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FIXED POINT THEOREMS FOR MULTIVALUED MAPPINGS IN RANDOM NORMED SPACES

Abstract In this paper we shall prove two fixed point theorems for multivalued mappings in random normed spaces. Theorem 1 is a generalization of the fixed point theorem from [1].

1. The notion of random normed space was introduced in [12] and in [1] and [5] some fixed point theorems in random normed spaces were proved. First, we shall give some definitions and notations.

Let Δ be the set of all distribution functions. A triplet (X, $\mathcal{F}$, t) of a real or complex linear space X, a mapping $\mathcal{F}: X \to \Delta$ and a T-norm $t \geqslant T_m$ ($T_m(u, v) = \max\{u+v-1, 0\}$, for every $u, v \in [0, 1]$), is called a *random normed space* iff it satisfies the following conditions in which F_p denotes the distribution function $\mathcal{F}(p)$:

(a) $F_p(0)=0$, for all p in X.

(b) $F_p=H$ iff $p=0$ X, were:

$$H(u)=\begin{cases} 1 & u>0 \\ 0 & u\leqslant 0 \end{cases}$$

(c) If λ is a nonzero scalar then:

$$F_{\lambda p}(u)=F_p\left(\frac{u}{|\lambda|}\right)$$

for all p in X and for all $u \in R$

(d) $F_{p+q}(u+v) \geqslant t(F_p(u), F_q(v))$, for all p, q in X and for all $u>0$, $v>0$.

The (ε, λ)-topology on X is defined to be a topology on X determined by the family of neighbourhoods:

$$\{U_v(\varepsilon, \lambda) \mid \varepsilon>0, \lambda \in (0, 1)\}$$

of each $v \in S$, where:

$$U_v(\varepsilon, \lambda)=\{u \mid F_{u-v}(\varepsilon)>1-\lambda\}$$

Let A be a subset of X. The function D_A on R defined by:

$$D_A(u)=\sup_{v<u} \inf_{p, q \in A} F_{p-q}(v)$$

is called the probabilistic diameter of A [2]. The set A is said to be a *probabilistic bounded* if [2]:

$$\sup_{u\in R} D_A(u)=1$$

Kuratowski's function for a probabilistic bounded subset A of X is the function α_A on R defined by:

$\alpha_A(u)=\sup\{\varepsilon>0$, there is a finite cover $\mathcal{A}$ of A such that $D_B(u)\geqslant\varepsilon$, for all $B\in\mathcal{A}\}$

The following are proved in [2]:

1. $\alpha_A\in\Delta$ for every probabilistic bounded subset A of X
2. If A is a probabilistic bounded subset of X then:

$$\alpha_A(u)\geqslant D_A(u), \qquad \text{for all } u\in R$$

3. If B is a probabilistic bounded subset of X and A is a nonempty subset of B then $\alpha_A(u)\geqslant\alpha_B(u)$, for all $u\in R$
4. If A and B are probabilistic bounded subsets of X then for all $u\in R$:

$$\alpha_{A\cup B}(u)=\min\{\alpha_A(u), \alpha_B(u)\}$$

5. If A is a probabilistic bounded subset of X then:

$$\alpha_{\overline{A}}(u)=\alpha_A(u), \text{ for all } u\in R$$

where $\overline{A}$ denotes the closure of A under the (ε, λ)-topology on X.

6. A probabilistic bounded subset A of X is a probabilistic precompact if and only if $\alpha_A=H$.
7. If $(X, \mathcal{F}, t)$ is a random normed space with T-norm $t=\min$ then:

$$\alpha_A=\alpha_{\overline{co}\,A}$$

If $K\subset X$, we shall denote by 2^K the family of all nonempty subsets C of K.

DEFINITION 1 [9] *Let X be a Hausdorff topological linear space and $M\subset X$. The set M is said to be admissible iff for every compact subset K of M and every neighbourhood V of zero in X there exists a continuous mapping $h:K\to M$ such that*:

(a) $\dim(\mathrm{span}(h(K))<\infty$

(b) $x-hx\in V$, *for every* $x\in K$

DEFINITION 2 [4] *A mapping $f:K\to 2^K$ $(K\subset X)$ has the almost continuous selection property iff for every neighbourhood V of zero in X there exists a continuous mapping $g_V:K\to K$ such that*:

$$g_V(x)\in(f(x)+V)\cap cof(K) \textit{ for every } x\in K$$

Now we shall prove a generalization of fixed point theorem from [1].

