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HYBRID COUPLED FIXED POINT THEOREMS FOR MAPS UNDER (CLRg) PROPERTY IN FUZZY METRIC SPACES

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Abstract. In this paper, we introduce the (CLRg) property for a hybrid pair of maps in fuzzy metric spaces and utilize the same to prove two unique common coupled fixed point theorems for two hybrid pairs of maps satisfying $\psi + \phi$ contractive condition in fuzzy metric spaces.

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1. Introduction

The concept of fuzzy sets was initiated by Zadeh [28] in 1965 which has inspired the fuzzification of almost all existing Mathematics. With similar quest, George and Veeramani [9] and Kramosil and Michalek [14] have introduced the concept of fuzzy topological spaces induced by fuzzy metrics which was required to be slightly manipulated to become Hausdorff. Thereafter, many authors proved fixed and common fixed point theorems in fuzzy metric spaces (e. g.[7, 8, 10, 11, 15, 17, 19, 22, 23, 26, 27]).
Now, we present the required preliminaries.

Definition 1.1. ([20]). A binary operation $* : [0,1] \times [0,1] \longrightarrow [0,1]$ is a continuous t-norm if it satisfies the following conditions:

1. $*$ is associative and commutative,
2. $*$ is continuous,
3. $a * 1 = a$ for all $a \in [0,1]$,
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Two natural examples of a continuous t-norm are $a * b = ab$ and $a * b = \min\{a, b\}$.

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Definition 1.2. ([9]). A 3-tuple $(X, M, *)$ is called a *fuzzy metric space* if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions (for each $x, y, z \in X$ and $t, s > 0$):

1. $M(x, y, t) > 0$,
2. $M(x, y, t) = 1$ if and only if $x = y$,
3. $M(x, y, t) = M(y, x, t)$,
4. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
5. $M(x, y, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the *open ball* $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by $B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}$. Let $(X, M, *)$ be a fuzzy metric space and τ the collection of all subsets $A \subset X$ with $x \in A$ if and only if there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Then τ forms a topology on X induced by the fuzzy metric M. This topology is Hausdorff as well as first countable.

A sequence $\{x_n\}$ in X converges to x if and only if $M(x_n, x, t) \to 1$ as $n \to \infty$, for each $t > 0$ and the same (sequence) is called a Cauchy sequence in the sense of [10] if $\lim_{n\to\infty} M(x_n, x_{n+p}, t_n) = 1$, for all $t > 0$ and each positive integer p. The fuzzy metric space $(X, M, *)$ is said to be *complete* if every Cauchy sequence in X is convergent. A subset A of X is said to be F-bounded if there exist $t > 0$ and $0 < r < 1$ such that $M(x, y, t) > 1 - r$ for all $x, y \in A$.

Example 1.3. Let $X = (-\infty, \infty)$. Put $a * b = ab$ for all $a, b \in [0, 1]$. For each $t \in (0, \infty)$, define $M(x, y, t) = \frac{t}{t+|x-y|}$ for all $x, y \in X$.

Example 1.4. Let $X = [0, 1]$ and $a * b = ab$ for all $a, b \in [0, 1]$ and let M be the fuzzy set on $X \times X \times (0, \infty)$ defined by

$$M(x, y, t) = e^{-\frac{|x-y|}{t}}$$

for all $t \geq 0$. Then $(X, M, *)$ is a fuzzy metric space.

Example 1.5. Let $X = [0, 1]$ and $a * b = ab$ for all $a, b \in [0, 1]$ and let M be the fuzzy set on $X \times X \times (0, \infty)$ defined by

$$M(x, y, t) = \left(\frac{t}{t+1}\right)^{|x-y|}$$

for all $t \geq 0$. Then $(X, M, *)$ is a fuzzy metric space.

Lemma 1.6. *([10]) Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing with respect to t, for all x, y in X.*

Definition 1.7. Let $(X, M, *)$ be a fuzzy metric space. Then M is said to be *continuous* on $X^2 \times (0, \infty)$ if $\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t)$, whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0, \infty)$ converges to a point $(x, y, t) \in X^2 \times (0, \infty)$, i.e. $\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1$ and $\lim_{n\to\infty} M(x, y, t_n) = M(x, y, t)$.

Lemma 1.8. *([18]). Let $(X, M, *)$ be a fuzzy metric space. Then M is a continuous function on $X^2 \times (0, \infty)$.*

Recently, Aamri and Moutawakil [1] introduced the property (E.A.) and proved common fixed point theorems under strict contractive condition. Thereafter, Sintunavarat and Kumam [24] introduced a new notion, namely: Common Limit Range property (in short CLRg). For some more references of this kind, one can be referred to [5, 6, 13, 21].
Very recently, Khan and Sumitra [3] extended the (CLRg) property for coupled maps (also see [25]) as follows.

Definition 1.9. Let $(X, M, *)$ be a fuzzy metric space. Two maps $F : X \times X \to X$ and $f : X \to X$ are said to satisfy the (CLRg) property if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} fx_n = f(p)$ and $\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} fy_n = f(q)$ for some $p, q \in X$.

Bhaskar and Lakshmikantham [4] introduced the concept of coupled fixed points. On the analogous lines Lakshmikantham and Ćirić [16] defined the common coupled fixed points. Later Xin-Qi Hu [12] defined the common fixed points for maps $F : X \times X \to X$ and $f : X \to X$. Abbas et al. [2] introduced the w-compatible pair of maps.

Definition 1.10. Let $F : X \times X \to X$ and $f : X \to X$.
(i)([4]). An element $(x, y) \in X \times X$ is called a coupled fixed point of F if $F(x, y) = x$ and $F(y, x) = y$.
(ii)([16]). An element $(x, y) \in X \times X$ is called a common coupled fixed point of F and f if $F(x, y) = fx = x$ and $F(y, x) = fy = y$.
(iii)([12]). A point $x \in X$ is called a common fixed polnt of F and f if $F(x, x) = x = fx$.
(iv)([2]). F and f are said to be w-compatible if $f(F(x, y)) = F(fx, fy)$ and $f(F(y, x)) = F(fy, fx)$ whenever $fx = F(x, y)$ and $fy = F(y, x)$ for all $x, y \in X$.

