(P_a F^a_s).\end{aligned}$$

In the following, we shall limit ourselves to the holomorphically semi-symmetric connexion (1.6). Then

$$P_i=-\omega S_a F^a_i,\qquad S_i=-P_a F^a_i,$$

and the preceding equation reduces to

$$R^a_{bst}F^t_aF^b_k+R^a_{kbt}F^b_sF^t_a=K^a_{bst}F^t_aF^b_k+K^a_{kqt}F^t_aF^q_s.$$

From this, substituting (3.3) and (3.4), we find

$$O^{ab}_{ij}K_{ab}=O^{ab}_{ij}R_{ab}. \tag{3.5}$$

On the other hand, for the connexion (1.6), (3.2) becomes

$$R^i_{rkj}=K^i_{rkj}+(\overset{\circ}{\nabla}_kS_j-\overset{\circ}{\nabla}_jS_k)\,\delta^i_r-\omega\,(F^a_j\overset{\circ}{\nabla}_kS_a-F^a_k\overset{\circ}{\nabla}_jS_a)\,F^i_r. \tag{3.6}$$

This can be written in the form

$$R^i_{rkj}=K^j_{rkj}+\overset{\circ}{\nabla}_kS_a\,(\delta^a_j\delta^i_r-\omega F^a_jF^i_r)-\overset{\circ}{\nabla}_jS_a\,(\delta^a_k\delta^i_r-\omega F^a_kF^i_r).$$

From this we find, by contraction with respect to i and j,

$$^*O^{ab}_{rk}\overset{\circ}{\nabla}_bS_a=\frac{1}{2}\,(K_{rk}-R_{rk}).$$

The condition (3.5) being satisfied, we can apply the lemma [2], p. 133. Thus the preceding equation permits a solution, and the general solution has the form

$$\overset{\circ}{\nabla}_kS_r=\frac{1}{2}\,(K_{rk}-R_{rk})+O^{ab}_{rk}\,W_{ab},$$

where W_{ab} is an arbitrary tensor.

Hence, we can put

$$\overset{\circ}{\nabla}_kS_r=\frac{1}{2}\,(K_{rk}-R_{rk}). \tag{3.7}$$

Substituting (3.7) into (3.6), we have

$$R^i_{rkj}+\frac{1}{2}\,(R_{kj}-R_{jk})\,\delta^i_r-\frac{1}{2}\,\omega\,(R_{ka}F^a_j-R_{ja}F^a_k)F^i_r= \tag{3.8}$$

$$=K^i_{rkj}-\frac{1}{2}\,\omega\,(K_{ka}F^a_j-K_{ja}F^a_k)\,F^i_r.$$

In the case of a Kähler space

$$K_{ij}+\omega K_{ab}F^a_iF^b_j=0,\quad \text{that is}\quad K_{aj}F^a_i=-K_{ia}F^a_j,$$

and (3.8) reduces, taking into account that K_{ij} is a symmetric tensor, to

$$R^i_{rkj}+\frac{1}{2}\,(R_{kj}-R_{jk})\,\delta^i_r+\frac{1}{2}\,(R_{ka}F^a_j-R_{ja}F^a_k)\,F^i_r=K^i_{rkj}+K_{ka}F^a_jF^i_r. \tag{3.9}$$

In the case of a locally decomposable Riemannian space, $\omega=+1$ and

$$K_{ij}-K_{ab}F_i^aF_j^b=0, \quad \text{that is} \quad K_{aj}F_i^a-K_{ia}F_j^a=0,$$

and (3.8) reduces to

$$R_{rkj}^i+\frac{1}{2}(R_{kj}-R_{jk})\,\delta_r^i-\frac{1}{2}(R_{ka}F_j^a-R_{ja}F_k^a)\,F_r^i=K_{rkj}^i. \tag{3.10}$$

(3.9) and (3.10) complete the proof of Theorem 2.

§4. Proof of Theorem 3. — In this section we shall consider the connexion (1.9). R_{rkj}^i is the curvature tensor of this connexion, K_{rkj}^i is the curvature tensor of the Riemannian connexion and $\overset{\circ}{\nabla}$ denotes hte covariant differentiation with respect to the Riemannian connexion. Then it is easy to see that

$$\begin{aligned} R_{rkj}^i=&K_{rkj}^i+\delta_r^i\overset{\circ}{\nabla}_kS_j+g_{jr}\overset{\circ}{\nabla}_kS^i-F_r^iF_j^a\overset{\circ}{\nabla}_kS_a+F_{jr}F_a^i\overset{\circ}{\nabla}_kS^a\\ &-\delta_i^r\overset{\circ}{\nabla}_jS_k-g_{kr}\overset{\circ}{\nabla}_jS^i+F_r^iF_k^a\overset{\circ}{\nabla}_jS_a-F_{kr}F_a^i\overset{\circ}{\nabla}_jS^a\\ &+g_{jr}S^iS_k-F_{rk}F_j^aS_aS^i-g_{kr}F_a^iF_j^bS^aS_b+F_{jr}F_a^iS^aS_k\\ &-g_{kr}S^iS_j+F_{jr}F_k^aS_aS^i+g_{jr}F_a^iF_k^bS^aS_b-F_{kr}F_a^iS^aS_j. \end{aligned}$$