THEOREM 1 *Let* $(X, \mathcal{F}, t)$ *be a random normed space with continuous T-norm* t, A *be a probabilistic bounded, closed and convex subset of* X *and* $f: A \to 2^A$ *be a closed mapping with the almost continuous selection property on every convex and compact subset of* A *and such that the following condition is satisfied:*

For every $M \subset A$ *such that* $\alpha_{\overline{co}f(M)} \leqslant \alpha_M$ *we have that* M *is relatively compact and for every* $M \subset A$ *the relation:*

$$M = \overline{co}\, f(M)$$

implies that M *is admissible.*

Then there exists at least one element $x \in A$ *such that:*

$$x \in f(x)$$

Proof: First, we shall show that there exists a nonempty set $K \subset A$ such that $K \subset f(K)$. Let us remark that if t is continuous then X is, in the (ε, λ)-topology, a linear topological space.

Let p be an arbitrary element from A and let us define the sequence $\{p_n\}_{n \in N \cup \{0\}}$ in the following way:

$$p_0 = p, \quad p_n \in f(p_{n-1}), \quad \text{for every} \quad n \in N.$$

Now we shall prove that K is the set of all the limit points of the set A_p where:

(1) $$A_p \overset{\text{def.}}{=\!=} \{p_n \mid n \in N\}$$

Suppose that A_p is not a relatively compact set. Then we have:

$$\alpha_{\overline{co}f(A_p)} > \alpha_{A_p}$$

On the other hand, from (1) it follows that $A_p \subset \{p_1\} \cup f(A_p)$ and so:

$$\alpha_{A_p} \geqslant \min\{\alpha_{\{p_1\}}, \alpha_{f(A_p)}\} = \alpha_{f(A_p)} \geqslant \alpha_{\overline{co}f(A_p)}$$

which is a contradiction. So we have that the set $\overline{A}_p$ is compact and $K \neq \emptyset$. Let us prove that $K \subset f(K)$. Suppose that $q \in K$. Then there exists a sequence $\{q_n\}_{n \in N} \subset A_p$ such that:

(2) $$\lim_{n \to \infty} q_n = q$$

since X is a metrisable topological space. Let $q_n = p_{m_n}$, for every $n \in N$. Then $\{p_{m_n - 1}\}_{n \in N} \subset A_p$ and since the set A_p is relatively compact there exists a subsequence $\{n_k\}_{k \in N} \subset N$ such that:

(3) $$\lim_{k \to \infty} p_{m_{n_k} - 1} = q' \in K$$

Further from (2) it follows that $\lim\limits_{k \to \infty} p_{m_{n_k}} = q$ and using the fact that $p_{m_{n_k}} \in f(p_{m_{n_k} - 1})$, for every $k \in N$, we conclude that $q \in f(q')$ because the mapping f is closed. So we have that $K \subset f(K) \subset \overline{co}\, f(K)$. As in [4], let us define the family $\mathcal{C}$ in the following way: $\mathcal{C} = \{Q \mid Q \subseteq A, K \subseteq Q, Q = \overline{co}\, Q, f(Q) \subset Q\}$.

Then $A \in \mathcal{C}$ and so $\mathcal{C} \neq \emptyset$ and similarly as in [4], it follows that there exists $C \neq \emptyset$, $C \in \mathcal{C}$ such that $C = \overline{co}\, f(C)$. From this we conclude that C is a convex, compact and admissible subset of A.

Further, the mapping $f \mid C: C \to C$ has the almost continuous selection property. So for every neighbourhood V of zero in X there exists a continuous mapping $g_V: C \to C$ such that:

$$g_V(x) \in f(x) + V, \quad \text{for every} \quad x \in C$$

From Theorem 1 in [9], it follows that there exists $x_V \in C$ such that: $x_V = g_V(x_V)$ and so:

$$x_V \in f(x_V) + V \tag{4}$$

From (4), as in [4], it is easy to prove, since C is compact, that there exists $x \in C$ such that $x \in f(x)$.

