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ON THE CONSTRUCTION OF ALL MATROIDS ON 7 ELEMENTS AT MOST

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Abstract. This paper describes briefly the methods of construction and gives the list of all non-isomorphic matroids whose carriers have 7 elements at most. A way for the verification of these results is pointed to also.

1. Definitions and denotations

For the definitions of matroids, flats, geometric lattices, hyperplanes, rank and dual matroids we refer the reader to [7] or to any introductory text on matroids.

An *n-set* is a set which has *n* elements.

The flats of rank 0,1 and *n* of *a* matroid of rank *n* are its *null, atoms* and *unit* respectively.

A geometric lattice which has at least one atom is *simple* if all its atoms are 1-sets, otherwise it is *non-simple*. The corresponding matroids are named in the same way.

We denote geometric lattice for short by *GL* or *G-lattice* and simple geometric lattice by *SGL* or *SG-lattice*.

A *semisimple G*-lattice is a non-simple *G*-lattice which has the empty null.

The addition of a new element z *to a flat* X of a matroid is the replacement of all flats *Y* which contain *X* by $Y\cup z$.

The dualization of a *G*-lattice is the construction of the *GL* of the dual matroid.

The *carrier* of a matroid (*G*-lattice) is the set on which it is defined.

2. The construction of simple matroids on 7 elements at most

The results of this section are just a small part of the results obtained in [3], but we could not use that approach and had to develop our own methods, which are rather different.

By $s(n, r)$ we denote the number of (non-isomorphic) simple matroids of rank r on an n-set.

The following results are easily deduced:

a) $s(n, r)=0$ for $n<r$

The rank of a matroid can never exceed the cardinality of its carrier.

b) $s(n, 0)=0$ for $n \geqslant 1$ and $s(n, 1)=0$ for $n \geqslant 2$

The *G*-lattices have the non-empty null in the first and the only atom with more than one element in the second case.

c) $s(n, 2)=1$ for $n \geqslant 2$

The correspoding *SG*-lattices have the empty null, n 1-atoms and the n-unit.

d) $s(n, n-1)=n-2$ for $n \geqslant 3$

It could be easily proved that these *SG*-lattices have exactly one flat which has (at least) two elements more than all flats which are its subsets. The rank of that flat may be between 2 and n.

e) $s(n, n)=1$ for $n \geqslant 0$

The *G*-lattices of these matroids are the lattices of power sets of their carriers.

All the above mentioned *G*-lattices are quite easy to construct. We shall now construct just the matroids counted by $s(5,3)$, $s(6,3)$, $s(7,3)$, $s(6,4)$, $s(7,4)$ and $s(7,5)$.

In order to deal with the *G*-lattices of rank 3, we have rather easily proved the following theorem:

The family F of at least two subsets of an n-set S is a family of flats of rank 2 of a *SG*-lattice of rank 3 iff there are at least two sets in F and each of $\binom{n}{2}$ 2-subsets of S is contained in exactly one subset from F.

The first condition is introduced only to avoid the case when there is just one flat of rank 2-the unit. The second condition can be explained by semimodularity and by the fact that any intersection of flats is again a flat ([7]).

Using this theorem, we adjoin a graph to each simple matroid of rank 3, so that the flats of rank 2, which have more than two elements, are mapped onto the vertices of the graph and two vertices are adjacent iff the corresponding flats have a non-empty intersection (of cardinality 1).

Exploring in detail the traits of these graphs, we have found that the values of $s(5,3)$, $s(6,3)$ and $s(7,3)$ are 4,9 and 23 respectively and learned the corresponding simple matroids. We point out that the isomorphic graphs do not always produce the isomorphic *SG*-lattices of rank 3. For example, a „triangle", whose all three of edges correspond to the same intersection element, produces a different *SG*-lattice from the one obtained by the triangle, whose edges correspond to different intersection elements.

The remaining SG-lattices of rank 4 and 5 have been constructed by exploring the possibilities for the addition of „new" families of hyperplanes between the „old" hyperplanes and the unit of SG-lattices of lower rank (that is, for the erecting of these matroids), in each single case. This problem was treated more fully by Crapo [4], but we have developed our own methods.

In a few words, we added each of the remaining elements to the old hyperplanes and made unions of the obtained sets, whenever the law of semimodularity (satisfied by G-lattices, [7]) so required. After that we investigated the possibilities for further incorporation of new hyperplanes (and construction of new simple matroids in this way).

In each case, the reason for the possible abandoning a possibility was the appearance of „forbidden" intersections of new hyperplanes (these intersections are different from all flats of a lower rank and they contradict the fact that any intersection of flats is a flat).

3. The construction of non-simple matroids on 7 elements at most

The leading idea is to look for the non-isomorphic possibilities for addition of new elements to nulls and atoms of simple geometric lattices (we use the fact that all flats of higher ranks are unions of atoms).

It is rather simple to count the non-isomorphic possibilities for the addition of new elements to the SG-lattices of rank 2. Let L be such a lattice having k atoms and let $L(t)$ denote the number of non-isomorphic geometric lattices obtained from L by the addition of t new elements. There is a bijection between the G-lattices, counted by $L(t)$ and having a non-empty null, and the G-lattices counted by $L(t-1)$. Namely, the lattices of the first class can be obtained by adding one new, always the same, element to all flats of the lattices of the second class.

The atoms of the geometric lattices, counted by $L(t)$ and having the empty null, have $k+t$ elements altogether (each of t new elements must be contained in an atom) and each two of these atoms have the empty intersection. So we can easily derive that

$$L(t)=L(t-1)+p(k+t, k) \tag{1}$$

where $p(k+t, k)$ is the number of partitions (without regard to order) of the natural number $k+t$ into a sum of k natural addends.

As $L(0)=1=p(k, k)$, we have that

$$L(t)=\sum_{j=0}^{t} p(k+j, k)=\sum_{j=k}^{k+t} p(j, k) \tag{2}$$

For each $k \geqslant 2$, there exists exactly one SG-lattice of rank 2 having k atoms. A G-lattice of rank 2 having n atoms can be obtained from a SG-lattice of rank 2 having k atoms, where $2 \leqslant k \leqslant n$.

Using this and (2) we obtain

$$m_2(n)=\sum_{k=2}^{n}\sum_{j=k}^{n}p(j,k) \tag{3}$$

where $m_2(n)$ denotes the number of non-isomorphic matroids of rank 2 on n elements.

The evaluation of $p(j,k)$ is rather difficult, although a generative function for it is given in [6]. Another formula for $m_2(n)$, much more convenient for evaluation, is given in [1].