From now on, $CB(X)$ denotes the set of all non-empty closed and bounded subsets of X. For $A, B \in CB(X)$ and for every $t > 0$, we write

$$\delta_M(A, B, t) = \inf\{M(a, b, t) : a \in A, b \in B\}.$$

If A consists of a single point a, we write $\delta_M(A, B, t) = \delta_M(a, B, t)$. If B also consists of a single point b, we write $\delta_M(A, B, t) = \delta_M(a, b, t) = M(a, b, t)$.
It follows immediately from the definition that

$$\delta_M(A, B, t) = \delta_M(B, A, t) \geq 0,$$

$$\delta_M(A, B, t) = 1 \Longleftrightarrow A = B = \{a \ singleton\},$$

for all $A, B \in CB(X)$.

Definition 1.11. A sequence $\{A_n\}$ in $CB(X)$ is said to be convergent to a set $A \in CB(X)$ if $\lim\limits_{n\to\infty} \delta_M(A_n, A, t) = 1$ for all $t > 0$.

Also, one can prove the following:

Lemma 1.12. *Let $\{A_n\}$ and $\{B_n\}$ be sequences in $CB(X)$ converging to A and B in $CB(X)$, respectively. Then $\lim\limits_{n\to\infty} \delta_M(A_n, B_n, t) = \delta_M(A, B, t)$ for all $t > 0$.*

In this paper, we give a new definition and utilize the same to prove two common fixed point theorems for two hybrid pairs of maps in the next section.

2. Main results

Firstly, we give the following definition.

Definition 2.1. Let $(X, M, *)$ be a fuzzy metric space. The hybrid pair of mappings $F : X \times X \to CB(X)$ and $S : X \to X$ is said to have the Common Limit Range property (in short CLRg) with respect to S if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} M(Sx_n, Sa, t) = 1, \quad \lim_{n\to\infty} \delta_M(F(x_n, y_n), A, t) = 1,$$

$$\lim_{n\to\infty} M(Sy_n, Sb, t) = 1, \quad \lim_{n\to\infty} \delta_M(F(y_n, x_n), B, t) = 1,$$

for some $a, b \in X$, $Sa \in A \in CB(X)$ and $Sb \in B \in CB(X)$.

Let Ψ be the class of monotonically increasing continuous functions $\psi : [0, 1] \to [0, 1]$ and Φ the class of monotonically increasing continuous functions $\phi : [0, 1] \to [0, 1]$ such that $\phi(t) > t$ for $0 < t < 1$.

In what follows, $(X, M, *)$ stands for a fuzzy metric space, $F, G : X \times X \to CB(X)$ and $S, T : X \to X$ besides

$$m^{x,\ y}_{u,\ v} = \min \left\{ \begin{array}{c} M(Sx, Tu, t), M(Sy, Tv, t), \delta_M(Sx, F(x, y), t), \\ \delta_M(Sy, F(y, x), t), \delta_M(Tu, G(u, v), t), \delta_M(Tv, G(v, u), t), \\ \delta_M(Sx, G(u, v), t), \delta_M(Sy, G(v, u), t), \\ \delta_M(Tu, F(x, y), t), \delta_M(Tv, F(y, x), t) \end{array} \right\}.$$

Now, we are equipped to prove our main result as follows.

Theorem 2.2. *Let $(X, M, *)$ be a fuzzy metric space. Assume that $F, G : X \times X \to CB(X)$ and $S, T : X \to X$ are maps which satisfy the following conditions:*

(2.2.1) *the pairs (F, S) and (G, T) satisfy the (CLRg) property with respect to S and T, respectively,* (2.2.2) *the pairs* (F,S) *and* (G,T) *are* w*-compatible,*

(2.2.3) $\psi\left(\delta_M\left(F(x,y),G(u,v),t\right)\right)\geq\psi\left(m_{u,\ v}^{x,\ y}\right)+\phi\left(m_{u,\ v}^{x,\ y}\right)$
for all $x,y,u,v\in X$, $t>0$, *where* $\psi\in\Psi$, $\phi\in\Phi$.

Then there exists a unique $x\in X$ *such that* $F(x,x)=\{Sx\}=\{x\}=\{Tx\}=G(x,x)$.

Proof. Since the pairs (F,S) and (G,T) satisfy the (CLRg) property with respect to S and T, respectively, therefore there exist sequences $\{x_n\}$, $\{y_n\}$, $\{u_n\}$ and $\{v_n\}$ in X such that

$$\lim_{n\to\infty}M(Sx_n,Sa,t)=1,\quad \lim_{n\to\infty}\delta_M(F(x_n,y_n),A,t)=1,$$

$$\lim_{n\to\infty}M(Sy_n,Sb,t)=1,\quad \lim_{n\to\infty}\delta_M(F(y_n,x_n),B,t)=1,$$

$$\lim_{n\to\infty}M(Tu_n,Ta^{'},t)=1,\quad \lim_{n\to\infty}\delta_M(G(u_n,v_n),P,t)=1,$$

and

$$\lim_{n\to\infty}M(Tv_n,Tb^{'},t)=1,\quad \lim_{n\to\infty}\delta_M(G(v_n,u_n),Q,t)=1,$$

for some $a,b,a^{'},b^{'}\in X$ and $Sa\in A\in CB(X)$, $Sb\in B\in CB(X)$, $Ta^{'}\in P\in CB(X)$, $Tb^{'}\in Q\in CB(X)$.
Suppose $0<\min\{\delta_M(A,P,t),\delta_M(B,Q,t)\}<1$ for some $t>0$. Consider

$$\psi\left(\delta_M(F(x_n,y_n),G(u_n,v_n),t)\right)\geq\psi\left(m_{u_n,\ v_n}^{x_n,\ y_n}\right)+\phi\left(m_{u_n,\ v_n}^{x_n,\ y_n}\right)\tag{2.1}$$

wherein

$$m_{u_n,\ v_n}^{x_n,\ y_n}=\min\left\{\begin{array}{c}M(Sx_n,Tu_n,t),M(Sy_n,Tv_n,t),\\ \delta_M(Sx_n,F(x_n,y_n),t),\delta_M(Sy_n,F(y_n,x_n),t),\\ \delta_M(Tu_n,G(u_n,v_n),t),\delta_M(Tv_n,G(v_n,u_n),t),\\ \delta_M(Sx_n,G(u_n,v_n),t),\delta_M(Sy_n,G(v_n,u_n),t),\\ \delta_M(Tu_n,F(x_n,y_n),t),\delta_M(Tv_n,F(y_n,x_n),t)\end{array}\right\}$$

and

$$\begin{aligned}\lim_{n\to\infty}m_{u_n,\ v_n}^{x_n,\ y_n}&=\min\left\{\begin{array}{c}M(Sa,Ta^{'},t),M(Sb,Tb^{'},t),1,1,1,1,\\ \delta_M(Sa,P,t),\delta_M(Sb,Q,t),\\ \delta_M(Ta^{'},A,t),\delta_M(Tb^{'},B,t)\end{array}\right\}\\ &\geq\min\left\{\delta_M(A,P,t),\delta_M(B,Q,t)\right\}.\end{aligned}$$