In the following Corollary we shall use the notation:

$$\Phi_n(t, x) = \underbrace{t(t(\ldots t(t}_{\text{n-times}}(x, x), x), \ldots), x), \qquad n \in N, \qquad x \in [0,1]$$

COROLLARY 1 *Let $(X, \mathcal{F}, t)$ be a random normed space with T-norm t such that the family $\{\Phi_n(t, x)\}_{n \in N}$ is equicontinuous at the point $x=1$. Further, let A be a probabilistic bounded, closed and convex subset of X and $f: A \to 2^A$ be a closed mapping which has the almost continuous selection property on every convex and compact subset of A. If:*

For every $M \subset A: \alpha_{\overline{co} f(M)} \leqslant \alpha_M$ implies that M is compact then there exists at least one element $x \in A$ such that $x \in f(x)$.

Proof: In [6] it is proved that X is, in the (ε, λ) topology, a locally convex topological linear space, since the family $\{\Phi_n(t, x)\}_{n \in N}$ is equicontinuous at the point $x=1$. In a locally convex space every convex subset is admissible and so all the conditions of the Theorem are satisfied which completes the proof.

COROLLARY 2 *Let $(X, \mathcal{F}, \min)$ be a random normed space, A be a probabilistic bounded, closed and convex subset of X and $f: A \to 2^A$ be a closed mapping which has the almost continuous selection property. If:*

For every $M \subset A: \alpha_{f(M)} \leqslant \alpha_M$ implies that M is compact then there exists at least one element $x \in A$ such that: $x \in f(x)$.

Proof: If $t = \min$, then the family $\{\Phi_n(t, x)\}_{n \in N}$ is equicontinuous at the point $x=1$ since in this case:

$\Phi_n(t, x) = x$, for every $n \in N$ and every $x \in [0,1]$. Further, if $t = \min$ then $\alpha_{A'} = \alpha_{\overline{co} A'}$ for every $A' \subset A$ and so all the conditions of Corollary 1 are satisfied.

Let us denote by 2^A_c the family of all convex subsets of A.

COROLLARY 3 *Let $(X, \mathcal{F}, t)$ be a random normed space with T-norm t such that the family $\{\Phi_n(t, x)\}_{n \in N}$ is equicontinuous at the point $x=1$. Further, let A be a probabilistic bounded, closed and convex subset of X and $f: A \to 2^A_c$ be a lower semicontinuous, closed mapping such that the following condition is satisfied:*

For every $M \subset A$, $\alpha_{\overline{co} f(M)} \leqslant \alpha_M$ implies that $\overline{M}$ is compact. Then there exists at least one element $x \in A$ such that $x \in f(x)$.

Proof: In [11] it is proved that the mapping f has the almost continuous selection property on A and so all the conditions of Corollary 1 are satisfied.

Similarly, we can prove the following Corollary.

COROLLARY 4 *Let* $(X, \mathcal{F}, \min)$ *be a random normed space, A be a probabilistic bounded, closed and convex subset of X and* $f: A \to 2^A_c$ *be a lower semicontinuous, closed mapping such that the following condition is satisfied*:

For every $M \subset A$, $\alpha_{f(M)} \leqslant \alpha_M$ *implies that* $\overline{M}$ *is compact. Then there exists at least one element* $x \in A$ *such that* $x \in f(x)$.

Remark: In [8] is given a nontrivial example of T-norm t ($t \neq \min$) such that the family $\{\Phi_n(t, x)\}_{n \in N}$ is equicontinuous at the point $x=1$.

2. In [5] the following fixed point theorem is given.

THEOREM A *Let* $(S, \mathcal{F}, t)$ *be a complete random normed space with continuous T-norm t, A be a closed subset of S and* $H: A \to A$ *be a continuous mapping such that the following two conditions are satisfied:*

1. *For every* $x \in A$ *there exists* $n(x) \in N$ *such that for every* $y \in A$, $F_{H^{n(x)}x - H^{n(x)}y}(q\varepsilon) \geqslant F_{x-y}(\varepsilon)$, *for every* $\varepsilon > 0$ *where* $q \in (0,1)$.

