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SCREEN PSEUDO-SLANT LIGHTLIKE SUBMANIFOLDS OF INDEFINITE KAEHLER MANIFOLDS

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Abstract. In this paper we introduce the notion of screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds giving characterization theorem with some non-trivial examples of such submanifolds. Integrability conditions of distributions D_1, D_2 and $RadTM$ on screen pseudo-slant lightlike submanifolds of an indefinite Kaehler manifold have been obtained. Further, we obtain necessary and sufficient conditions for foliations determined by above distributions to be totally geodesic.

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1. Introduction

The theory of lightlike submanifolds of a semi-Riemannian manifold was introduced by Duggal and Bejancu [3]. In [2], B.Y. Chen defined slant immersions in complex geometry as a natural generalization of both holomorphic immersions and totally real immersions. In [5], A. Lotta introduced the concept of slant immersion of a Riemannian manifold into an almost contact metric manifold. A. Carriazo defined and studied bi-slant submanifolds of almost Hermitian and almost contact metric manifolds and further gave the notion of pseudo-slant submanifolds [1]. On other hand, the theory of invariant, screen slant, screen real, screen Cauchy-Riemann lightlike submanifolds have been studied in [4]. Thus motivated sufficiently, we introduce the notion of screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds. This new class of lightlike submanifolds of an indefinite Kaehler manifold includes invariant, screen slant, screen real, screen Cauchy-Riemann lightlike submanifolds as its subcases. The paper is arranged as follows. There are some basic results in section 2. In section 3, we study screen pseudo-slant lightlike submanifolds of an indefinite Kaehler manifold, giving some examples. Section 4 is devoted to the study of foliations determined by distributions on screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds.

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2. Preliminaries

A submanifold (M^m, g) immersed in a semi-Riemannian manifold $(\overline{M}^{m+n}, \overline{g})$ is called a lightlike submanifold [3] if the metric g induced from $\overline{g}$ is degenerate and the radical distribution $RadTM$ is of rank r, where $1 \leq r \leq m$. Let $S(TM)$ be a screen distribution which is a semi-Riemannian complementary distribution of $RadTM$ in TM, that is

$$TM = RadTM \oplus_{orth} S(TM). \tag{2.1}$$

Now consider a screen transversal vector bundle $S(TM^{\perp})$, which is a semi-Riemannian complementary vector bundle of $RadTM$ in $TM^{\perp}$. Since for any local basis $\{\xi_i\}$ of $RadTM$, there exists a local null frame $\{N_i\}$ of sections with values in the orthogonal complement of $S(TM^{\perp})$ in $[S(TM)]^{\perp}$ such that $\overline{g}(\xi_i, N_j) = \delta_{ij}$ and $\overline{g}(N_i, N_j) = 0$, it follows that there exists a lightlike transversal vector bundle $ltr(TM)$ locally spanned by $\{N_i\}$. Let $tr(TM)$ be complementary (but not orthogonal) vector bundle to TM in $T\overline{M}|_M$. Then

$$tr(TM) = ltr(TM) \oplus_{orth} S(TM^{\perp}), \tag{2.2}$$

$$T\overline{M}|_M = TM \oplus tr(TM), \tag{2.3}$$

$$T\overline{M}|_M = S(TM) \oplus_{orth} [RadTM \oplus ltr(TM)] \oplus_{orth} S(TM^{\perp}). \tag{2.4}$$

Following are four cases of a lightlike submanifold $\big(M, g, S(TM), S(TM^{\perp})\big)$:
Case.1 r-lightlike if $r < min\,(m, n)$,
Case.2 co-isotropic if $r = n < m$, $S\left(TM^{\perp}\right) = \{0\}$,
Case.3 isotropic if $r = m < n$, $S\left(TM\right) = \{0\}$,
Case.4 totally lightlike if $r = m = n$, $S(TM) = S(TM^{\perp}) = \{0\}$.
The Gauss and Weingarten formulae are given as

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \tag{2.5}$$

$$\overline{\nabla}_X V = -A_V X + \nabla^t_X V, \tag{2.6}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(tr(TM))$, where $\nabla_X Y, A_V X$ belong to $\Gamma(TM)$ and $h(X, Y), \nabla^t_X V$ belong to $\Gamma(tr(TM))$. ∇ and ∇^t are linear connections on M and on the vector bundle $tr(TM)$, respectively. The second fundamental form h is a symmetric $F(M)$-bilinear form on $\Gamma(TM)$ with values in $\Gamma(tr(TM))$ and the shape operator A_V is a linear endomorphism of $\Gamma(TM)$. From (2.5) and (2.6), for any $X, Y \in \Gamma(TM)$, $N \in \Gamma(ltr(TM))$ and $W \in \Gamma(S(TM^{\perp}))$, we have

$$\overline{\nabla}_X Y = \nabla_X Y + h^l\left(X, Y\right) + h^s\left(X, Y\right), \tag{2.7}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^l_X N + D^s\left(X, N\right), \tag{2.8}$$