On letting $n\to\infty$ in (2.1), we get

$$\psi\left(\delta_M(A,P,t)\right)\geq\psi\left(\min\left\{\begin{array}{c}\delta_M(A,P,t),\\ \delta_M(B,Q,t)\end{array}\right\}\right)+\phi\left(\min\left\{\begin{array}{c}\delta_M(A,P,t),\\ \delta_M(B,Q,t)\end{array}\right\}\right).$$

Similarly, we can also show that

$$\psi\left(\delta_M(B,Q,t)\right)\geq\psi\left(\min\left\{\begin{array}{c}\delta_M(A,P,t),\\ \delta_M(B,Q,t)\end{array}\right\}\right)+\phi\left(\min\left\{\begin{array}{c}\delta_M(A,P,t),\\ \delta_M(B,Q,t)\end{array}\right\}\right).$$

Thus, in all we have

$$\psi\left(\min\left\{\begin{array}{c}\delta_M(A,P,t),\\ \delta_M(B,Q,t)\end{array}\right\}\right)\geq\psi\left(\min\left\{\begin{array}{c}\delta_M(A,P,t),\\ \delta_M(B,Q,t)\end{array}\right\}\right)$$
$$+\phi\left(\min\left\{\begin{array}{c}\delta_M(A,P,t),\\ \delta_M(B,Q,t)\end{array}\right\}\right),$$

which in turn yields that

$$\begin{array}{l}0\geq\phi\left(\min\left\{\ \delta_M(A,P,t),\delta_M(B,Q,t)\ \right\}\right)\\ \quad>\min\left\{\ \delta_M(A,P,t),\delta_M(B,Q,t)\ \right\},\end{array}$$

a contradiction. Hence for all $t>0$, we have

$$\min\left\{\delta_M(A,P,t),\delta_M(B,Q,t)\right\}=1$$

so that $A=P=\{a\ singleton\}$ and $B=Q=\{a\ singleton\}$.
Since $Sa\in A$ and $Sb\in B$ we have

$$\begin{array}{c}A=P=\{Sa\}=\{Ta'\}\\ B=Q=\{Sb\}=\{Tb'\}.\end{array}\tag{2.2}$$

Now, suppose that $0<M(Sa,Sb,t)<1$ for some $t>0$.
Consider

$$\psi\left(\delta_M(F(y_n,x_n),G(u_n,v_n),t)\right)\geq\psi\left(m^{y_n,\ x_n}_{u_n,\ v_n}\right)+\phi\left(m^{y_n,\ x_n}_{u_n,\ v_n}\right)\tag{2.3}$$

$$m^{y_n,\ x_n}_{u_n,\ v_n}=\min\left\{\begin{array}{c}M(Sy_n,Tu_n,t),M(Sx_n,Tv_n,t),\\ \delta_M(Sy_n,F(y_n,x_n),t),\delta_M(Sx_n,F(x_n,y_n),t),\\ \delta_M(Tu_n,G(u_n,v_n),t),\delta_M(Tv_n,G(v_n,u_n),t),\\ \delta_M(Sy_n,G(u_n,v_n),t),\delta_M(Sx_n,G(v_n,u_n),t),\\ \delta_M(Tu_n,F(y_n,x_n),t),\delta_M(Tv_n,F(x_n,y_n),t)\end{array}\right\}.$$

$$\begin{array}{rl}\lim\limits_{n\to\infty}m^{y_n,\ x_n}_{u_n,\ v_n}&=\min\left\{\begin{array}{c}M(Sb,Ta',t),M(Sa,Tb',t),1,1,1,1,\\ \delta_M(Sb,P,t),\delta_M(Sa,Q,t),\\ \delta_M(Ta',A,t),\delta_M(Tb',B,t)\end{array}\right\}\\ &=M(Sb,Sa,t),\quad\text{due to (2.2)}\end{array}$$

Letting $n\to\infty$ in (2.3), we get
$\psi(M(Sb,Sa,t))\geq\psi(M(Sb,Sa,t))+\phi(M(Sb,Sa,t))$
$0\geq\phi(M(Sb,Sa,t))>M(Sb,Sa,t)$.
It is a contradiction. Hence $M(Sa,Sb,t)=1$ for all $t>0$ so that $Sa=Sb$.
Thus

$$Ta'=Sa=Sb=Tb'.\tag{2.4}$$

Suppose that $0<\min\left\{\delta_M(F(a,b),Sa,t),\delta_M(F(b,a),Sb,t)\right\}<1$ for some $t>0$.
Consider,

$$\psi\left(\delta_M(F(a,b),G(u_n,v_n),t)\right)\geq\psi\left(m^{a,\ b}_{u_n,\ v_n}\right)+\phi\left(m^{a,\ b}_{u_n,\ v_n}\right)\tag{2.5}$$

wherein

$$m^{a,\ b}_{u_n,\ v_n} = \min\left\{\begin{array}{c} M(Sa,Tu_n,t), M(Sb,Tv_n,t), \\ \delta_M(Sa,F(a,b),t), \delta_M(Sb,F(b,a),t), \\ \delta_M(Tu_n,G(u_n,v_n),t), \delta_M(Tv_n,G(v_n,u_n),t), \\ \delta_M(Sa,G(u_n,v_n),t), \delta_M(Sb,G(v_n,u_n),t), \\ \delta_M(Tu_n,F(a,b),t), \delta_M(Tv_n,F(b,a),t) \end{array}\right\}$$