The addition of new elements is by far more complicated in the case of SG-lattices of a higher rank. For example, if the flats of rank 2 of a SG-lattice of rank 3 are $\{a, b, c\}$, $\{a, d\}$, $\{b, d\}$ and $\{c, d\}$, then it is obvious that the atom $\{d\}$ has a special position. Adding one new element, e, to the atoms $\{a\}$, $\{b\}$ and $\{c\}$, we obtain the geometric lattices of three isomorphic matroids, but adding e to $\{d\}$ we obtain a new matroid. Therefore we define a „weak" partition among the atoms of a geometric lattice:

$x\sim y\longleftrightarrow$ the matroids whose geometric lattices arise by the addition of one new element to the atoms x and y are isomorphic

The classes of the weak partition in the example above are $\{a, b, c\}$ and $\{d\}$.

We assert that it is sufficient „to keep in mind" this partition, in order to construct all geometric lattices on at most 7 elements, with one single exception. Here is the exception:

We examine the SG-lattice of rank 3, whose flats of rank 2 are $\langle a, b, c\rangle$, $\{a, d, e\}$, $\{b, d\}$, $\{b, e\}$, $\{c, d\}$ and $\{c, e\}$. The classes of the weak partition are $\{a, b, c, d\}$ and $\{e\}$.

Keeping in mind this partition, we arrive at four non-isomorphic possibilities for the addition of two new elements, f and g, directly to the atoms (we write down just the corresponding families of atoms):

1. $\{a, f, g\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{e\}$
2. $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{e, f, g\}$
3. $\{a, f\}$, $\{b, g\}$, $\{c\}$, $\{d\}$, $\{e\}$
4. $\{a, f\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{e, g\}$

A thorough inspection, however, shows that in this way we omit the matroid with the isomorphic G-lattice and the following family of atoms:

5. $\{a, f\}$, $\{b\}$, $\{c, g\}$, $\{d\}$, $\{e\}$

It is because of this example that we introduce another, „strong", partition among the atoms of a SG-lattice on S:

$$x\approx y\longleftrightarrow(\{z_1, \ldots, z_n, x\} \text{ is flat}\longleftrightarrow\{z_1, \ldots, z_n, y\} \text{ is flat}$$
$$\text{for each } \{z_1, \ldots, z_n\}\subseteq S/\{x, y\})$$

It is easy to prove that adding new elements to the atoms in all possible ways, according to the strong partition, we cannot omit a matroid. Namely, two atoms in the same class of the strong partition can be transposed, the matroid remaining the same, and these transpositions can be iterated. Unfortunately, the strong partition is too crude and, using it when adding new elements to the atoms, we often obtain the same matroid several times.

It is routine to show that the classes of the weak and the strong partition coincide with all *SG*-lattices on at most 7 elements, excluding the mentioned exception. This completes the proof of the assertion above.

It would probably be very hard to generalize this theory and use it for the construction of matroids on 8 (or more) elements. In such cases, we would have to consider several different partitions at once. What is more, there are ([3]) 950 *SG*-lattices on 8 elements and we would have to use a computer.

We have obtained the following table of the values of $m_r(n)$, the number of non-isomorphic matroids of rank r an n-set, $0 \leqslant r, n \leqslant 7$;

$r \backslash n$	0	1	2	3	4	5	6	7
0	1	1	1	1	1	1	1	1
1		1	2	3	4	5	6	7
2			1	3	7	13	23	37
3				1	4	13	38	108
4					1	5	23	108
5						1	6	37
6							1	7
7								1

We immediately read from this table the values of $m(n)$, the number of non--isomorphic matroids on an n-set, for $n \leqslant 7$:

n	0	1	2	3	4	5	6	7
$m(n)$	1	2	4	8	17	38	98	306

4. The dualization of geometric lattices

The table of values of $m_r(n)$ was obtained without the use of duality and so its symmetry ($m_{n-r}(n) = m_r(n)$) strongly confirms the obtained results. A still better confirmation is reached by identifying the pairs of dual matroids (although we cannot avoid leaving a very small possibility that two dual mistakes are made). As the matroids in our investigation were given in terms of geometric lattices, we had to develop methods for the construction of geometric lattices of dual matroids (i.e., for the *dualization* of geometric lattices).

We have developed nine variants of the dualization algorithms. Eight of them are based on the close connection (complementarity) of the bases of mutually dual matroids, while the ninth is based on the direct connection (described in [2]) between mutually dual geometric lattices. We shall give a brief description of the first eight algorithms.

Up to the end of this section we shall consider that the dual geometric lattice (which is to be constructed) is of rank r on an n-set S. The family of flats of rank k of that lattice is denoted by F_k. By G/i is denoted the family of all intersections of i subsets of a family G. The algorithms are denoted by *Alg* (accompanied by a capital letter). By S_k is denoted the family of all k-subsets of S.

The first four algorithms have three main steps, which are shown by the following scheme:

$$\begin{array}{ccc} \mathcal{B} & \xrightarrow{\quad\quad} & \mathcal{B}^* \\ 1.\uparrow & 2. & \downarrow 3. \\ GL & & (GL)^* \end{array}$$

By $\mathcal{B}$ is denoted the family of bases, while the asterisk is used for the denotation of the dual objects.

The first two steps are trivial and common for these algorithms:

1. step: The bases from $\mathcal{B}$ are exactly those $(n-r)-$subsets of S which are not contained in a hyperplane of the first GL.

2. step: $X \in \mathcal{B} \Leftrightarrow S \setminus X \in \mathcal{B}^*$

It is because of the third step that we differentiate four variants of the dualization algorithm with the given scheme.

Alg A: The family F_{r-1} is composed of all maximal supersets, which do not contain any base, of all $(r-1)$-subsets of S. Having F_{r-1}, we construct the families F_{r-1}/i for $2 \leq i \leq r$. Then the family F_k is composed of all different maximal sets of the family F_{r-1}/k, which are not the sets of a family F_i, where $i > k$.

Alg B: The family F_{r-1} is constructed in the same way as with Alg A. The family F_k is composed of all different maximal sets of the family $F_{k+1}/2$ for each k, $r-2 \geq i \geq 0$.

Alg C: Let E_k denote the family of all k-subsets of S, which are not contained in a base. The family F_k is composed of all maximal subsets of S, such that all their $(k+1)$-subsets belong to E_{k+1}, and of all k-subsets of S which are not included in any of these maximal subsets, for each k, $0 \leq k \leq r-1$.

Alg D: F_0 /the null/ is the set of those elements of S, which are not contained in any base. The family F_k is composed of all different maximal subsets of S, subject to the condition that no flat of F_k contains a $(k+1)$-subset of a base and constructed by addition of new elements of the flats of the family F_{k-1}, for each k, $1 \leq k \leq r$.