$$(2.9)\qquad \overline{\nabla}_X W = -A_W X + \nabla^s_X W + D^l\,(X, W)\,,$$

where $h^l(X,Y) = L\,(h(X,Y))$, $h^s(X,Y) = S\,(h(X,Y))$, $D^l(X,W) = L(\nabla^t_X W)$, $D^s(X,N) = S(\nabla^t_X N)$. L and S are the projection morphisms of $tr(TM)$ on $ltr(TM)$ and $S(TM^{\perp})$, respectively. ∇^l and ∇^s are linear connections on $ltr(TM)$ and $S(TM^{\perp})$ called the lightlike connection and screen transversal connection on M, respectively.
Now by using (2.5), (2.7)-(2.9) and metric connection $\overline{\nabla}$, we obtain

$$(2.10)\qquad \overline{g}(h^s(X,Y),W) + \overline{g}(Y, D^l(X,W)) = g(A_W X, Y),$$

$$(2.11)\qquad \overline{g}(D^s(X,N),W) = \overline{g}(N, A_W X).$$

Denote the projection of TM on $S(TM)$ by $\overline{P}$. Then from the decomposition of the tangent bundle of a lightlike submanifold, for any $X, Y \in \Gamma(TM)$ and $\xi \in \Gamma(RadTM)$, we have

$$(2.12)\qquad \nabla_X \overline{P}Y = \nabla^*_X \overline{P}Y + h^*(X, \overline{P}Y),$$

$$(2.13)\qquad \nabla_X \xi = -A^*_\xi X + \nabla^{*t}_X \xi,$$

By using above equations, we obtain

$$(2.14)\qquad \overline{g}(h^l(X,\overline{P}Y),\xi) = g(A^*_\xi X, \overline{P}Y),$$

$$(2.15)\qquad \overline{g}(h^*(X,\overline{P}Y),N) = g(A_N X, \overline{P}Y),$$

$$(2.16)\qquad \overline{g}(h^l(X,\xi),\xi) = 0, \qquad A^*_\xi \xi = 0.$$

It is important to note that in general ∇ is not a metric connection. Since $\overline{\nabla}$ is a metric connection, by using (2.7), we get

$$(2.17)\qquad (\nabla_X g)(Y,Z) = \overline{g}(h^l(X,Y),Z) + \overline{g}(h^l(X,Z),Y).$$

An indefinite almost Hermitian manifold $(\overline{M}, \overline{g}, \overline{J})$ is a 2m-dimensional semi-Riemannian manifold $\overline{M}$ with semi-Riemannian metric $\overline{g}$ of constant index q, $0 < q < 2m$ and a (1, 1) tensor field $\overline{J}$ on $\overline{M}$ such that following conditions are satisfied:

$$(2.18)\qquad \overline{J}^2 X = -X,$$

$$(2.19)\qquad \overline{g}(\overline{J}X, \overline{J}Y) = \overline{g}(X,Y),$$

for all $X, Y \in \Gamma(T\overline{M})$.
An indefinite almost Hermitian manifold $(\overline{M}, \overline{g}, \overline{J})$ is called an indefinite Kaehler manifold if $\overline{J}$ is parallel with respect to $\overline{\nabla}$, i.e.,

$$(2.20)\qquad (\overline{\nabla}_X \overline{J})Y = 0,$$

for all $X, Y \in \Gamma(T\overline{M})$, where $\overline{\nabla}$ is Levi-Civita connection with respect to $\overline{g}$.

3. Screen Pseudo-Slant Lightlike Submanifolds

In this section, we introduce the notion of screen pseudo-slant lightlike submanifolds of indefinite Kaehler manifolds. At first, we state the following Lemma for later use:

Lemma 3.1. *Let M be a $2q$-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$ such that $2q < dim(M)$. Then the screen distribution $S(TM)$ of lightlike submanifold M is Riemannian.*

The proof of above Lemma follows as in Lemma 3.1 of [6], so we omit it.

Definition 1. Let M be a $2q$-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ of index $2q$ such that $2q < dim(M)$. Then we say that M is a screen pseudo-slant lightlike submanifold of $\overline{M}$ if the following conditions are satisfied:
(i) $RadTM$ is invariant with respect to $\overline{J}$, i.e. $\overline{J}(RadTM) = RadTM$,
(ii) there exists non-degenerate orthogonal distributions D_1 and D_2 on M such that $S(TM) = D_1 \oplus_{orth} D_2$,
(iii) the distribution D_1 is anti-invariant, i.e. $\overline{J}D_1 \subset S(TM^\perp)$,
(iv) the distribution D_2 is slant with angle $\theta(\neq \pi/2)$, i.e. for each $x \in M$ and each non-zero vector $X \in (D_2)_x$, the angle θ between $\overline{J}X$ and the vector subspace $(D_2)_x$ is a constant$(\neq \pi/2)$, which is independent of the choice of $x \in M$ and $X \in (D_2)_x$.
This constant angle θ is called the slant angle of distribution D_2. A screen pseudo-slant lightlike submanifold is said to be proper if $D_1 \neq \{0\}$, $D_2 \neq \{0\}$ and $\theta \neq 0$.