and

$$\begin{aligned} \lim_{n\to\infty} m^{a,\ b}_{u_n,\ v_n} &= \min\left\{\begin{array}{c} M(Sa,Ta',t), M(Sb,Tb',t), \delta_M(Sa,F(a,b),t), \\ \delta_M(Sb,F(b,a),t), \delta_M(Ta',P,t), \delta_M(Tb',Q,t), \\ \delta_M(Sa,P,t), \delta_M(Sb,Q,t), \\ \delta_M(Ta',F(a,b),t), \delta_M(Tb',F(b,a),t) \end{array}\right\} \\ &\geq \min\left\{\begin{array}{c} 1,1,\delta_M(Sa,F(a,b),t), \delta_M(Sb,F(b,a),t), 1,1,1,1, \\ \delta_M(Sa,F(a,b),t), \delta_M(Sb,F(b,a),t) \end{array}\right\} \\ &= \min\left\{\delta_M(Sa,F(a,b),t), \delta_M(Sb,F(b,a),t)\right\}. \end{aligned}$$

Letting $n\to\infty$ in (2.5), we get

$$\begin{aligned} \psi\left(\delta_M(Sa,F(a,b),t)\right) \geq \psi&\left(\min\left\{\begin{array}{c} \delta_M(Sa,F(a,b),t), \\ \delta_M(Sb,F(b,a),t) \end{array}\right\}\right) \\ &+\phi\left(\min\left\{\begin{array}{c} \delta_M(Sa,F(a,b),t), \\ \delta_M(Sb,F(b,a),t) \end{array}\right\}\right). \end{aligned}$$

Similarly we can show that

$$\begin{aligned} \psi\left(\delta_M(Sb,F(b,a),t)\right) \geq \psi&\left(\min\left\{\begin{array}{c} \delta_M(Sa,F(a,b),t), \\ \delta_M(Sb,F(b,a),t) \end{array}\right\}\right) \\ &+\phi\left(\min\left\{\begin{array}{c} \delta_M(Sa,F(a,b),t), \\ \delta_M(Sb,F(b,a),t) \end{array}\right\}\right). \end{aligned}$$

Thus, we have

$$\begin{aligned} \psi\left(\min\left\{\begin{array}{c} \delta_M(Sa,F(a,b),t), \\ \delta_M(Sb,F(b,a),t) \end{array}\right\}\right) &\geq \psi\left(\min\left\{\begin{array}{c} \delta_M(Sa,F(a,b),t), \\ \delta_M(Sb,F(b,a),t) \end{array}\right\}\right) \\ &\quad +\phi\left(\min\left\{\begin{array}{c} \delta_M(Sa,F(a,b),t), \\ \delta_M(Sb,F(b,a),t) \end{array}\right\}\right), \end{aligned}$$

which in turn yields that

$$0 \geq \phi\left(\min\left\{\begin{array}{c} \delta_M(Sa,F(a,b),t), \\ \delta_M(Sb,F(b,a),t) \end{array}\right\}\right) > \min\left\{\begin{array}{c} \delta_M(Sa,F(a,b),t), \\ \delta_M(Sb,F(b,a),t) \end{array}\right\},$$

a contradiction. Hence for every $t>0$, we have

$$\min\left\{\ \delta_M(Sa,F(a,b),t), \delta_M(Sb,F(b,a),t)\ \right\} = 1$$

so that

$$F(a,b) = \{Sa\} \quad \textit{and} \quad F(b,a) = \{Sb\}. \tag{2.6}$$

Similarly, by taking $x = x_n$, $y = y_n$, $u = a^{'}$, $v = b^{'}$ and $x = y_n$, $y = x_n$, $u = b^{'}$, $v = a^{'}$ in (2.2.3) and letting $n \to \infty$, we can show that

$$G(a^{'}, b^{'}) = \{Ta^{'}\} \quad and \quad G(b^{'}, a^{'}) = \{Tb^{'}\}. \tag{2.7}$$

Let $x = Sa$. Then from (2.4), $Sa = Sb = Ta^{'} = Tb^{'} = x$.
Since (F, S) and (G, T) are w-compatible, from (2.6) and (2.7), it follows that

$$Sx = SSa = SF(a, b) = F(Sa, Sb) = F(x, x). \tag{2.8}$$

and

$$Tx = TTa = TG(a^{'}, b^{'}) = G(Ta^{'}, Tb^{'}) = G(x, x). \tag{2.9}$$

Suppose $0 < M(Sx, x, t) < 1$ for some $t > 0$.
Consider

$$\begin{aligned}\psi(M(Sx,x,t)) &= \psi(M(F(x,x), G(a^{'}, b^{'}, t) \text{ from (2.7) and (2.8)},\\ &= \psi(\delta_M(F(x,x), G(a^{'}, b^{'}, t), \text{ since } F(x,x) = \{Sx\} \text{ and}\\ &\qquad\qquad\qquad\qquad G(a^{'}, b^{'}) = \{Ta^{'}\} = \{x\}\\ &= \psi\left(m^{x;\ x}_{a^{'},\ b^{'}}\right) + \phi\left(m^{x;\ x}_{a^{'},\ b^{'}}\right)\end{aligned}$$

$$\begin{aligned} m^{x;\ x}_{a^{'},\ b^{'}} &= \min\left\{\begin{array}{c} M(Sx, Ta^{'}, t), M(Sx, Tb^{'}, t),\\ \delta_M(Sx, F(x,x), t), \delta_M(Sx, F(x,x), t),\\ \delta_M(Ta^{'}, G(a^{'}, b^{'}), t), \delta_M(Tb^{'}, G(b^{'}, a^{'}), t),\\ \delta_M(Sx, G(a^{'}, b^{'}), t), \delta_M(Sx, G(b^{'}, a^{'}), t),\\ \delta_M(Ta^{'}, F(x,x), t), \delta_M(Tb^{'}, F(x,x), t) \end{array}\right\}\\ &= \min\left\{\begin{array}{c} M(Sx,x,t), M(Sx,x,t), 1, 1, 1, 1, M(Sx,x,t),\\ M(Sx,x,t), M(x,Sx,t), M(x,Sx,t) \end{array}\right\}\\ &= M(Sx,x,t).\end{aligned}$$

Thus

$$\psi(M(Sx,x,t)) \geq \psi(M(Sx,x,t)) + \phi(M(Sx,x,t))$$
$$0 \geq \phi(M(Sx,x,t)) > M(Sx,x,t),$$

a contradiction. Hence $M(Sx, x, t) = 1$ for every $t > 0$ so that $Sx = x$.
Similarly we can show that $Tx = x$. Thus from (2.8) and (2.9), we have

$$F(x, x) = \{Sx\} = \{x\} = \{Tx\} = G(x, x).$$

Hence (x, x) is a common coupled fixed point of F, G, S and T.
Uniqueness of x follows easily from (2.2.3). □

One can prove the following along the similar lines as Theorem 2.2.