The next four variants of the dualization algorithm are more convenient in the case when the majority of $(n-r)$-subsets of S are the bases from $\mathcal{B}$. These algorithms also have three steps, shown by the following scheme:

$$\begin{array}{ccc} S_{n-r} \setminus \mathcal{B} & \xrightarrow{\quad\quad} & S_r \setminus \mathcal{B}^* \\ 1.\uparrow & 2. & \downarrow 3. \\ GL & & (GL)^* \end{array}$$

The first two steps are again trivial and common for these four algorithms:

1. step: The sets of $S_{n-r} \setminus B$ are exactly all $(n-r)$-subsets of hyperplanes.

2. step: $X \in S_{n-r} \setminus \mathcal{B} \Leftrightarrow S \setminus X \in S_r \setminus \mathcal{B}^*$

3. step: Each of the algorithmas A, B, C and D can be, after a slight modification, turned to the corresponding variant having the second scheme. These variants are denoted by adding an index 1.

Alg A_1 and Alg B_1: The family F_{r-1} is composed of

a) all $(r-1)$-subsets of S which are not contained in a set from $S_r \setminus \mathcal{B}^*$

b) all maximal subsets of S such that all their r-subsets belong to $S_r \setminus \mathcal{B}^*$

The rest is the same as with Alg A and Alg B.

Alg C_1: The family E_k is composed of all k-subsets of S, which are contained in $\binom{n-k}{r-k}$ sets of $S_r \setminus \mathcal{B}^*$. The rest is the same as with Alg C.

Alg D_1: The null is the maximal subset of S, such that each its element is contained in $\binom{n-1}{r-1}$ sets of $S_r \setminus \mathcal{B}^*$. The condition in Alg D „no flat of F_k contains a $(k+1)$-subset of a base" is replaced by „all $(k+1)$-subsets of those flats of F_k, which have more than k elements, are contained in $\binom{n-k-1}{r-k-1}$ sets of $S_r \setminus \mathcal{B}^*$". All the rest remains the same as with Alg D.

5. On the list of all matroids on 7 elements at most

We adopt the convention to denote sets of numbers /subsets of $\{1, 2, 3, 4, 5, 6, 7\}$/ without brackets and commas.

We give primarily two definitions. The first of them is equivalent to those given in [2] and [5].

A flat X of rank r is *essential* if it is the null or has at least two elements more than all its subsets which are flats of rank $r-1$.

An essential flat of rank r is a *real essential flat* /shortly-*RE*-flat/ if it is non--empty, different from the unit, and cannot be „predicted" using the flats of rank $r-1$ (that is, its existence cannot be guaranteed by semimodularity applied on the flats of lower ranks)*.

The essential flats together with their ranks, as it is shown in [5], completely determine the matroid of given rank. It is easy to show that the family of essential flats can be reduced to the family of real essential flats, the matroid of given rank and cardinality of carrier still being completely determined.

The matroids in our list are given by means of the real essential flats, the rank and the cardinality of the carrier.

A simple /non-simple/ matroid of rank r an n-set is denoted by $\{n, r, t\}$ /(n, r, t)/. The number t denotes the order of the matroid among the matroids of the same rank and cardinality of carrier. This order is subject to change, but simple matroids are always listed in front of the non-simple matroids of the same rank and the cardinality of carrier.

* More precisely, if a flat has to be essential due to a semimodularity, then it is a *RE*-flat only if it is not a minimal set that satisfies the semimodularity.

If the family of *RE*-flats is empty, we write just a short line instead of it. If there are essential flats, different from the empty set and the carrier, which are, not *RE*-flats, then we write three points after the family of *RE*-flats.

The *RE*-flats of a matroid are listed in order according to increasing ranks. The ranks are denoted in brackets after all *RE*-flats of the same rank.

The designation of the considered matroid in each row is in front of the family of *RE*-flats, while the designation of the dual matroid is behind that family.

Example: Let us look at the row

(7, 3, 90) 27/1/, 1237, 145/2/ ⟨7, 4, 25⟩

We start from $F_0=\emptyset$. Each family F_k is obtained by addition of one new element from $1234567/X$ to all X from F_{k-1}, except in the case of the unit and the essential flats. In this way we quickly obtain that the geometric lattice is

1234567
1237 145 247 257 267 16 34 35 36 46 56
1 27 3 4 5 6
∅

Dualizing this *G*-lattice we obtain the *G*-lattice, isomorphic /but, in general, not necessarily the same/ with the *G*-lattice of ⟨7, 4, 25⟩.

Another example: Let us look at the matroid (7, 3, 40). The semimodularity, applied to the flats 156 and 27 guarantees the existence of the non-real essential flat 12567 or its superset of rank 2. As the list of *RE*-flats does not contain a superset of 12567, so 12567 itself is a non-real essential flat.

THE LIST OF ALL MATROIDS ON 7 ELEMENTS AT MOST

Matroid	Real essential flats	Dual matroid
[0,0,1]	—	[0,0,1]
(1,0,1)	—	⟨1,1,1∼⟩
⟨1,1,1⟩	—	(1,0,1)
(2,0,1)	—	⟨2,2,1⟩
(2,1,1)	2/0/	(2,1,1∼)
(2,1,2)	—	(2,1,2)
⟨2,2,1⟩	—	(2,0,1)
(3,0,1)	—	⟨3,3,1⟩
(3,1,1)	23/0/	(3,2,2)
(3,1,2)	3/0/	(3,2,3)
(3,1,3)	—	⟨3,2,1⟩
⟨3,2,1⟩	—	(3,1,3)

Matroid	Real essential flats	Dual matroid
(3,2,2)	3/0/	(3,1,1)
(3,2,3)	13/1/	(3,1,2)
⟨3,3,1⟩	—	(3,0,1)
(4,0,1)	—	⟨4,4,1⟩
(4,1,1)	234/0/	(4,3,3)
(4,1,2)	34/0/	(4,3,4)
(4,1,3)	4/0/	⟨4,3,2⟩
(4,1,4)	—	⟨4,3,1⟩
⟨4,2,1⟩	—	⟨4,2,1⟩
(4,2,2)	34/0/	(4,2,2)
(4,2,3)	4/0/,134/1/	(4,2,3)
(4,2,4)	134/1/	(4,2,6)
(4,2,5)	13,24/1/	(4,2,5)
(4,2,6)	4/0/	(4,2,4)
(4,2,7)	14/1/	(4,2,7)
⟨4,3,1⟩	—	(4,1,4)
⟨4,3,2⟩	123/2/	(4,1,3)
(4,3,3)	4/0/	(4,1,1)
(4,3,4)	14/1/	(4,1,2)
⟨4,4,1⟩	—	(4,0,1)
(5,0,1)	—	⟨5,5,1⟩
(5,1,1)	2345/0/	(5,4,4)
(5,1,2)	345/0/	(5,4,5)
(5,1,3)	45/0/	⟨5,4,3⟩
(5,1,4)	5/0/	⟨5,4,2⟩
(5,1,5)	—	⟨5,4,1⟩
⟨5,2,1⟩	—	(5,3,1)
(5,2,2)	345/0/	(5,3,5)
(5,2,3)	45/0/,1345/1/	(5,3,6)
(5,2,4)	5/0/,1345/1/	(5,3,7)
(5,2,5)	5/0/,135,245/1/	(5,3,8)
(5,2,6)	1345/1/	(5,3,11)
(5,2,7)	134,25/1/	(5,3,12)
(5,2,8)	45/0/	(5,3,13)
(5,2,9)	5/0/,145/1/	(5,3,9)
(5,2,10)	145/1/	(5,3,10)
(5,2,11)	14, 25/1/	⟨5,3,4⟩
(5,2,12)	5/0/	⟨5,3,3⟩
(5,2,13)	15/1/	⟨5,3,2⟩
⟨5,3,1⟩	—	⟨5,2,1⟩
⟨5,3,2⟩	123/2/	(5,2,13)
⟨5,3,3⟩	125, 345/2/	(5,2,12)
⟨5,3,4⟩	1234/2/	(5,2,11)
(5,3,5)	45/0/	(5,2,2)