2. *There exists* $x_0 \in A$ *such that* $\sup\limits_{\varepsilon} G_{x_0}(\varepsilon) = 1$, *where*:

$G_{x_0}(\varepsilon) = \inf\{F_{H^s x_0 - x_0}(\varepsilon) \mid s \in N\}$, *for every* $\varepsilon > 0$.

Then Fix $H = \{x\}$, *where* $x = \lim\limits_{n \to \infty} x_n$, $x_n = H^{n(x_{n-1})} x_{n-1}$, $n \in N$.

Let us denote by $\mathcal{R}(M)$ the family of all nonempty and convex subsets of M where M is a subset of a topological vector space.

THEOREM B [3] *Let E be a Hausdorff topological vector space, M be a nonempty, convex and compact subset of E,* $\Phi: M \to \mathcal{R}(M)$ *be an upper semicontinuous mapping such that for every* $y \in M$ *the set*:

$$\Phi^{-1}(y) = \{x \mid y \in \Phi(x)\}$$

is open. Then there exists at least one fixed point of the mapping Φ.

Now we shall prove a fixed point theorem for mapping $H + \Phi$ where H is a singlevalued and Φ is a multivalued mapping.

THEOREM 2 *Let* $(S, \mathcal{F}, t)$ *be a complete random normed space with continuous T-norm t, M be a nonempty, convex and compact subset of S, H be a linear mapping from S into S,* $\Phi: M \to \mathcal{R}(S)$ *be an upper semicontinuous mapping such that* $HM + \Phi M \subseteq M$ *and that the following conditions are satisfied*:

(i) *One of the following two conditions is satisfied:*

a) *For every* $x \in M$ *there exists* $n(x) \in N$ *such that for every* $y \in M$ *and every* $\varepsilon > 0$:

$$F_{H^{n(x)}(x) - H^{n(x)}y}(q\varepsilon) \geqslant F_{x-y}(\varepsilon), \text{ where } q \in (0,1).$$

b) *There exists* $n \in N$ *such that*:

$$F_{H^n x - H^n y}(\varepsilon) \geqslant F_{x-y}(\varepsilon), \text{ for every } x, y \in M \text{ and every } \varepsilon > 0.$$

(ii) *For every* $y \in S$ *is the set* $\Phi^{-1}(y)$ *open.*

Then there exists at least one fixed point of the mapping $H + \Phi$.

Proof: For every $y \in \overline{\Phi(M)}$, we shall define the mapping $G_y: M \to M$ in the following way:

$$G_y(x) = Hx + y, \quad \text{for every} \quad y \in \overline{\Phi(M)}$$

Since:

$$G_y^n u = H^n u + \sum_{k=0}^{n-1} H^k y, \text{ for every } n \in N,\ u \in M,\ y \in \overline{\Phi(M)}$$

we have that for every $x \in M$, every $y \in \overline{\Phi(M)}$ and $\varepsilon > 0$ (i) a) implies:

$$F_{G_y^{n(x)}(x) - G_y^{n(x)}(u)}(q\varepsilon) \geqslant F_{x-u}(\varepsilon), \text{ for every } u \in M$$

for every $(x_1, x_2, \varepsilon) \in M \times M \times (0, \infty)$. First, we shall suppose that $k \in (0,1)$. Since T-norm t is continuous, S is a Hausdorff topological linear space and so it follows that the compact set M is bounded in the (ε, λ)-topology. We shall show that for every $y \in \Phi(M)$ all the conditions of Theorem A are satisfied. Let V be an arbitrary neighbourhood of zero in S in the (ε, λ)-topology. Then there exists $\delta > 0$ such that $M \subseteq \delta V$. Let us suppose that:

$$V = \{x \mid F_x(\varepsilon) > 1 - \lambda\}$$

Then from $\frac{x}{\delta} \in V$, for every $x \in M$, it follows that:

$$F_{\frac{x}{\delta}}(\varepsilon) > 1 - \lambda, \quad \text{for every} \quad x \in M$$

and so:

$$F_x(\delta\varepsilon) > 1 - \lambda \quad \text{for every} \quad x \in M$$

From this we conclude that for every $x_0 \in M$:

$$\sup_{\varepsilon} G_{x_0}(\varepsilon) = 1, \quad \text{for every} \quad y \in \Phi(M)$$

where $G_{x_0, y}(\varepsilon) = \inf \{F_{G_y^s(x_0) - x_0}(\varepsilon) \mid s \in N\}$

every $y \in \overline{\Phi(M)}$ there exists one and only one element $Ry \in M$ such that:

$$Ry = HRy + y$$

Since the set M is compact it is easy to prove that the mapping $R: \overline{\Phi(M)} \to M$ is continuous, similarly as in [7]. Now we shall define the mapping $R^*: M \to 2^M$ in the following way:

$$R^* x = \bigcup_{y \in \Phi x} Ry, \quad \text{for every} \quad x \in M$$

The mapping R^* is obviously upper semicontinuous since Φ is upper semicontinuous and R is continuous. It remains to prove that $R^* x$ is a convex set, for every $x \in M$. Since the mapping H is affine, it follows that for every $x \in M$ and every $\alpha, \beta \geqslant 0$ such that $\alpha + \beta = 1$ we have:

$$(5) \qquad R(\alpha y_1 + \beta y_2) = \alpha R y_1 + \beta R y_2, \quad \text{for every} \quad y_1, y_2 \in \Phi x$$

Since Φ^* is convex, it is easy to show that $R^* x$ is convex using relation (5). Since the mapping R is a one to one mapping there exists $R^{-1}: R(\Phi(M)) \to \Phi(M)$. Further $(R^*)^{-1} y = \{x \mid y \in R^* x\}$ and we shall prove that:

$$(R^*)^{-1} y = \Phi^{-1}(R^{-1} y)$$

Suppose that $x \in (R^*)^{-1}y$. Then $y \in R^*x$ which means that:

$$y \in \bigcup_{z \in \Phi x} Rz$$

So there exists $z \in \Phi x$ such that $y = Rz$, $z \in \Phi x$. Then:

$$R^{-1}y = z \in \Phi x$$

and so:

$$x \in \Phi^{-1}(R^{-1}y)$$

Suppose now that $x \in \Phi^{-1}(R^{-1}y)$. Then $R^{-1}y \in \Phi x$ i. e. $R^{-1}y = z$, $z \in \Phi x$. This implies that $y = Rz$, $z \in \Phi x$ and so $y \in R^*x$, which means that $x \in (R^*)^{-1}y$. Since $\Phi^{-1}y$ is open for every $y \in S$, we conclude that $\mathrm{Fix}(R^*) \neq \emptyset$. Since $\mathrm{Fix}(R^*) \subseteq \mathrm{Fix}(H+\Phi)$, it follows that for $k \in (0,1)$ is $\mathrm{Fix}(H+\Phi) \neq \emptyset$. Now, suppose (i) b). For every $n \in N$ we shall define the mappings H_n and Φ_n in the following way:

$$H_n x = \lambda_n H x, \quad \Phi_n x = \lambda_n \Phi(x) + (1-\lambda_n) x_0, \quad \text{for every} \quad x \in M$$

where $\{\lambda_n\}_{n \in n} \subseteq (0,1)$ and $\lim_{n\to\infty} \lambda_n = 1$. Then for every $m \in N$ and every $x \in M$, $H_m^n x = \lambda_m^n H^n x$ and so:

$$F_{H_m^n x - H_m^n y}(\lambda_m^n \varepsilon) = F_{\lambda_m^n (H^n x - H^n y)}(\lambda_m^n \varepsilon) = F_{H^n x - H^n y}(\varepsilon) \geq F_{x-y}(\varepsilon)$$