Theorem 2.3. *Let $(X, M, *)$ be a fuzzy metric space. If $F, G : X \times X \to CB(X)$ and $S, T : X \to X$ are maps which satisfy the following conditions:*

(2.3.1) *the pairs (F, S) and (G, T) satisfy the (CLRg) property with respect to S and T, respectively,* (2.3.2) *the pairs* (F,S) *and* (G,T) *are* w*-compatible,*

(2.3.3) $\delta_M(F(x,y),G(u,v),kt) \geq m^{x;\ y}_{u,\ v}$ *for all* $x,y,u,v \in X$, $t>0$, *where* $k\in(0,1)$ *and* $\lim_{t\to\infty} M(x,y,t)=1$ *for all* $x,y\in X$.

Then there exists a unique $x \in X$ *such that* $F(x,x)=\{Sx\}=\{x\}=\{Tx\}=G(x,x)$.

Theorem 2.4. *Let* $(X,M,*)$ *be a fuzzy metric space,* $F,G: X\times X\to X$ *and* $S,T: X\to X$ *be mappings satisfying*

(2.4.1) *the pairs* (F,S) *and* (G,T) *satisfy the (CLRg) property with respect to* S *and* T*, respectively,*

(2.4.2) *the pairs* (F,S) *and* (G,T) *are* w*-compatible,*

(2.4.3) $M(F(x,y),G(u,v),kt) \geq m^{x;\ y}_{u,\ v}$ *for all* $x,y,u,v\in X$, $t>0$, *where* $k\in(0,1)$ *and*

$$m^{x;\ y}_{u,\ v} = \min\left\{\begin{array}{c} M(Sx,Tu,t), M(Sy,Tv,t), \\ M(Sx,F(x,y),t), M(Sy,F(y,x),t), \\ M(Tu,G(u,v),t), M(Tv,G(v,u),t), \\ M(Sx,G(u,v),t), M(Sy,G(v,u),t), \\ M(Tu,F(x,y),t), M(Tv,F(y,x),t) \end{array}\right\},$$

(2.4.4) $\lim_{t\to\infty} M(x,y,t)=1$ *for all* $x,y\in X$.

Then there exists $x\in X$ *such that* $F(x,x)=Sx=x=Tx=G(x,x)$.

Now, we give two examples to illustrate Theorem 2.4.

Example 2.5. Let $X=[0,1]$ and $a*b=ab$ for all $a,b\in[0,1]$ and let M be the fuzzy set on $X\times X\times(0,\infty)$ defined by

$$M(x,y,t)=e^{-\frac{|x-y|}{t}}$$

for all $t\geq 0$. Then $(X,M,*)$ is a fuzzy metric space.
Define $F,G: X\times X\to X$ and $S,T: X\to X$ by $F(x,y)=\frac{x+y}{8}$, $G(x,y)=\frac{x+y}{16}$, $Sx=\frac{x}{2}$ and $Tx=\frac{x}{4}$. Then $|\frac{x+y}{8}-\frac{u+v}{16}| = \frac{1}{16}|2x-u+2y-v| \leq \frac{1}{2}\max\{\frac{|2x-u|}{4},\frac{|2y-v|}{4}\}$. Now,

$$\begin{aligned} M(F(x,y),G(u,v),\frac{1}{2}t) &= e^{-\frac{\left|\frac{x+y}{8}-\frac{u+v}{16}\right|}{\frac{1}{2t}}} \\ &\geq e^{-\frac{\frac{1}{2}\max\left\{\frac{|2x-u|}{4},\frac{|2y-v|}{4}\right\}}{\frac{1}{2t}}} \\ &= e^{-\frac{\max\left\{\frac{|2x-u|}{4},\frac{|2y-v|}{4}\right\}}{t}} \\ &\geq \min\left\{e^{-\frac{|2x-u|}{4t}}, e^{-\frac{|2y-v|}{4t}}\right\} \\ &= \min\{M(Sx,Tu,t),M(Sy,Tv,t)\} \\ &\geq m^{x;\ y}_{u,\ v}. \end{aligned}$$

Also (F,S) and (G,T) satisfy the (CLRg) property with respect to S and T, respectively with sequences $\{x_n\} = \{\frac{1}{\sqrt{n}}\}, \{y_n\} = \{\frac{1}{n}\}, \{u_n\} = \{\frac{1}{n^2}\}$ and $\{v_n\} = \{\frac{1}{n}\}$, respectively. Clearly, the pairs (F,S) and (G,T) are w-compatible. Clearly $(0,0)$ is the unique common fixed point of F, G, S and T.

Example 2.6. Let $X = [0,1]$ and $a * b = ab$ for all $a, b \in [0,1]$ and let M be the fuzzy set on $X \times X \times (0,\infty)$ defined by

$$M(x,y,t) = \left(\frac{t}{t+1}\right)^{|x-y|}$$

for all $t \geq 0$. Then $(X, M, *)$ is a fuzzy metric space.

Define $F, G : X \times X \to X$ and $S, T : X \to X$ by $F(x,y) = \frac{x^2+y^2}{16}$, $G(x,y) = \frac{x+y}{16}, Sx = \frac{x^2}{4}$ and $Tx = \frac{x}{4}$.