Matroid	Real essential flats	Dual matroid
(5,3,6)	5/0/, 145/1/	(5,2,3)
(5,3,7)	145/0/	(5,2,4)
(5,3,8)	14, 25/1/, . . .	(5,2,5)
(5,3,9)	5/0/	(5,2,9)
(5,3,10)	15/1/	(5,2,10)
(5,3,11)	5/0/, 1235/2/	(5,2,6)
(5,3,12)	15/1/, 1235/2/	(5,2,7)
(5,3,13)	45/1/, 123/2/	(5,2,8)
⟨5,4,1⟩	—	(5,1,5)
⟨5,4,2⟩	1234/3/	(5,1,4)
⟨5,4,3⟩	123/2/	(5,1,3)
(5,4,4)	5/0/	(5,1,1)
(5,4,5)	12/1/	(5,1,2)
⟨5,5,1⟩	—	(5,0,1)
(6,0,1)	—	⟨6,6,1⟩
(6,1,1)	23456/0/	(6,5,5)
(6,1,2)	3456/0/	(6,5,6)
(6,1,3)	456/0/	⟨6,5,4⟩
(6,1,4)	56/0/	⟨6,5,3⟩
(6,1,5)	6/0/	⟨6,5,2⟩
(6,1,6)	—	⟨6,5,1⟩
⟨6,2,1⟩	—	⟨6,4,1⟩
(6,2,2)	3456/0/	(6,4,12)
(6,2,3)	456/0/, 13456/1/	(6,4,13)
(6,2,4)	56/0/, 13456/1/	(6,4,21)
(6,2,5)	56/0/, 1356, 2456/1/	(6,4,15)
(6,2,6)	6/0/, 13456/1/	(6,4,18)
(6,2,7)	6/0/, 1346/, 256/1/	(6,4,23)
(6,2,8)	13456/1/	(6,4,16)
(6,2,9)	1345, 26/1/	(6,4,20)
(6,2,10)	134, 256/1/	⟨6,4,7⟩
(6,2,11)	465/0/	(6,4,14)
(6,2,12)	56/0/, 1456/1/	(6,4,22)
(6,2,13)	6/0/, 1456/1/	(6,4,19)
(6,2,14)	6/0/, 146, 256/1/	⟨6,4,10⟩
(6,2,15)	1456/1/	(6,4,17)
(6,2,16)	145, 26/1/	⟨6,4,6⟩
(6,2,17)	14, 25, 36/1/	⟨6,4,5⟩
(6,2,18)	56/0/	⟨6,4,11⟩
(6,2,19)	6/0/, 156/1/	⟨6,4,9⟩
(6,2,20)	156/1/	⟨6,4,4⟩
(6,2,21)	15, 26/1/	⟨6,4,3⟩
(6,2,22)	6/0/	⟨6,4,8⟩
(6,2,23)	16/0/	⟨6,4,2⟩

Matroid	Real essential flats	Dual matroid
⟨6,3,1⟩	—	⟨6,3,1⟩
⟨6,3,2⟩	123/2/	⟨6,3,2⟩
⟨6,3,3⟩	123, 145/2/	⟨6,3,3⟩
⟨6,3,4⟩	1234/2/	(6,3,29)
⟨6,3,5⟩	123, 456/2/	⟨6,3,5⟩
⟨6,3,6⟩	124, 135, 236/2/	⟨6,3,6⟩
⟨6,3,7⟩	123, 145, 246, 356/2/	⟨6,3,7⟩
⟨6,3,8⟩	123, 1456/2/	(6,3,32)
⟨6,3,9⟩	12345/2/	(6,3,28)
(6,3,10)	456/0/	(6,3,10)
(6,3,11)	56/0/, 1456/1/	(6,3,11)
(6,3,12)	6/0/, 1456/1/	(6,3,21)
(6,3,13)	6/0/, 146, 256/1/, . . .	(6,3,13)
(6,3,14)	1456/1/	(6,3,17)
(6,3,15)	145, 26/1/, . . .	(6,3,23)
(6,3,16)	14, 25, 36/1/, . . .	(6,3,16)
(6,3,17)	56/0/	(6,3,14)
(6,3,18)	6/0/, 156/1/	(6,3,24)
(6,3,19)	156/1/	(6,3,19)
(6,3,20)	15, 26/1/, . . .	(6,3,35)
(6,3,21)	56/0/, 12356/2/	(6,3,12)
(6,3,22)	6/0/, 156/1/, 12356/2/	(6,3,22)
(6,3,23)	6/0/, 456/1/, 1236/2/	(6,3,15)
(6,3,24)	156/1/, 12356/2/	(6,3,18)
(6,3,25)	456/1/, 123/2/	(6,3,25)
(6,3,26)	15, 26/1/, 12356/2/	(6,3,33)
(6,3,27)	15, 46/1/, 1235/2/, . . .	(6,3,27)
(6,3,28)	6/0/	⟨6,3,9⟩
(6,3,29)	16/1/	⟨6,3,4⟩
(6,3,30)	6/0/, 1236/2/	(6,3,37)
(6,3,31)	16/1/, 1236/2/	(6,3,31)
(6,3,32)	46/1/, 123/2/	⟨5,3,8⟩
(6,3,33)	6/0/, 1256, 3456/2/	(6,3,26)
(6,3,34)	16/1/, 1256, 345/2/	(6,3,34)
(6,3,35)	56/1/, 1256, 3456/2/	(6,3,20)
(6,3,36)	6/0/, 12346/2/	(6,3,36)
(6,3,37)	16/1/, 12346/2/	(6,3,30)
(6,3,38)	56/1/, 1234/2/	(6,3,38)
⟨6,4,1⟩	—	⟨6,2,1⟩
⟨6,4,2⟩	1234/3/	(6,2,23)
⟨6,4,3⟩	1234, 1256/3/	(6,2,21)
⟨6,4,4⟩	123/2/	(6,2,20)
⟨6,4,5⟩	1234, 1256, 3456/3/	(6,2,17)
⟨6,4,6⟩	123/2/, 1456/3/	(6,2,16)