From the above definition, we have the following decomposition

$$TM = RadTM \oplus_{orth} D_1 \oplus_{orth} D_2. \tag{3.1}$$

In particular, we have
(i) if $D_1 = 0$, then M is a screen slant lightlike submanifold,
(ii) if $D_2 = 0$, then M is a screen real lightlike submanifold,
(iii) if $D_1 = 0$ and $\theta = 0$, then M is an invariant lightlike submanifold,
(iv) if $D_1 \neq 0$ and $\theta = 0$, then M is a screen CR-lightlike submanifold.
Thus the above new class of lightlike submanifolds of an indefinite Kaehler manifold includes invariant, screen slant, screen real, screen Cauchy-Riemann lightlike submanifolds as its sub-cases which have been studied in [4].
Let $(\mathbb{R}^{2m}_{2q}, \overline{g}, \overline{J})$ denote the manifold $\mathbb{R}^{2m}_{2q}$ with its usual Kaehler structure given by
$\overline{g} = \frac{1}{4}(-\sum_{i=1}^{q} dx^i \otimes dx^i + dy^i \otimes dy^i + \sum_{i=q+1}^{m} dx^i \otimes dx^i + dy^i \otimes dy^i)$,
$\overline{J}(\sum_{i=1}^{m}(X_i \partial x_i + Y_i \partial y_i)) = \sum_{i=1}^{m}(Y_i \partial x_i - X_i \partial y_i)$,
where (x^i, y^i) are the cartesian coordinates on $\mathbb{R}^{2m}_{2q}$. Now, we construct some examples of screen pseudo-slant lightlike submanifolds of an indefinite Kaehler manifold.

Example 1. Let $(\mathbb{R}_2^{12}, \overline{g}, \overline{J})$ be an indefinite Kaehler manifold, where $\overline{g}$ is of signature $(-,+,+,+,+,+,-,+,+,+,+,+)$ with respect to the canonical basis $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6\}$.
Suppose M is a submanifold of $\mathbb{R}_2^{12}$ given by $x^1 = y^2 = u_1$, $x^2 = -y^1 = u_2$, $x^3 = u_3 \cos\beta$, $y^3 = u_3 \sin\beta$, $x^4 = u_4 \sin\beta$, $y^4 = u_4 \cos\beta$, $x^5 = u_5 \cos\theta$, $y^5 = u_6 \cos\theta$, $x^6 = u_6 \sin\theta$, $y^6 = u_5 \sin\theta$.
The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6\}$, where

$Z_1 = 2(\partial x_1 + \partial y_2)$, $\quad Z_2 = 2(\partial x_2 - \partial y_1)$,

$Z_3 = 2(\cos\beta \partial x_3 + \sin\beta \partial y_3)$, $Z_4 = 2(\sin\beta \partial x_4 + \cos\beta \partial y_4)$,

$Z_5 = 2(\cos\theta \partial x_5 + \sin\theta \partial y_6)$, $Z_6 = 2(\sin\theta \partial x_6 + \cos\theta \partial y_5)$.

Hence $RadTM = span\{Z_1, Z_2\}$ and $S(TM) = span\{Z_3, Z_4, Z_5, Z_6\}$.
Now $ltr(TM)$ is spanned by $N_1 = -\partial x_1 + \partial y_2$, $\quad N_2 = -\partial x_2 - \partial y_1$. $S(TM^\perp)$ is spanned by

$W_1 = 2(\sin\beta \partial x_3 - \cos\beta \partial y_3)$, $W_2 = 2(\cos\beta \partial x_4 - \sin\beta \partial y_4)$,

$W_3 = 2(\sin\theta \partial x_5 - \cos\theta \partial y_6)$, $W_4 = 2(\cos\theta \partial x_6 - \sin\theta \partial y_5)$.

It follows that $\overline{J}Z_1 = Z_2$ and $\overline{J}Z_2 = -Z_1$, which implies that $RadTM$ is invariant, i.e. $\overline{J}RadTM = RadTM$. On the other hand, we can see that $D_1 = span\{Z_3, Z_4\}$ such that $\overline{J}Z_3 = W_1$ and $\overline{J}Z_4 = W_2$, which implies that D_1 is anti-invariant with respect to $\overline{J}$ and $D_2 = span\{Z_5, Z_6\}$ is a slant distribution with slant angle 2θ. Hence M is a screen pseudo-slant 2-lightlike submanifold of $\mathbb{R}_2^{12}$.

Example 2. Let $(\mathbb{R}_2^{12}, \overline{g}, \overline{J})$ be an indefinite Kaehler manifold, where $\overline{g}$ is of signature $(-,+,+,+,+,+,-,+,+,+,+,+)$ with respect to the canonical basis $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6\}$.
Suppose M is a submanifold of $\mathbb{R}_2^{12}$ given by $x^1 = u_1$, $y^1 = -u_2$, $x^2 = -u_1 \cos\alpha - u_2 \sin\alpha$, $y^2 = -u_1 \sin\alpha + u_2 \cos\alpha$, $x^3 = y^4 = u_3$, $x^4 = y^3 = u_4$, $x^5 = u_5 \cos u_6$, $y^5 = u_5 \sin u_6$, $x^6 = \cos u_5$, $y^6 = \sin u_5$.
The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6\}$, where

$Z_1 = 2(\partial x_1 - \cos\alpha \partial x_2 - \sin\alpha \partial y_2)$,

$Z_2 = 2(-\partial y_1 - \sin\alpha \partial x_2 + \cos\alpha \partial y_2)$,

$Z_3 = 2(\partial x_3 + \partial y_4)$, $\quad Z_4 = 2(\partial x_4 + \partial y_3)$,

$Z_5 = 2(\cos u_6 \partial x_5 + \sin u_6 \partial y_5 - \sin u_5 \partial x_6 + \cos u_5 \partial y_6)$,

$Z_6 = 2(-u_5 \sin u_6 \partial x_5 + u_5 \cos u_6 \partial y_5)$.