We have $\frac{\frac{t}{2}}{\frac{t}{2}+1} \geq \left(\frac{t}{t+1}\right)^2$ for all $t \geq 0$. Now,

$$\begin{aligned}
M(F(x,y), G(u,v), \frac{1}{2}t) &= \left(\frac{\frac{t}{2}}{\frac{t}{2}+1}\right)^{\left|\frac{x^2+y^2}{16} - \frac{u+v}{16}\right|} \\
&\geq \left(\frac{t}{t+1}\right)^{\left|\frac{x^2-u+y^2-v}{8}\right|} \\
&\geq \left(\frac{t}{t+1}\right)^{\frac{\left|x^2-u\right|+\left|y^2-v\right|}{8}} \\
&= \left(\frac{t}{t+1}\right)^{\frac{|fx-gu|+|fy-gv|}{2}} \\
&\geq \left(\frac{t}{t+1}\right)^{\max\{|fx-gu|,|fy-gv|\}} \\
&\geq \min\left\{\left(\frac{t}{t+1}\right)^{|fx-gu|}, \left(\frac{t}{t+1}\right)^{|fy-gv|}\right\} \\
&= \min\{M(fx,gu,t), M(fy,gv,t)\} \\
&\geq m^{x,y}_{u,v}.
\end{aligned}$$

Also (F,S) and (G,T) satisfy the (CLRg) property with respect to S and T, respectively with sequences $\{x_n\} = \{\frac{1}{\sqrt{n}}\}, \{y_n\} = \{\frac{1}{n}\}, \{u_n\} = \{\frac{1}{n^2}\}$ and $\{v_n\} = \{\frac{1}{n}\}$, respectively. Clearly, the pairs (F,S) and (G,T) are w-compatible. Clearly $(0,0)$ is the unique common fixed point of F, G, S and T.

Remark 2.7. Recently, Sumitra et al. [25] proved a unique coupled common fixed point theorem for four self mappings (see Theorem 3.2 of [25]). Inherently they used the condition $\lim_{t\to\infty} M(x,y,t) = 1$ for all $x, y \in X$ in the proof of their Theorem 3.2. Moreover, the condition $a*b \geq ab, \forall a, b \in [0,1]$ is redundant. Our Theorem 2.3 with H-type t-norm is a generalization and extension of Theorem 3.2 of [25].

Theorem 2.8. *Let $(X, M, *)$ be a fuzzy metric space. If $F, G : X \times X \to CB(X)$ and $S, T : X \to X$ are maps which satisfy the following conditions:*

(2.8.1) *the pairs (F, S) and (G, T) satisfy the (CLRg) property with respect to S and T, respectively,*

(2.8.2) *the pairs (F, S) and (G, T) are w-compatible,*

(2.8.3) $\delta_M(F(x, y), G(u, v), t) \geq \phi(m_{u,v}^{x,y})$ *for all $x, y, u, v \in X$, $t > 0$, where $\phi : [0, 1] \to [0, 1]$ is continuous, monotonically increasing and $\phi(t) > t$ for $0 < t < 1$.*

Then there exists a unique $x \in X$ such that $F(x, x) = \{Sx\} = \{x\} = \{Tx\} = G(x, x)$.

Finally we prove the following.

Theorem 2.9. *Let $(X, M, *)$ be a fuzzy metric space. If $F, G : X \times X \to CB(X)$ and $S, T : X \to X$ are maps which satisfy the following conditions:*

(2.9.1)(a) *the pair (F, S) satisfies the (CLRg) property with respect to S and $F(X \times X \subseteq T(X)$,*

or

(2.9.1)(b) *the pair (G, T) satisfies the (CLRg) property with respect to T and $G(X \times X) \subseteq S(X)$,*

(2.9.2) *the pairs (F, S) and (G, T) are w-compatible,*

(2.9.3) $\psi\left(\delta_M\left(F(x, y), G(u, v), t\right)\right) \geq \psi\left(m_{u,\ v}^{x,\ y}\right) + \phi\left(m_{u,\ v}^{x,\ y}\right)$
for all $x, y, u, v \in X$, $t > 0$, where $\psi \in \Psi$, $\phi \in \Phi$.

Then there exists a unique $x \in X$ such that $F(x, x) = \{Sx\} = \{x\} = \{Tx\} = G(x, x)$.

Proof. Suppose (2.9.1)(a) holds.
Then there exist sequences $\{x_n\}$, $\{y_n\}$ in X such that

$$\lim_{n\to\infty} M(Sx_n, Sa, t) = 1, \quad \lim_{n\to\infty} \delta_M(F(x_n, y_n), A, t) = 1,$$

$$\lim_{n\to\infty} M(Sy_n, Sb, t) = 1, \quad \lim_{n\to\infty} \delta_M(F(y_n, x_n), B, t) = 1$$

for some $a, b \in X$ and $Sa \in A \in CB(X)$, $Sb \in B \in CB(X)$.
Since $F(x_n, y_n) \subseteq F(X \times X) \subseteq T(X)$, there exist $\alpha_n \in F(x_n, y_n)$ and $u_n \in X$ such that $\alpha_n = Tu_n$ for all n.
Also $M(Tu_n, Sa, t) = M(\alpha_n, Sa, t) \geq \delta_M(F(x_n, y_n), A, t) \to 1$ as $n \to \infty$.
Hence $\lim_{n\to\infty} M(Tu_n, Sa, t) = 1$.
Similarly, there exists $v_n \in X$ such that $\lim_{n\to\infty} M(Tv_n, Sb, t) = 1$.
Let $\lim_{n\to\infty} G(u_n, v_n) = P$ and $\lim_{n\to\infty} G(v_n, u_n) = Q$.