Matroid	Real essential flats	Dual matroid
⟨6,4,7⟩	123, 456/2/	(6,2,10)
⟨6,4,8⟩	12345/3/	(6,2,22)
⟨6,4,9⟩	123/2/, 12345/3/	(6,2,19)
⟨6,4,10⟩	123, 145/2/, . . .	(6,2,14)
⟨6,4,11⟩	1234/2/	(6,2,18)
(6,4,12)	56/0/	(6,2,2)
(6,4,13)	6/0/, 156/1/	(6,2,3)
(6,4,14)	156/1/	(6,2,11)
(6,4,15)	15, 26/1/, . . .	(6,2,5)
(6,4,16)	6/0/	(6,2,8)
(6,4,17)	16/1/	(6,2,15)
(6,4,18)	6/0/, 12346/3/	(6,2,6)
(6,4,19)	16/1/, 12346/3/	(6,2,13)
(6,4,20)	56/1/, 1234/3/	(6,2,9)
(6,4,21)	6/0/, 1236/2/	(6,2,4)
(6,4,22)	16/1/, 1236/2/	(6,2,12)
(6,4,23)	46/1/, 123/2/	(6,2,7)
⟨6,5,1⟩	—	(6,1,6)
⟨6,5,2⟩	12345/4/	(6,1,5)
⟨6,5,3⟩	1234/3/	(6,1,4)
⟨6,5,4⟩	123/2/	(6,1,3)
(6,5,5)	6/0/	(6,1,1)
(6,5,6)	16/1/	(6,1,2)
⟨6,6,1⟩	—	(6,0,1)
(7,0,1)	—	⟨7,7,1⟩
(7,1,1)	234567/0/	(7,6,6)
(7,1,2)	34567/0/	(7,6,7)
(7,1,3)	4567/0/	⟨7,6,5⟩
(7,1,4)	567/0/	⟨7,6,4⟩
(7,1,5)	67/0/	⟨7,6,3⟩
(7,1,6)	7/0/	⟨7,6,2⟩
(7,1,7)	—	⟨7,6,1⟩
⟨7,2,1⟩	—	⟨7,5,1⟩
(7,2,2)	34567/0/	(7,5,23)
(7,2,3)	4567/0/, 134567/1/	(7,5,24)
(7,2,4)	567/0/, 134567/1/	(7,5,35)
(7,2,5)	567/0/, 13567, 24567/1/	(7,5,26)
(7,2,6)	67/0/, 134567/1/	(7,5,32)
(7,2,7)	67/0/, 13467, 2567/1/	(7,5,37)
(7,2,8)	7/0/, 134567/1/	(7,5,29)
(7,2,9)	7/0/, 13457, 267/1/	(7,5,34)
(7,2,10)	70/, 1347, 2567/1/	⟨7,5,18⟩
(7,2,11)	134567/1/	(7,5,27)
(7,2,12)	13456, 27/1/	(7,5,31)

Matroid	Real essential flats	Dual matroid
(7,2,13)	1345, 267/1/	⟨7,5,11⟩
(7,2,14)	4567/0/	(7,5,25)
(7,2,15)	567/0/, 14567/1/	(7,5,36)
(7,2,16)	67/0/, 14567/1/	(7,5,33)
(7,2,17)	67/0/, 1467, 2567/1/	⟨7,5,21⟩
(7,2,18)	7/0/, 14567/1/	(7,5,30)
(7,2,19)	7/0/, 1457, 267/1/	⟨7,5,17⟩
(7,2,20)	7/0/, 147, 257, 367/1/	⟨7,5,15⟩
(7,2,21)	14567/1/	(7,5,28)
(7,2,22)	1456, 27/1/	⟨7,5,10⟩
(7,2,23)	145, 267/1/	⟨7,5,9⟩
(7,2,24)	145, 26, 37/1/	⟨7,5,7⟩
(7,2,25)	567/0/	⟨7,5,22⟩
(7,2,26)	67/0/, 1567/1/	⟨7,5,20⟩
(7,2,27)	7/0/, 1567/1/	⟨7,5,16⟩
(7,2,28)	7/0/, 157, 267/1/	⟨7,5,14⟩
(7,2,29)	1567/1/	⟨7,5,8⟩
(7,2,30)	156, 27/1/	⟨7,5,6⟩
(7,2,31)	15, 26, 37/1/	⟨7,5,5⟩
(7,2,32)	67/0/	⟨7,5,19⟩
(7,2,33)	7/0/, 167/1/	⟨7,5,13⟩
(7,2,34)	167/1/	⟨7,5,4⟩
(7,2,35)	16, 27/1/	⟨7,5,3⟩
(7,2,36)	7/0/	⟨7,5,12⟩
(7,2,37)	16/1/	⟨7,5,2⟩
⟨7,3,1⟩	—	⟨7,4,1⟩
⟨7,3,2⟩	123/2/	⟨7,4,2⟩
⟨7,3,3⟩	123, 145/2/	⟨7,4,3⟩
⟨7,3,4⟩	1234/2/	⟨7,4,8⟩
⟨7,3,5⟩	123, 456/2/	⟨7,4,4⟩
⟨7,3,6⟩	123, 145, 246/2/	⟨7,4,6⟩
⟨7,3,7⟩	123, 145, 246, 356/2/	⟨7,4,9⟩
⟨7,3,8⟩	1234, 156/2/	⟨7,4,12⟩
⟨7,3,9⟩	12345/2/	(7,4,77)
⟨7,3,10⟩	123, 145, 167/2/	⟨7,4,7⟩
⟨7,3,11⟩	123, 147, 456/2/	⟨7,4,5⟩
⟨7,3,12⟩	123, 145, 167, 246/2/	⟨7,4,11⟩
⟨7,3,13⟩	125, 234, 147, 567/2/	⟨7,4,10⟩
⟨7,3,14⟩	123, 145, 167, 246, 257/2/	⟨7,4,14⟩
⟨7,3,15⟩	123, 147, 257, 367, 456/2/	⟨7,4,15⟩
⟨7,3,16⟩	123, 145, 167, 246, 257, 356/2/	⟨7,4,17⟩
⟨7,3,17⟩	123, 147, 156, 246, 257, 345, 367/2/	⟨7,4,19⟩
⟨7,3,18⟩	1234, 567/2/	⟨7,4,13⟩
⟨7,3,19⟩	1234, 156, 257/2/	⟨7,4,16⟩