Hence $RadTM = span\{Z_1, Z_2\}$ and $S(TM) = span\{Z_3, Z_4, Z_5, Z_6\}$.
Now $ltr(TM)$ is spanned by $N_1 = -\partial x_1 - \cos\alpha \partial x_2 - \sin\alpha \partial y_2$, $N_2 = \partial y_1 - \sin\alpha \partial x_2 + \cos\alpha \partial y_2$. $S(TM^\perp)$ is spanned by

$W_1 = 2(\partial x_3 - \partial y_4)$, $\quad W_2 = 2(\partial x_4 - \partial y_3)$,

$W_3 = 2(\cos u_6 \partial x_5 + \sin u_6 \partial y_5 + \sin u_5 \partial x_6 - \cos u_5 \partial y_6)$,

$W_4 = 2(u_5 \cos u_5 \partial x_6 + u_5 \sin u_5 \partial y_6)$.

It follows that $\overline{J}Z_1 = Z_2$ and $\overline{J}Z_2 = -Z_1$, which implies that $RadTM$ is invariant, i.e. $\overline{J}RadTM = RadTM$. On the other hand, we can see that $D_1 = span\{Z_3, Z_4\}$ such that $\overline{J}Z_3 = W_2$ and $\overline{J}Z_4 = W_1$, which implies that D_1 is anti-invariant with respect to $\overline{J}$ and $D_2 = span\{Z_5, Z_6\}$ is a slant distribution with slant angle $\pi/4$. Hence M is a screen pseudo-slant 2-lightlike submanifold of $\mathbb{R}_2^{12}$.

Now, for any vector field X tangent to M, we put $\overline{J}X = PX + FX$, where PX and FX are tangential and transversal parts of $\overline{J}X$, respectively. We denote the projections on $RadTM$, D_1 and D_2 in TM by P_1, P_2 and P_3, respectively. Similarly, we denote the projections of $tr(TM)$ on $ltr(TM)$, $\overline{J}(D_1)$ and D' by Q_1, Q_2 and Q_3, respectively, where D' is non-degenerate orthogonal complementary subbundle of $\overline{J}(D_1)$ in $S(TM^\perp)$. Then, for any $X \in \Gamma(TM)$, we get

$$X = P_1X + P_2X + P_3X. \tag{3.2}$$

Now applying $\overline{J}$ to (3.2), we have

$$\overline{J}X = \overline{J}P_1X + \overline{J}P_2X + \overline{J}P_3X, \tag{3.3}$$

which gives

$$\overline{J}X = \overline{J}P_1X + \overline{J}P_2X + fP_3X + FP_3X, \tag{3.4}$$

where fP_3X (resp. FP_3X) denotes the tangential (resp. transversal) component of $\overline{J}P_3X$. Thus we get $\overline{J}P_1X \in \Gamma(RadTM)$, $\overline{J}P_2X \in \Gamma(\overline{J}(D_1)) \subset \Gamma(S(TM^\perp))$, $fP_3X \in \Gamma(D_2)$ and $FP_3X \in \Gamma(D')$. Also, for any $W \in \Gamma(tr(TM))$, we have

$$W = Q_1W + Q_2W + Q_3W. \tag{3.5}$$

Applying $\overline{J}$ to (3.5), we obtain

$$\overline{J}W = \overline{J}Q_1W + \overline{J}Q_2W + \overline{J}Q_3W, \tag{3.6}$$

which gives

$$\overline{J}W = \overline{J}Q_1W + \overline{J}Q_2W + BQ_3W + CQ_3W, \tag{3.7}$$

where BQ_3W (resp. CQ_3W) denotes the tangential (resp. transversal) component of $\overline{J}Q_3W$. Thus we get $\overline{J}Q_1W \in \Gamma(ltr(TM))$, $\overline{J}Q_2W \in \Gamma(D_1)$, $BQ_3W \in \Gamma(D_2)$ and $CQ_3W \in \Gamma(D')$.
Now, by using (2.20), (3.4), (3.7) and (2.7)-(2.9) and identifying the components on $RadTM$, D_1, D_2, $ltr(TM)$, $\overline{J}(D_1)$ and D', we obtain

$$\begin{aligned}\nabla^{*t}_X\overline{J}P_1Y + P_1(\nabla_X fP_3Y) =& P_1(A_{FP_3Y}X) + P_1(A_{\overline{J}P_2Y}X)\\ &+ \overline{J}P_1\nabla_XY,\end{aligned} \tag{3.8}$$