Suppose $0 < \min\{\delta_M(A,P,t), \delta_M(B,Q,t)\} < 1$ for some $t > 0$.
Consider

$$\psi\left(\delta_M(F(x_n,y_n),G(u_n,v_n),t)\right) \geq \psi\left(m_{u_n,\ v_n}^{x_n,\ y_n}\right) + \phi\left(m_{u_n,\ v_n}^{x_n,\ y_n}\right) \tag{2.10}$$

$$m_{u_n,\ v_n}^{x_n,\ y_n} = \min\left\{\begin{array}{c} M(Sx_n,Tu_n,t), M(Sy_n,Tv_n,t), \\ \delta_M(Sx_n,F(x_n,y_n),t), \delta_M(Sy_n,F(y_n,x_n),t), \\ \delta_M(Tu_n,G(u_n,v_n),t), \delta_M(Tv_n,G(v_n,u_n),t), \\ \delta_M(Sx_n,G(u_n,v_n),t), \delta_M(Sy_n,G(v_n,u_n),t), \\ \delta_M(Tu_n,F(x_n,y_n),t), \delta_M(Tv_n,F(y_n,x_n),t) \end{array}\right\}$$

$$\begin{aligned} \lim_{n\to\infty} m_{u_n,\ v_n}^{x_n,\ y_n} &= \min\left\{\begin{array}{c} 1,1,1,1,\delta_M(Sa,P,t),\delta_M(Sb,Q,t), \\ \delta_M(Sa,P,t),\delta_M(Sb,Q,t),1,1 \end{array}\right\} \\ &\geq \min\left\{\delta_M(A,P,t),\delta_M(B,Q,t)\right\}. \end{aligned}$$

Letting $n \to \infty$ in (2.10), we get

$$\begin{aligned} \psi\left(\delta_M(A,P,t)\right) \geq \psi&\left(\min\left\{\begin{array}{c} \delta_M(A,P,t), \\ \delta_M(B,Q,t) \end{array}\right\}\right) \\ &+\phi\left(\min\left\{\begin{array}{c} \delta_M(A,P,t), \\ \delta_M(B,Q,t) \end{array}\right\}\right). \end{aligned}$$

Similarly we can show that

$$\begin{aligned} \psi\left(\delta_M(B,Q,t)\right) \geq \psi&\left(\min\left\{\begin{array}{c} \delta_M(A,P,t), \\ \delta_M(B,Q,t) \end{array}\right\}\right) \\ &+\phi\left(\min\left\{\begin{array}{c} \delta_M(A,P,t), \\ \delta_M(B,Q,t) \end{array}\right\}\right). \end{aligned}$$

Thus we have

$$\begin{aligned} \psi\left(\min\left(\begin{array}{c} \delta_M(A,P,t), \\ \delta_M(B,Q,t) \end{array}\right)\right) \geq \psi&\left(\min\left\{\begin{array}{c} \delta_M(A,P,t), \\ \delta_M(B,Q,t) \end{array}\right\}\right) \\ &+\phi\left(\min\left\{\begin{array}{c} \delta_M(A,P,t), \\ \delta_M(B,Q,t) \end{array}\right\}\right), \end{aligned}$$

which in turn yields that

$$0 \geq \phi\left(\min\left\{\delta_M(A,P,t),\delta_M(B,Q,t)\right\}\right) > \min\left\{\delta_M(A,P,t),\delta_M(B,Q,t)\right\}.$$

It is a contradiction. Hence for all $t > 0$, we have

$$\min\left\{\delta_M(A,P,t),\delta_M(B,Q,t)\right\} = 1.$$

Hence $A = P = \{a\ singleton\}$ and $B = Q = \{a\ singleton\}$.
Since $Sa \in A$ and $Sb \in B$ we have $A = P = \{Sa\}$ and $B = Q = \{Sb\}$.
Thus $\lim_{n\to\infty} G(u_n,v_n) = \{Sa\}$ and $\lim_{n\to\infty} G(v_n,u_n) = \{Sb\}$.
Now by taking $x = x_n$, $y = y_n$, $u = v_n$, $v = u_n$ in (2.9.3) and letting $n \to \infty$, we can show that

$$Sa = Sb. \tag{2.11}$$

Taking $x = a$, $y = b$, $u = u_n$, $v = v_n$ and $x = b$, $y = a$, $u = v_n$, $v = u_n$ in (2.9.3) and letting $n \to \infty$, we can show that $F(a, b) = \{Sa\}$ and $F(b, a) = \{Sb\}$.
Since $\{Sa\} = F(a, b) \subseteq F(X \times X) \subseteq T(X)$, there exists $a^{'} \in X$ such that $Sa = Ta^{'}$.
Since $\{Sb\} = F(b, a) \subseteq F(X \times X) \subseteq T(X)$, there exists $b^{'} \in X$ such that $Sb = Tb^{'}$.
From (2.11), we have $Ta^{'} = Sa = Sb = Tb^{'}$.
Now taking $x = x_n$, $y = y_n$, $u = a^{'}$, $v = b^{'}$ and $x = y_n$, $y = x_n$, $u = b^{'}$, $v = a^{'}$ in (2.9.3) and letting $n \to \infty$, we can show that $G(a^{'}, b^{'}) = \{Ta^{'}\}$ and $G(b^{'}, a^{'}) = \{Tb^{'}\}$.
The rest of the proof follows as in Theorem 2.2.
Similarly we can prove Theorem 2.9 if (2.9.1)(b) holds. □

3. An Application

As an application of Theorem 2.4, we prove a theorem on the existence and uniqueness of the solution of a Fredholm nonlinear integral equation. To accomplish this purpose, we consider the following integral equation:

$$x(p) = \int_a^b (K_1(p, q) + K_2(p, q)) \left[f(q, x(q)) + g(q, x(q))\right] dq + h(p), \tag{3.1}$$

for all $p \in I = [a, b]$, $K_1, K_2 \in C(I \times I, R)$ and $h \in C(I, R)$.

Let Θ be the set of all functions $\theta : R^+ \to R^+$ satisfying the following conditions:

(i_θ) θ is non-decreasing,
(ii_θ) $\theta(p) \le p$.

We also require the functions K_1, K_2, f and g to satisfy the following conditions:

Assumption (3.1)

(i) $K_1(p, q) \ge 0$ and $K_2(p, q) \le 0$ for all $p, q \in I$,

(ii) there exist positive numbers λ, μ and $\theta \in \Theta$ such that for all $x, y \in C(I, R)$ with $x \ge y$, the following conditions hold:

$$0 \le f(q, x) - f(q, y) \le \lambda\theta(x - y) - \mu\theta(x - y) \tag{3.2}$$

$$\lambda\theta(x - y) - \mu\theta(x - y) \le g(q, x) - g(q, y) \le 0, \tag{3.3}$$

(iii)

$$\max\{\lambda, \mu\} \sup_{p \in I} \int_a^b [K_1(p, q) - K_2(p, q)] dq \le \frac{1}{4}. \tag{3.4}$$

Now, we are equipped to prove the following theorem:

Theorem 3.1. *Consider the integral equation* (3.1) *with* K_1, $K_2 \in C(I \times I, R)$ *and* $h \in C(I, R)$. *If all the conditions embodied in the Assumption* (3.1) *are satisfied, then the integral equation* (3.1) *has a unique solution in* $C(I, R)$.