Since:

$$H_n M + \Phi_n M = \lambda_n (HM + \Phi M) + (1-\lambda_n) x_0 \subset M$$

and:

$$\Phi_n^{-1} y = \{x \mid y \in \Phi_n x\} = \{x \mid y \in \lambda_n \Phi x + (1-\lambda_n) x_0\} =$$

$$= \left\{x \,\middle|\, \frac{y-(1-\lambda_n) x_0}{\lambda_n} \in \Phi x\right\} = \Phi^{-1}\left(\frac{y-(1-\lambda_n) x_0}{\lambda_n}\right)$$

there exists, for every $n \in N$, $x_n \in M$ such that $x_n \in H_n x_n + \Phi_n x_n$
This means that there exists $y_n \in \Phi x_n$ so that:

$$x_n = \lambda_n H x_n + \lambda_n y_n + (1-\lambda_n) x_0$$

Then we have:

$$\lim_{n\to\infty} x_n - Hx_n - y_n = \lim_{n\to\infty} (\lambda_n - 1) Hx_n + (\lambda_n - 1) y_n + (1-\lambda_n) x_0 =$$

$$= \lim_{n\to\infty} (\lambda_n - 1)(Hx_n + y_n) + (1-\lambda_n) x_0 = 0$$

since $Hx_n + y_n \subseteq M$ and is bounded. Since M is compact there exists a subsequence $\{n_k\}_{k \in N}$ such that:

$$\lim_{k\to\infty} Hx_{n_k} + y_{n_k} = y^*$$

and so $\lim_{k\to\infty} x_{n_k} = y^*$. Since $y_{n_k} \in \Phi x_{n_k}$ and:

$$\lim_{k\to\infty} y_{n_k} = y^* - Hy^*$$

we have that $y^* - Hy^* \in \Phi y^*$, which means that $y^* \in \mathrm{Fix}(\Phi + H)$ and the proof is complete.

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TEOREME O NEPOKRETNOJ TAČKI ZA VIŠEZNAČNA PRESLIKAVANJA U SLUČAJNIM NORMIRANIM PROSTORIMA

U ovom radu su dokazane sledeće dve teoreme.

TEOREMA 1 *Neka je* $(X, \mathcal{F}, t)$ *slučajan normirani prostor sa neprekidnom* T*-normom* t, A *verovatnosno ograničen, zatvoren i konveksan podskup od* X *i* $f: A \to 2^A$ *zatvoreno preslikavanje sa osobinom gotovo neprekidne selekcije nad svakim konveksnim i kompaktnim podskupom od* A *i tako da je sledeći uslov zadovoljen: Za svako* $M \subset A$ *tako da je* $\alpha_{\overline{co}\, f(M)} \leq \alpha_M$ *sledi da je* M *relativno kompaktan i za svako* $M \subset A$ *relacija* $M = \overline{co}\, f(M)$ *implicira da je* M *dopustiv. Tada postoji bar jedan elemenat* $x \in A$ *takav da je:*

$$x \in f(x)$$

TEOREMA 2 *Neka je* $(S, \mathcal{F}, t)$ *kompletan normirani prostor sa neprekidnom* T*-normom* t, M *je neprazan, konveksan i kompaktan podskup od* S, H *linearno preslikavanje* S *u* S, $\Phi: M \to \mathcal{R}(S)$ *od gore poluneprekidno preslikavanje tako da je* $H(M) + \Phi(M) \subseteq M$ *i zadovoljeni su sledeći uslovi:*

(i) *Jedan od sledeća dva uslova je zadovoljen:*

a) *Za svako* $x \in M$ *postoji* $n(x) \in N$ *tako da je za svako* $y \in M$ *i svako* $\varepsilon > 0$:

$$F_{H^{n(x)}x - H^{n(x)}y}(\varepsilon) \geqslant F_{x-y}(\varepsilon), \text{ gde je } q \in (0,1)$$

b) *Postoji* $n \in N$ *tako da je:*

$$F_{H^n x - H^n y}(\varepsilon) \geqslant F_{x-y}(\varepsilon) \text{ za svako } x, y \in M \text{ i svako } \varepsilon > 0$$

(ii) *Za svako* $y \in S$ *skup* $\Phi^{-1}(y)$ *je otvoren.*

Tada postoji bar jedna nepokretna tačka preslikavanja $H + \Phi$.