$$\begin{aligned}P_2(A^*_{\overline{J}P_1Y}X) + P_2(A_{\overline{J}P_2Y}X) + P_2(A_{FP_3Y}X) =& P_2(\nabla_X fP_3Y)\\ &- \overline{J}Q_2h^s(X,Y),\end{aligned} \tag{3.9}$$

$$\begin{aligned}P_3(A^*_{\overline{J}P_1Y}X)+P_3(A_{\overline{J}P_2Y}X) + P_3(A_{FP_3Y}X) =& P_3(\nabla_X fP_3Y)\\ &- fP_3(\nabla_XY) - BQ_3h^s(X,Y),\end{aligned} \tag{3.10}$$

$$\begin{aligned}h^l(X,\overline{J}P_1Y) + D^l(X,\overline{J}P_2Y) + h^l(X,fP_3Y) + D^l(X,FP_3Y)&\\ = \overline{J}h^l(X,Y),&\end{aligned} \tag{3.11}$$

$$Q_2\nabla^s_X\overline{J}P_2Y + Q_2\nabla^s_X FP_3Y = \overline{J}P_2\nabla_X Y - Q_2h^s(X,\overline{J}P_1Y) - Q_2h^s(X,fP_3Y), \tag{3.12}$$

$$Q_3\nabla^s_X\overline{J}P_2Y + Q_3\nabla^s_X FP_3Y - FP_3\nabla_X Y = CQ_3h^s(X,Y) - Q_3h^s(X,fP_3Y) - Q_3h^s(X,\overline{J}P_1Y). \tag{3.13}$$

Theorem 3.2. *Let M be a $2q$-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then M is a screen pseudo-slant lightlike submanifold of $\overline{M}$ if and only if*

(i) $ltr(TM)$ is invariant and D_1 is anti-invariant with respect to $\overline{J}$,

(ii) there exists a constant $\lambda \in (0,1]$ such that $P^2X = -\lambda X$.

Moreover, there also exists a constant $\mu \in [0,1)$ such that $BFX = -\mu X$, for all $X \in \Gamma(D_2)$, where D_1 and D_2 are non-degenerate orthogonal distributions on M such that $S(TM) = D_1 \oplus_{orth} D_2$ and $\lambda = \cos^2\theta$, θ is slant angle of D_2.

Proof. Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 is anti-invariant and $RadTM$ is invariant with respect to $\overline{J}$. For any $N \in \Gamma(ltr(TM))$ and $X \in \Gamma(S(TM))$, using (2.19) and (3.4), we obtain $\overline{g}(\overline{J}N,X) = -\overline{g}(N,\overline{J}X) = -\overline{g}(N,\overline{J}P_2X + fP_3X + FP_3X) = 0$. Thus $\overline{J}N$ does not belong to $\Gamma(S(TM))$. For any $N \in \Gamma(ltr(TM))$ and $W \in \Gamma(S(TM^{\perp}))$, from (2.19) and (3.7), we have $\overline{g}(\overline{J}N,W) = -\overline{g}(N,\overline{J}W) = -\overline{g}(N,\overline{J}Q_2W + BQ_3W + CQ_3W) = 0$. Hence, we conclude that $\overline{J}N$ does not belong to $\Gamma(S(TM^{\perp}))$.

Now suppose that $\overline{J}N \in \Gamma(RadTM)$. Then $\overline{J}(\overline{J}N) = \overline{J}^2N = -N \in \Gamma(ltrTM)$, which contradicts that $RadTM$ is invariant. Thus $ltr(TM)$ is invariant with respect to $\overline{J}$. Now for any $X \in \Gamma(D_2)$ we have $|PX| = |\overline{J}X|\cos\theta$, which implies

$$\cos\theta = \frac{|PX|}{|\overline{J}X|}. \tag{3.14}$$

In view of (3.14), we get $\cos^2\theta = \dfrac{|PX|^2}{|\overline{J}X|^2} = \dfrac{g(PX,PX)}{g(\overline{J}X,\overline{J}X)} = \dfrac{g(X,P^2X)}{g(X,\overline{J}^2X)}$, which gives

$$g(X,P^2X) = \cos^2\theta\, g(X,\overline{J}^2X). \tag{3.15}$$

Since M is a screen pseudo-slant lightlike submanifold, $\cos^2\theta = \lambda(constant) \in (0,1]$ and therefore from (3.15), we get $g(X,P^2X) = \lambda g(X,\overline{J}^2X) = g(X,\lambda\overline{J}^2X)$, which implies

$$g(X,(P^2 - \lambda\overline{J}^2)X) = 0. \tag{3.16}$$

Since $(P^2 - \lambda\overline{J}^2)X \in \Gamma(D_2)$ and the induced metric $g = g|_{D_2\times D_2}$ is non-degenerate (positive definite), from (3.16), we have $(P^2 - \lambda\overline{J}^2)X = 0$, which implies

$$P^2X = \lambda\overline{J}^2X = -\lambda X. \tag{3.17}$$

Now, for any vector field $X \in \Gamma(D_2)$, we have

$$\overline{J}X = PX + FX, \tag{3.18}$$

where PX and FX are tangential and transversal parts of $\overline{J}X$, respectively. Applying $\overline{J}$ to (3.18) and taking tangential component, we get