Matroid	Real essential flats	Dual matroid
⟨7,3,20⟩	1234, 156, 257, 367/2/	⟨7,4,18⟩
⟨7,3,21⟩	12345, 167/2/	(7,4,80)
⟨7,3,22⟩	1234, 1567/2/	⟨7,4,20⟩
⟨7,3,23⟩	123456/2/	(7,4,76)
(7,3,24)	4567/0/	(7,4,50)
(7,3,25)	567/0/, 14567/1/	(7,4,51)
(7,3,26)	67/0/, 14567/1/	(7,4,68)
(7,3,27)	67/0/, 1467, 2567/1/, . . .	(7,4,53)
(7,3,28)	7/0/, 14567/1/	(7,4,61)
(7,3,29)	7/0/, 1457, 267/1/, . . .	(7,4,70)
(7,3,30)	7/0/, 147, 257, 367/1/, . . .	(7,4,56)
(7,3,31)	14567/1/	(7,4,57)
(7,3,32)	1456, 27/1/, . . .	(7,4,63)
(7,3,33)	145, 267/1/, . . .	(7,4,93)
(7,3,34)	145, 26, 37/1/, . . .	(7,4,75)
(7,3,35)	567/0/	(7,4,54)
(7,3,36)	67/0/, 1567/1/	(7,4,71)
(7,3,37)	7/0/, 1567/1/	(7,4,64)
(7,3,38)	7/0/, 157, 267/1/, . . .	(7,4,103)
(7,3,39)	1567/1/	(7,4,59)
(7,3,40)	156, 27/1/, . . .	(7,4,90)
(7,3,41)	15, 26, 37/1/, . . .	⟨7,4,38⟩
(7,3,42)	567/0/, 123567/2/	(7,4,52)
(7,3,43)	67/0/, 1567/1/, 123567/2/	(7,4,69)
(7,3,44)	67/0/, 4567/1/, 12367/2/	(7,4,55)
(7,3,45)	7/0/, 1567/1/, 123567/2/	(7,4,62)
(7,3,46)	7/0/, 4567/1/, 1237/2/	(7,4,72)
(7,3,47)	7/0/, 157, 267/1/, 123567/2/	(7,4,102)
(7,3,48)	7/0/, 157, 467, /1/12357/1/, . . .	(7,4,74)
(7,3,49)	1567/1/, 123567/2/	(7,4,58)
(7,3,50)	4567/1/, 123/2/	(7,4,65)
(7,3,51)	156, 27/1/, 123567/2/	(7,4,89)
(7,3,52)	156, 47/1/, 12356/2/, . . .	(7,4,67)
(7,3,53)	15, 467/1/, 1235/2/, . . .	(7,4,94)
(7,3,54)	15, 26, 37/1/, 123567/2/	(7,4,87)
(7,3,55)	15, 26, 47/1/, 12356/2/	(7,4,105)
(7,3,56)	67/0/	⟨7,4,49⟩
(7,3,57)	7/0/, 167/1/	⟨7,4,47⟩
(7,3,58)	167/1/	⟨7,4,37⟩
(7,3,59)	16, 27/1/	⟨7,4,33⟩
(7,3,60)	67/0/, 12367/2/	(7,4,107)
(7,3,61)	7/0/, 167/1/, 12367/2/	(7,4,99)
(7,3,62)	7/0/, 467/1/, 1237/2/	⟨7 4,48⟩
(7,3,63)	167/1/, 12367/2/	(7,4,85)

Matroid	Real essential flats	Dual matroid
(7,3,64)	467/1/, 123/2/	⟨7,4,39⟩
(7,3,65)	16, 27/1/, 12367/2/	(7,4,82)
(7,3,66)	16, 47/1/, 1236/2/, . . .	⟨7,4,35⟩
(7,3,67)	46, 57/1/, 123/2/, . . .+	⟨7,4,34⟩
(7,3,68)	67/0/, 12567, 34567/2/	(7,4,73)
(7,3,69)	7/0/, 167/1/, 12567, 3457/2/	(7,4,104)
(7,3,70)	7/0/, 567/1/, 12567, 34567/2/	(7,4,66)
(7,3,71)	167/1/, 12567, 345/2/	(7,4,91)
(7,3,72)	567/1/, 12567, 34567/2/	(7,4,60)
(7,3,73)	16, 27/1/, 12567, 345/2/	(7,4,88)
(7,3,74)	16, 37/1/, 1256, 3457/2/	⟨7,4,36⟩
(7,3,75)	16, 57/1/, 12567, 3457/2/	(7,4,92)
(7,3,76)	67/0/, 123467/2/	(7,4,106)
(7,3,77)	7/0/, 167/1/, 123467/2/	(7,4,98)
(7,3,78)	7/0/, 567/1/, 12347/2/	(7,4,108)
(7,3,79)	167/1/, 123467/2/	(7,4,84)
(7,3,80)	567/1/, 1234/2/	⟨7,4,40⟩
(7,3,81)	16, 27/1/, 123467/2/	(7,4,81)
(7,3,82)	16, 57/1/, 12346/2/, . . .	(7,4,101)
(7,3,83)	7/0/	⟨7,4,41⟩
(7,3,84)	17/1/	⟨7,4,21⟩
(7,3,85)	7/0/, 1237/2/	⟨7,4,42⟩
(7,3,86)	17/1/, 1237/2/	⟨7,4,23⟩
(7,3,87)	47/1/, 123/2/	⟨7,4,22⟩
(7,3,88)	7/0/, 1237, 1457/2/	⟨7,4,43⟩
(7,3,89)	17/1/, 1237, 1457/2/	⟨7,4,27⟩
(7,3,90)	27/1/, 1237, 145/2/	⟨7,4,25⟩
(7,3,91)	67/1/, 123, 145/2/	⟨7,4,24⟩
(7,3,92)	7/0/, 12347/2/	(7,4,96)
(7,3,93)	17/1/, 12347/2/	(7,4,79)
(7,3,94)	57/1/, 1234/2/	⟨7,4,28⟩
(7,3,95)	7/0/, 1237, 4567/2/	⟨7,4,44⟩
(7,3,96)	17/1/, 1237, 456/2/	⟨7,4,26⟩
(7,3,97)	7/0/, 1247, 1357, 2367/2/	⟨7,4,45⟩
(7,3,98)	17/1/, 1247, 1357, 236/2/	⟨7,4,30⟩
(7,3,99)	47/1/, 1247, 135, 236/2/	⟨7,4,29⟩
(7,3,100)	7/0/, 1237, 1457, 2467, 3567/2/	⟨7,4,46⟩
(7,3,101)	17/1/, 1237, 1457, 246, 356/2/	⟨7,4,31⟩
(7,3,102)	7/0/, 1237, 14567/1/	(7,4,100)
(7,3,103)	17/1/, 1237, 14567/2/	(7,4,86)
(7,3,104)	27/1/, 1237, 1456/2/	⟨7,4,32⟩
(7,3,105)	47/1/, 123, 14567/2/	(7,4,83)
(7,3,106)	7/0/, 123457/2/	(7,4,95)
(7,3,107)	17/1/, 123457/2/	(7,4,78)