Proof. It is well known that $X = C(I, R)$ is a complete metric space with respect to the sup metric

$$d(x,y) = \sup_{p \in I} |x(p) - y(p)| .$$

It is straightforward to check that $(X, M, *)$ is a fuzzy metric space if we define

$$M(x,y,t) = e^{-\frac{d(x,y)}{t}}, \text{for all } x, y \in C(I,R) \text{ and } t > 0,$$

wherein $*$ is defined by $x * y = xy$ (for all x, $y \in I$). Now, define a mapping $F : X \times X \to X$ by

$$\begin{aligned} F(x,y)(p) &= \int_a^b K_1(p,q)[f(q,x(q)) + g(q,y(q))]dq \\ &\quad + \int_a^b K_2(p,q)[f(q,y(q)) + g(q,x(q))]dq + h(p), \end{aligned}$$

for all $p \in I$. On using (3.2) and (3.3), we have (for x, y, u, $v \in X$)

$$\begin{aligned} &F(x,y)(p) - F(u,v)(p) \\ &= \int_a^b K_1(p,q)\,[f(q,x(q)) + g(q,y(q))]\,dq \\ &\quad + \int_a^b K_2(p,q)\,[f(q,y(q)) + g(q,x(q))]\,dq \qquad (3.5) \\ &\quad - \int_a^b K_1(p,q)\,[f(q,u(q)) + g(q,v(q))]\,dq \\ &\quad - \int_a^b K_2(p,q)\,[f(q,v(q)) + g(q,u(q))]\,dq \\ &= \int_a^b K_1(p,q)[(f(q,x(q)) - f(q,u(q))) - (g(q,v(q)) - g(q,y(q)))]dq \end{aligned}$$

$$
\begin{aligned}
&-\int_a^b K_2(p,q)[(f(q,v(q))-f(q,y(q)))-(g(q,x(q))-g(q,u(q)))]dq\\
\leq\ &\int_a^b K_1(p,q)\left[\lambda\theta\left(x(q)-u(q)\right)+\mu\theta\left(v(q)-y(q)\right)\right]dq\\
&-\int_a^b K_2(p,q)\left[\lambda\theta\left(v(q)-y(q)\right)+\mu\theta\left(x(q)-u(q)\right)\right]dq.
\end{aligned}
$$

As the function θ is non-decreasing, we have

$$
\begin{aligned}
\theta\left(x(q)-u(q)\right) &\leq \theta\left(\sup_{q\in I}|x(q)-u(q)|\right)=\theta(d(x,u)),\\
\theta\left(v(q)-y(q)\right) &\leq \theta\left(\sup_{q\in I}|v(q)-y(q)|\right)=\theta(d(y,v)).
\end{aligned}
$$

Appealing to (3.5) and making use of the fact that $K_2(p,\, q)\leq 0$, we obtain

$$
\begin{aligned}
&|F(x,y)(p)-F(u,v)(p)|\\
&\leq \int_a^b K_1(p,q)\left[\lambda\theta(d(x,u))+\mu\theta(d(y,v))\right]dq\\
&\quad-\int_a^b K_2(p,q)\left[\lambda\theta(d(y,v))+\mu\theta(d(x,u))\right]dq,\\
&\leq \int_a^b K_1(p,q)\left[\max\{\lambda,\mu\}\theta(d(x,u))+\max\{\lambda,\mu\}\theta(d(y,v))\right]dq\\
&\quad-\int_a^b K_2(p,q)\left[\max\{\lambda,\mu\}\theta(d(y,v))+\max\{\lambda,\mu\}\theta(d(x,u))\right]dq.
\end{aligned}
$$

Now, taking the supremum with respect to p and making use of (3.4), we get

$$
\begin{aligned}
&d(F(x,y),F(u,v)) \\
\leq\ &\max\{\lambda,\mu\}\sup_{p\in I}\int_a^b\left(K_1(p,q)-K_2(p,q)\right)dq.\left[\theta(d(x,u))+\theta(d(y,v))\right]\\
\leq\ &\frac{\theta(d(x,u))+\theta(d(y,v))}{4}.
\end{aligned}
\tag{3.6}
$$

Since θ is non-decreasing, we have

$$
\theta(d(x,u)) \leq \theta\left(\max\left\{d(x,u),d(y,v)\right\}\right),
$$

$$\theta(d(y,v)) \quad \leq \quad \theta\left(\max\left\{d(x,u), d(y,v)\right\}\right),$$

which implies (due to (ii_θ)) that

$$\begin{aligned} \frac{\theta(d(x,u)) + \theta(d(y,v))}{2} &\leq \theta\left(\max\left\{d(x,u), d(y,v)\right\}\right) \\ &\leq \max\left\{d(x,u), d(y,v)\right\}, \end{aligned}$$

so that (owing to (3.6), we have

$$d(F(x,y), F(u,v)) \leq \frac{1}{2}\max\left\{d(x,u), d(y,v)\right\}. \tag{3.7}$$

Now, on making use of (3.7), it follows that

$$\begin{aligned} M(F(x,y), F(u,v), \frac{t}{2}) &= e^{-\frac{d(F(x,y),F(u,v))}{\frac{t}{2}}} \\ &\geq e^{-\frac{\frac{1}{2}\max\{d(x,u),d(y,v)\}}{\frac{t}{2}}} \\ &= e^{-\frac{\max\{d(x,u),d(y,v)\}}{t}} \\ &\geq \min\left\{e^{-\frac{d(x,u)}{t}}, e^{-\frac{d(y,v)}{t}}\right\} \\ &= \min\left\{M(Sx,Tu,t), M(Sy,Tv,t)\right\} \\ &\geq m^{x,y}_{u,v}. \end{aligned}$$

Thus the involved contractive condition of Theorem 2.4 is satisfied if we set $F = G$ and $Sx = Tx = x$, Also, it is straightforward to notice that all the hypotheses of Theorem 2.4 are satisfied and henceforth F has a coupled fixed point $(x,x) \in X^2$ which also remains the solution of the integral equation (3.1). □

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