$$-X = P^2X + BFX. \tag{3.19}$$

From (3.17) and (3.19), we get

$$BFX = -\mu X, \qquad \forall X \in \Gamma(D_2), \tag{3.20}$$

where $1 - \lambda = \mu(constant) \in [0, 1)$. This proves (ii).
Conversely suppose that conditions (i) and (ii) are satisfied. We can show that $RadTM$ is invariant in similar way that $ltr(TM)$ is invariant. From (3.19), for any $X \in \Gamma(D_2)$, we get

$$-X = P^2X - \mu X, \tag{3.21}$$

which implies

$$P^2X = -\lambda X, \tag{3.22}$$

where $1 - \mu = \lambda(constant) \in (0, 1]$.
Now $\cos\theta = \frac{g(\overline{J}X,PX)}{|\overline{J}X||PX|} = -\frac{g(X,\overline{J}PX)}{|\overline{J}X||PX|} = -\frac{g(X,P^2X)}{|\overline{J}X||PX|} = -\lambda\frac{g(X,\overline{J}^2X)}{|\overline{J}X||PX|} = \lambda\frac{g(\overline{J}X,\overline{J}X)}{|\overline{J}X||PX|}$.
From above equation, we get

$$\cos\theta = \lambda\frac{|\overline{J}X|}{|PX|}. \tag{3.23}$$

Therefore (3.14) and (3.23) give $\cos^2\theta = \lambda(constant)$.
Hence M is a screen pseudo-slant lightlike submanifold. □

Corollary 3.1. *Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$ with slant angle θ, then for any $X, Y \in \Gamma(D_2)$, we have*
(i) $g(PX, PY) = \cos^2\theta\, g(X, Y)$,
(ii) $g(FX, FY) = \sin^2\theta\, g(X, Y)$.

The proof of above Corollary follows by using similar steps as in proof of Corollary 3.2 of [6].

Theorem 3.3. *Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $RadTM$ is integrable if and only if*
(i) $Q_2h^s(Y, \overline{J}P_1X) = Q_2h^s(X, \overline{J}P_1Y)$,
(ii) $Q_3h^s(Y, \overline{J}P_1X) = Q_3h^s(X, \overline{J}P_1Y)$,
(iii) $P_3A^*_{\overline{J}P_1X}Y = P_3A^*_{\overline{J}P_1Y}X$, for all $X, Y \in \Gamma(RadTM)$.

Proof. Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Let $X, Y \in \Gamma(RadTM)$. From (3.12), we have $Q_2h^s(X, \overline{J}P_1Y) = \overline{J}P_2\nabla_XY$, which gives $Q_2h^s(X, \overline{J}P_1Y) - Q_2h^s(Y, \overline{J}P_1X) = \overline{J}P_2[X, Y]$. In view of (3.13), we get $Q_3h^s(X, \overline{J}P_1Y) = CQ_3h^s(X, Y) + FP_3\nabla_XY$, which implies $Q_3h^s(X, \overline{J}P_1Y) - Q_3h^s(Y, \overline{J}P_1X) = FP_3[X, Y]$. Also from (3.10), we have $P_3A^*_{\overline{J}P_1Y}X = fP_3\nabla_XY + BQ_3h^s(X, Y)$, which gives $P_3A^*_{\overline{J}P_1Y}X - P_3A^*_{\overline{J}P_1X}Y = fP_3[X, Y]$. Thus, we obtain the required results. □

Theorem 3.4. *Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 is integrable if and only if*

(i) $P_1A_{\overline{J}P_2X}Y = P_1A_{\overline{J}P_2Y}X$ and $P_3A_{\overline{J}P_2X}Y = P_3A_{\overline{J}P_2Y}X$,

(ii) $Q_3(\nabla^s_Y\overline{J}P_2X) = Q_3(\nabla^s_X\overline{J}P_2Y)$, for all $X, Y \in \Gamma(D_1)$.

Proof. Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Let $X, Y \in \Gamma(D_1)$. From (3.8), we have $P_1A_{\overline{J}P_2Y}X = -\overline{J}P_1\nabla_XY$, which gives $P_1A_{\overline{J}P_2X}Y - P_1A_{\overline{J}P_2Y}X = \overline{J}P_1[X, Y]$. In view of (3.10), we obtain $P_3A_{\overline{J}P_2Y}X + BQ_3h^s(X, Y) = -fP_3\nabla_XY$, which implies $P_3A_{\overline{J}P_2X}Y - P_3A_{\overline{J}P_2Y}X = fP_3[X, Y]$. Also from (3.13), we get $Q_3(\nabla^s_X\overline{J}P_2Y) + CQ_3h^s(X, Y) = -FP_3\nabla_XY$, which gives $Q_3(\nabla^s_Y\overline{J}P_2X) - Q_3(\nabla^s_X\overline{J}P_2Y) = FP_3[X, Y]$. This proves the theorem. □

Theorem 3.5. *Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_2 is integrable if and only if*

(i) $P_1(\nabla_XfP_3Y - \nabla_YfP_3X) = P_1(A_{FP_3Y}X - A_{FP_3X}Y)$,

(ii) $Q_2(\nabla^s_XFP_3Y - \nabla^s_YFP_3X) = Q_2(h^s(Y, fP_3X) - h^s(X, fP_3Y))$,

for all $X, Y \in \Gamma(D_2)$.