Matroid	Real essential flats	Dual matroid
(7,3,108)	67/1/, 12345/2/	(7,4,97)
⟨7,4,1⟩	—	⟨7,3,1⟩
⟨7,4,2⟩	4567/3/	⟨7,3,2,⟩
⟨7,4,3⟩	2367, 4567/3/	⟨7,3,3⟩
⟨7,4,4⟩	1237, 4567/3/	⟨7,3,5⟩
⟨7,4,5⟩	1237, 4567, 2356/3/	⟨7,3,11⟩
⟨7,4,6⟩	1357, 2367, 4567/3/	⟨7,3,6⟩
⟨7,4,7⟩	2345, 2367, 4567/3/	⟨7,3,10⟩
⟨7,4,8⟩	123/2/	⟨7,3,4⟩
⟨7,4,9⟩	1247, 1357, 2367, 4567/3/	⟨7,3,7⟩
⟨7,4,10⟩	1234, 1567, 2356, 3467/3/	⟨7,3,13⟩
⟨7,4,11⟩	1357, 2345, 2367, 4567/3/	⟨7,3,12⟩
⟨7,4,12⟩	123/3/, 1456/2/	⟨7,3,8⟩
⟨7,4,13⟩	123/2/, 4567/3/	⟨7,3,18⟩
⟨7,4,14⟩	1346, 1357, 2345, 2367, 4567/3/	⟨7,3,14⟩
⟨7,4,15⟩	1237, 4567, 1245, 1346, 2356/3/	⟨7,3,15⟩
⟨7,4,16⟩	123/2/, 1456, 2457/3/	⟨7,3,19⟩
⟨7,4,17⟩	1247, 1346, 1357, 2345, 2367, 4567/3/	⟨7,3,16⟩
⟨7,4,18⟩	123/2/, 1456, 2457, 3467/3/	⟨7,3,20⟩
⟨7,4,19⟩	1245, 1267, 1346, 1357, 2347, 2356, 4567/3/	⟨7,3,17⟩
⟨7,4,20⟩	123, 456/2/	⟨7,3,22⟩
⟨7,4,21⟩	12345/3/	(7,3,84)
⟨7,4,22⟩	12345, 4567/3/	(7,3,87)
⟨7,4,23⟩	123/2/, 12345/3/	(7,3,86)
⟨7,4,24⟩	12345,2367, 4567/3/	(7,3,91)
⟨7,4,25⟩	123/2/, 12345, 1567/3/	(7,3,90)
⟨7,4,26⟩	123/2/, 12345, 4567/3/	(7,3,96)
⟨7,4,27⟩	123, 145/2/, . . .	(7,3,89)
⟨7,4,28⟩	123/2/, 14567/3/	(7,3,94)
⟨7,4,29⟩	123/2/, 12345, 1467, 2567/3/	(7,3,99)
⟨7,4,30⟩	123, 145/2/, 2467/3/, . . .	(7,3,98)
⟨7,4,31⟩	123, 145/2/, 2467, 3567/3/, . . .	(7,3,101)
⟨7,4,32⟩	123, 456/2/, 12347/3/	(7,3,104)
⟨7,4,33⟩	123/2/, 12345, 12367/3/	(7,3,59)
⟨7,4,34⟩	123/2/, 12345, 12367, 4567/3/	(7,3,67)
⟨7,4,35⟩	123, 145/2/, 12367/3/, . . .	(7,3,66)
⟨7,4,36⟩	123, 456, 147/2/, . . .	(7,3,74)
⟨7,4,37⟩	1234/2/	(7,3,58)
⟨7,4,38⟩	123, 145, 167/2/, . . .	(7,3,41)
⟨7,4,39⟩	1234,/2/1567/3/	(7,3,64)
⟨7,4,40⟩	1234, 567/2/	(7,3,80)
⟨7,4,41⟩	123456/3/	(7,3,83)
⟨7,4,42⟩	123/2/, 123456/3/	(7,3,85)
⟨7,4,43⟩	123, 145/2/, 123456/3/	(7,3,88)

Matroid	Real essential flats	Dual matroid
⟨7,4,44⟩	123, 456/2/	(7,3,95)
⟨7,4,45⟩	123, 145, 246/2/, 123456/3/	(7,3,97)
⟨7,4,46⟩	123, 146, 256, 345/2/, 123456/3/	(7,3,100)
⟨7,4,47⟩	1234/2/, 123456/3/	(7,3,57)
⟨7,4,48⟩	1234, 156/2/, . . .	(7,3,62)
⟨7,4,49⟩	12345/2/	(7,3,56)
(7,4,50)	567/0/	(7,3,24)
(7,4,51)	67/0/, 1567/1/	(7,3,25)
(7,4,52)	7/0/, 1567/1/	(7,3,42)
(7,4,53)	7/0/, 157, 267/1/, . . .	(7,3,27)
(7,4,54)	1567/1/	(7,3,35)
(7,4,55)	156, 27/1/, . . .	(7,3,44)
(7,4,56)	15, 26, 37/1/, . . .	(7,3,30)
(7,4,57)	67/0/	(7,3,31)
(7,4,58)	7/0/, 167/1/	(7,3,49)
(7,4,59)	167/1/	(7,3,39)
(7,4,60)	17, 27/1/, . . .	(7,3,72)
(7,4,61)	67/0/, 123467/3/	(7,3,28)
(7,4,62)	7/0/, 167/1/, 123467/3/	(7,3,45)
(7,4,63)	7/0/, 567/1/, 12347/3/	(7,3,32)
(7,4,64)	167/1/, 123467/3	(7,3,37)
(7,4,65)	567/1/, 1234/3/	(7,3,50)
(7,4,66)	16, 27/1/, 123467/3/	(7,3,70)
(7,4,67)	16, 57/1/, 12346/3/	(7,3,52)
(7,4,68)	67/0/, 12367/2/	(7,3,26)
(7,4,69)	7/0/, 167/1/, 12367/2/	(7,3,43)
(7,4,70)	7/0/, 467/1/, 1237/2/	(7,3,29)
(7,4,71)	167/1/, 12367/2/	(7,3,36)
(7,4,72)	467/1/, 123/2/	(7,3,46)
(7,4,73)	16, 27/1/, 12367/2/	(7,3,68)
(7,4,74)	16, 47/1/, 1236/2/, . . .	(7,3,48)
(7,4,75)	46, 57/1/, 123/2/, . . .	(7,3,34)
(7,4,76)	7/0/	⟨7,3,23⟩
(7,4,77)	17/1/	⟨7,3,9⟩
(7,4,78)	7/0/, 12347/3/	(7,3,107)
(7,4,79)	17/1/, 12347/3/	(7,3,93)
(7,4,80)	57/1/, 1234/2/	⟨7,3,21⟩
(7,4,81)	7/0/, 12347, 12567/3/	(7,3,81)
(7,4,82)	17/1/, 12347, 12567/3/	(7,3,65)
(7,4,83)	37/1/, 12347, 1256	(7,3,105)
(7,4,84)	7/0/, 1237/2/	(7,3,79)
(7,4,85)	17/1/, 1237/2/	(7,3,63)
(7,4,86)	47/1/, 123/2/, . . .	(7,3,103)