If we put

$$\psi_{jk}=\overset{\circ}{\nabla}_kS_j+S_jS_k+F_j^aF_k^bS_aS_b, \qquad \psi_k^i=g^{ij}\psi_{jk},$$

the tensor R_{rkj}^i may be written in the form

$$\begin{aligned} R_{rkj}^i=&K_{rkj}^i+\delta_r^i\psi_{jk}-\delta_r^i\psi_{kj}-F_r^iF_j^a\psi_{ak}+F_r^iF_k^a\psi_{aj}\\ &+g_{jr}\psi_k^i-g_{kr}\psi_j^i+F_{jr}F_a^i\psi_k^a-F_{kr}F_a^i\psi_j^a. \end{aligned} \tag{4.1}$$

Contracting (4.1) with respect to i and j, we obtain

$$R_{rk}=K_{rk}+2\psi_{rk}-\psi_{kr}+F_k^aF_r^b\psi_{ab}-g_{kr}\psi_a^a-F_{kr}F_a^b\psi_b^a. \tag{4.2}$$

We now introduce the notations

$$R_{rk}g^{rk}=R, \qquad K_{rk}g^{rk}=K, \qquad R_{rk}F^{rk}=R^* \tag{4.3}$$

$$K_{rk}F^{rk}=K^*, \qquad F^{rk}=F_a^rg^{ak}, \qquad F_{rk}g^{rk}=F_a^a=\varphi=p-q$$

and suppose $p\geqslant 2$, $q\geqslant 2$.

Contracting (4.2) by g^{rk} and F^{rk}, we find that

$$\left\{\begin{aligned} \psi_a^a&=\frac{n-2}{\varphi^2-(n-2)^2}(R-K)-\frac{\varphi}{\varphi^2-(n-2)^2}(R^*-k^*),\\ F_b^a\psi_a^b&=-\frac{\varphi}{\varphi^2-(n-2)^2}(R-K)+\frac{n-2}{\varphi^2-(n-2)^2}(R^*-k^*). \end{aligned}\right. \tag{4.4}$$

We note that according to the supposition $p \geqslant 2$, $q \geqslant 2$, $\varphi^2-(n-2)^2 \neq 0$.

On the other hand, we have from (4.1)

$$R_{rk}+R_{ab}F_r^aF_k^b=K_{rk}+K_{ab}F_r^aF_k^b+2\psi_{ab}(\delta_r^a\delta_k^b+F_r^aF_k^b)-2g_{kr}\psi_a^a-2F_{kr}F_b^a\psi_a^b,$$

or

$$\psi_{ab}O_{rk}^{ab}=\frac{1}{2}(R_{ab}O_{rk}^{ab}-K_{ab}O_{rk}^{ab}+g_{kr}\psi_a^a+F_{kr}F_b^a\psi_a^b). \tag{4.5}$$

It is easy to see that

$$(R_{ab}O_{pq}^{ab}-K_{ab}O_{pq}^{ab}+g_{pq}\psi_a^a+F_{pq}F_b^a\psi_a^b)^*O_{rk}^{pq}=0.$$

Consequently, the linear equation (4.5) with the unknown tensor ψ_{rk} permits a solution

$$\psi_{rk}=\frac{1}{2}(R_{ab}O_{rk}^{ab}-K_{ab}O_{rk}^{ab}+g_{rk}\psi_a^a+F_{rk}F_b^a\psi_a^b).$$

Taking into account that for the locally decomposable Riemannian space $K_{ab}O_{rk}^{ab}=K_{rk}$, this solution reduces to

$$\psi_{rk}=\frac{1}{2}(R_{ab}O_{rk}^{ab}-K_{rk}+g_{rk}\psi_a^a+F_{rk}F_b^a\psi_a^b). \tag{4.6}$$

Substituting (4.6) and (4.4) into (4.1), we find (1.10).

REFERENCES

[1] E. Bartolotti — *Sulla geometria della varieta a connesione affine*, Ann. di Mat., 1930 (4), VIII, pp. 53—101

[2] K. Yano — *Differential geometry on complex and almost complex spaces*, Pergamon Press, 1965

[3] K. Yano and T. Imai — *On the semi-symmetric metric F-connexion*, Tensor, vol. 29, No 2, pp. 134—138

[4] M. Prvanović — *Product semi-symmetric connections of the locally decomposable Riemannian spaces*, Bull. Acad. Serbe Sci. Arts LXIV, Sci. math. 10 (1979), 17—27

Mileva Prvanović

HOLOMORFNO SEMI-SIMETRIČNE KONEKSIJE

Rezime

Holomorfno semi-simetrična koneksija mnogostrukosti je ona koneksija čiji tenzor torzije zadovoljava uslov (1.5). Do tog smo uslova došli tako što smo na prostore sa kompleksnom odnosno lokalnom produkt strukturom proširili geometrijsku interpretaciju semi-simetrične koneksije koju je dao E. Bartolloti [1]. Zatim smo posmatrali dve specijalne holomorfno semi-simetrične koneksije: (1.6) i (1.9). Pokazali smo da tenzori krivina tih koneksija zadovoljavaju uslove (1.7) i (1.8), odnosno (1.10).