Proof. Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Let $X, Y \in \Gamma(D_2)$. In view of (3.8), we get $P_1(\nabla_XfP_3Y) = P_1(A_{FP_3Y}X) + \overline{J}P_1\nabla_XY$, thus $P_1(\nabla_XfP_3Y) - P_1(\nabla_YfP_3X) - P_1(A_{FP_3Y}X) + P_1(A_{FP_3X}Y) = \overline{J}P_1[X, Y]$. From (3.12), we get $Q_2\nabla^s_XFP_3Y + Q_2h^s(X, fP_3Y) = \overline{J}P_2\nabla_XY$, which implies $Q_2\nabla^s_XFP_3Y - Q_2\nabla^s_YFP_3X + Q_2h^s(X, fP_3Y) - Q_2h^s(Y, fP_3X) = \overline{J}P_2[X, Y]$. This concludes the theorem. □

Theorem 3.6. *Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then the induced connection ∇ is a metric connection if and only if*

(i) $\overline{J}Q_2h^s(X, Y) = 0$ and $BQ_3h^s(X, Y) = 0$,

(ii) A^*_Y vanishes on $\Gamma(TM)$, for all $X \in \Gamma(TM)$ and $Y \in \Gamma(RadTM)$.

Proof. Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then the induced connection ∇ on M is a metric connection if and only if $RadTM$ is a parallel distribution with respect to ∇ [6]. From (2.7), (2.13) and (2.20), for any $X \in \Gamma(TM)$ and $Y \in \Gamma(RadTM)$, we have $\overline{\nabla}_X\overline{J}Y = \overline{J}\nabla^{*t}_XY - \overline{J}A^*_YX + \overline{J}h^l(X, Y) + \overline{J}Q_2h^s(X, Y) + \overline{J}Q_3h^s(X, Y)$. On comparing tangential components of both sides of the above equation, we get $\nabla_X\overline{J}Y = \overline{J}\nabla^{*t}_XY - \overline{J}A^*_YX + \overline{J}Q_2h^s(X, Y) + BQ_3h^s(X, Y)$, which completes the proof. □

4. Foliations Determined by Distributions

In this section, we obtain necessary and sufficient conditions for foliations determined by distributions on a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold to be totally geodesic.

Definition 2. A screen pseudo-slant lightlike submanifold M of an indefinite Kaehler manifold $\overline{M}$ is said to be a mixed geodesic screen pseudo-slant lightlike submanifold if its second fundamental form h satisfies $h(X,Y) = 0$, for all $X \in \Gamma(D_1)$ and $Y \in \Gamma(D_2)$. Thus M is mixed geodesic screen pseudo-slant lightlike submanifold if $h^l(X,Y) = 0$ and $h^s(X,Y) = 0$, for all $X \in \Gamma(D_1)$ and $Y \in \Gamma(D_2)$.

Theorem 4.1. *Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $RadTM$ defines a totally geodesic foliation if and only if $\overline{g}(D^l(X,P_2Z) + D^l(X,FP_3Z),\overline{J}Y) = -\overline{g}(h^l(X,fP_3Z),\overline{J}Y)$, for all $X,Y \in \Gamma(RadTM)$ and $Z \in \Gamma(S(TM))$.*

Proof. Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. To prove the distribution $RadTM$ defines a totally geodesic foliation, it is sufficient to show that $\nabla_X Y \in \Gamma(RadTM)$, for all $X,Y \in \Gamma(RadTM)$. Since $\overline{\nabla}$ is a metric connection, using (2.7), (2.19), (2.20) and (3.4), for any $X,Y \in \Gamma(RadTM)$ and $Z \in \Gamma(S(TM))$, we get $\overline{g}(\nabla_X Y,Z) = -\overline{g}(\overline{\nabla}_X(\overline{J}P_2Z + fP_3Z + FP_3Z),\overline{J}Y)$, which gives $\overline{g}(\nabla_X Y,Z) = -\overline{g}(D^l(X,\overline{J}P_2Z)+h^l(X,fP_3Z)+D^l(X,FP_3Z),\overline{J}Y)$. This completes the proof. □

Theorem 4.2. *Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 defines a totally geodesic foliation if and only if*

(i) $\overline{g}(h^s(X,fZ),\overline{J}Y) = -\overline{g}(\nabla^s_X FZ,\overline{J}Y)$,

(ii) $D^s(X,\overline{J}N)$ has no component in $\overline{J}(D_1)$,

for all $X,Y \in \Gamma(D_1)$, $Z \in \Gamma(D_2)$ and $N \in \Gamma(ltr(TM))$.