Matroid	Real essential flats	Dual matroid
(7,4,87)	7/0/, 12347, 12567, 34,67/3/	(7,3,54)
(7,4,88)	17/1/, 12347, 12567, 3456/3/	(7,3,73)
(7,4,89)	7/0/, 1237/2/, 14567/3/	(7,3,51)
(7,4,90)	17/1/, 1237/2/, 14567/3/	(7,3,40)
(7,4,91)	27/1/, 1237/2/, 1456/3/	(7,3,71)
(7,4,92)	47/1/, 123/2/, 14567/3/	(7,3,75)
(7,4,93)	7/0/, 1237, 4567/2/	(7,3,33)
(7,4,94)	17/1/, 1237, 456/2/	(7,3,53)
(7,4,95)	7/0/, 123457/3/	(7,3,106)
(7,4,96)	17/1/, 123457/3/	(7,3,92)
(7,4,97)	67/1/, 12345/3/	(7,3,108)
(7,4,98)	7/0/, 1237/2/, 123457/3/	(7,3,77)
(7,4,99)	17/1/, 1237/2/, 123457/3/	(7,3,61)
(7,4,100)	47/1/, 123/2/, 123457/3/	(7,3,102)
(7,4,101)	67/1/, 123/2/, 12345/3/, . . .	(7,3,82)
(7,4,102)	7/0/, 1237, 1457/2/, . . .	(7,3,47)
(7,4,103)	17/1/, 1237, 1457/2/, . . .	(7,3,38)
(7,4,104)	27/1/, 1237, 145/2/, . . .	(7,3,69)
(7,4,105)	67/1/, 123, 145/2/, . . .	(7,3,55)
(7,4,106)	7/0/, 12347/2/	(7,3,76)
(7,4,107)	17/1/, 12347/2/	(7,3,60)
(7,4,108)	57/1/, 1234/2/	(7,3,78)
⟨7,5,1⟩	—	⟨7,2,1⟩
⟨7,5,2⟩	12345/4/	(7,2,37)
⟨7,5,3⟩	12345, 12367/4/	(7,2,35)
⟨7,5,4⟩	4567/3/	(7,2,34)
⟨7,5,5⟩	12345, 12367, 14567/4/	(7,2,31)
⟨7,5,6⟩	4567/3/ 12345/4/	(7,2,30)
⟨7,5,7⟩	4567/3/. 12345, 12367/4/	(7,2,24)
⟨7,5,8⟩	123/2/	(7,2,29)
⟨7,5,9⟩	1237, 4567/3/	(7,2,23)
⟨7,5,10⟩	123/2/, 14567/4/	(7,2,22)
⟨7,5,11⟩	123/2/, 4567/3/	(7,2,13)
⟨7,5,12⟩	123456/4/	(7,2,36)
⟨7,5,13⟩	4567/3/, 124567/4/	(7,2,33)
⟨7,5,14⟩	2367, 4567/3/	(7,2,28)
⟨7,5,15⟩	2345, 2367, 4567/3/, . . .	(7,2,20)
⟨7,5,16⟩	123/2/, 123456/4/	(7,2,27)
⟨7,5,17⟩	123/2/, 1456/3/, . . .	(7,2,19)
⟨7,5,18⟩	123, 456/2/, . . .	(7,2,10)
⟨7,5,19⟩	12345/3/	(7,2,32)
⟨7,5,20⟩	123/2/, 12345/3/	(7,2,26)
⟨7,5,21⟩	123, 145/2/	(7,2,17)
⟨7,5,22⟩	1234/2/	(7,2,25)

Matroid	Real essential flats	Dual matroid
(7,5,23)	67/0/	(7,2,2)
(7,5,24)	7/0/, 167/1/	(7,2,3)
(7,5,25)	167/1/	(7,2,14)
(7,5,26)	16, 27/1/, . . .	(7,2,5)
(7,5,27)	7/0/	(7,2,11)
(7,5,28)	17/1/	(7,2,21)
(7,5,29)	7/0/, 123457/4/	(7,2,8)
(7,5,30)	17/1/, 123457/4/	(7,2,18)
(7,5,31)	67/1/, 12345/4/	(7,2,12)
(7,5,32)	7/0/, 12347/3/	(7,2,6)
(7,5,33)	17/1/, 12347/3/	(7,2,16)
(7,5,34)	57/1/, 1234/3/, . . .	(7,2,9)
(7,5,35)	7/0/, 1237/2/	(7,2,4)
(7,5,36)	17/1/, 1237/2/	(7,2,15)
(7,5,37)	47/1/, 123/2/	(7,2,7)
⟨7,6,1⟩	—	(7,1,7)
⟨7,6,2⟩	123456/5/	(7,1,6)
⟨7,6,3⟩	12345/4/	(7,1,5)
⟨7,6,4⟩	1234/3/	(7,1,4)
⟨7,6,5⟩	123/2/	(7,1,3)
(7,6,6)	7/0/	(7,1,1)
(7,6,7)	17/1/	(7,1,2)
⟨7,7,1⟩	—	(7,0,1)

REFERENCES

[1] Acketa D. M., *On the enumeration of matroids of rank 2* (to appear)

[2] Acketa, D. M., *On the essential flats of geometric lattices* (to appear)

[3] Blackburn, J. A., Crapo, H.H., Higgs, D.A., *A Catalogue of Combinatorial Geometries*, Math. Comput. 27 (1973), 155–166.

[4] Crapo, H. H., *Erecting Geometries*, Proceedings of 2nd Chapel Hill Conference on Combinatorial Math. (1970), 74–99.

[5] Crapo, H. H., *Constructions in combinatorial geometries*. (N. S. F. Advanced Science Seminar in Combinatorial Theory) (Notes, Bodwoin College) (1971)

[6] Graver, J. E., Watkins, M. E., *Combinatorics with Emphasis on the Theory of Graphs*, Springer-Verlag (1977)

17] Welsh, D. J. A., *Matroid Theory*, London Math. Soc. Monographs No. 8, Academic Press (1976)

Dragan M. Acketa

O KONSTRUKCIJI SVIH MATROIDA NA NAJVIŠE 7 ELEMENATA

Rezime

U radu se ukratko opisuju metodi konstrukcije i daje spisak svih matroida sa nosačima od najviše 7 elemenata. Ukazuje se i na put verifikacije dobijenih rezultata.