Proof. Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. It is easy to see that the distribution D_1 defines a totally geodesic foliation if and only if $\nabla_X Y \in \Gamma(D_1)$, for all $X,Y \in \Gamma(D_1)$. Since $\overline{\nabla}$ is a metric connection, using (2.7), (2.19) and (2.20), for any $X,Y \in \Gamma(D_1)$ and $Z \in \Gamma(D_2)$, we obtain $\overline{g}(\nabla_X Y,Z) = -\overline{g}(\overline{\nabla}_X\overline{J}Z,\overline{J}Y)$, which gives $\overline{g}(\nabla_X Y,Z) = \overline{g}(h^s(X,fZ) + \nabla^s_X FZ,\overline{J}Y)$. Now, from (2.7), (2.19) and (2.20), for all $X,Y \in \Gamma(D_1)$ and $N \in \Gamma(ltr(TM))$, we get $\overline{g}(\nabla_X Y,N) = -\overline{g}(\overline{J}Y,\overline{\nabla}_X\overline{J}N)$, which implies $\overline{g}(\nabla_X Y,N) = -\overline{g}(\overline{J}Y,D^s(X,\overline{J}N))$. Thus, the theorem is completed. □

Theorem 4.3. *Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_2 defines a totally geodesic foliation if and only if*

(i) $\overline{g}(fY,A_{\overline{J}Z}X) = \overline{g}(FY,\nabla^s_X\overline{J}Z)$,

(ii) $\overline{g}(fY,A_{\overline{J}N}X) = \overline{g}(FY,D^s(X,\overline{J}N))$,

for all $X,Y \in \Gamma(D_2)$, $Z \in \Gamma(D_1)$ and $N \in \Gamma(ltr(TM))$.

Proof. Let M be a screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then the distribution D_2 defines a totally geodesic foliation if and only if $\nabla_X Y \in \Gamma(D_2)$, for all $X, Y \in \Gamma(D_2)$. Since $\overline{\nabla}$ is a metric connection, using (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_2)$ and $Z \in \Gamma(D_1)$, we get $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}Z)$, which gives $\overline{g}(\nabla_X Y, Z) = \overline{g}(fY, A_{\overline{J}Z}X) - \overline{g}(FY, \nabla^s_X \overline{J}Z)$. In view of (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_2)$ and $N \in \Gamma(ltr(TM))$, we obtain $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}N)$, which implies $\overline{g}(\nabla_X Y, N) = \overline{g}(fY, A_{\overline{J}N}X) - \overline{g}(FY, D^s(X, \overline{J}N))$. Thus, we obtain the required results. □

Theorem 4.4. *Let M be a mixed geodesic screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 defines a totally geodesic foliation if and only if $\nabla^s_X FZ$ and $D^s(X, \overline{J}N)$ have no components in $\overline{J}(D_1)$, for all $X \in \Gamma(D_1)$, $Z \in \Gamma(D_2)$ and $N \in \Gamma(ltr(TM))$.*

Proof. Let M be a mixed geodesic screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then the distribution D_1 defines a totally geodesic foliation if and only if $\nabla_X Y \in \Gamma(D_1)$, for all $X, Y \in \Gamma(D_1)$. Since $\overline{\nabla}$ is a metric connection, using (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $Z \in \Gamma(D_2)$, we get $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}Z)$, which gives $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\nabla^s_X FZ + h^s(X, fZ), \overline{J}Y)$. Now, from (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $N \in \Gamma(ltr(TM))$, we obtain $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X \overline{J}N)$, which implies $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, D^s(X, \overline{J}N))$. This concludes the theorem. □

Theorem 4.5. *Let M be a mixed geodesic screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then the induced connection ∇ on $S(TM)$ is a metric connection if and only if*

(i) $\overline{g}(fW, A^*_{\overline{J}\xi} Z) = \overline{g}(FW, h^s(Z, \overline{J}\xi))$,

(ii) $h^s(X, \overline{J}\xi)$ has no component in $\overline{J}(D_1)$,

for all $X \in \Gamma(D_1)$, $Z, W \in \Gamma(D_2)$ and $\xi \in \Gamma(RadTM)$.

Proof. Let M be a mixed geodesic screen pseudo-slant lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $h^l(X, Z) = 0$, for all $X \in \Gamma(D_1)$ and $Z \in \Gamma(D_2)$. Since $\overline{\nabla}$ is a metric connection, using (2.7), (2.19) and (2.20), for any $X, Y \in \Gamma(D_1)$ and $\xi \in \Gamma(RadTM)$, we obtain $\overline{g}(h^l(X, Y), \xi) = -g(\overline{J}Y, \overline{\nabla}_X \overline{J}\xi)$, which implies $\overline{g}(h^l(X, Y), \xi) = -g(\overline{J}Y, h^s(X, \overline{J}\xi))$. In view of (2.7), (2.19) and (2.20), for any $Z, W \in \Gamma(D_2)$ and $\xi \in \Gamma(RadTM)$, we get $\overline{g}(h^l(Z, W), \xi) = -\overline{g}(fW, \nabla_Z \overline{J}\xi) - \overline{g}(FW, h^s(Z, \overline{J}\xi))$, thus $\overline{g}(h^l(Z, W), \xi) = \overline{g}(fW, A^*_{\overline{J}\xi} Z) - \overline{g}(FW, h^s(Z, \overline{J}\xi))$. This completes the proof.

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