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K^{th} ROOT TRANSFORMATIONS FOR SOME SUBCLASSES OF ALPHA CONVEX FUNCTIONS DEFINED THROUGH CONVOLUTION

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Abstract. In this paper we introduce a new subclass of analytic functions defined through convolution. We obtain the sharp upper bounds for the coefficient functional corresponding to the k^{th} root transformation for the function f in this class. Similar problems are investigated for the inverse function and $\frac{z}{f(z)}$. The results of this paper generalise the work of earlier researchers in this direction.

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1. Introduction

Let $\mathcal{A}$ be the class of all functions $f(z)$ analytic in the open unit disk $\triangle = [z \in C :| z |< 1]$ normalized by $f(0) = 0$ and $f'(0) = 1$. Let $f(z)$ be a function in the class $\mathcal{A}$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n; \tag{1.1}$$

Let $\mathbb{S}$ be the subclass of $\mathcal{A}$, consisting of univalent functions. For a univalent function $f(z)$ of the form (1.1), the k^{th} root transformation is defined by

$$F(z) = [f(z^k)]^{\frac{1}{k}} = z + \sum_{n=1}^{\infty} b_{nk+1} z^{nk+1} \tag{1.2}$$

Let B_o be the family of analytic functions $w(z)$ in $\triangle$ with $w(0) = 0$ and $|w(z)| \leq 1$. We write $f \prec g$ if there exists a Schwartz function $w(z)$ in B_o such that $f(z) = g(w(z)) \forall\, z \in \Delta$.

During the last century a lot of work has been done in the direction of finding upper bounds for a_2, a_3 and $|a_3 - \mu a_2^2|$ for the function f in certain subclasses of $\mathcal{A}$, for some real or complex μ. This work was initiated by Fekete

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and Szego [3]. A classical result of Fekete - Szego [3] determines the maximum value of $|a_3-\mu a_2^2|$ as a function of real parameter μ for the subclass $\mathbb{S}$ of $\mathcal{A}$. This is known as Fekete - Szego inequality. Here $|a_3-\mu a_2^2|$ is called as Fekete - Szego coefficient functional. Pfluger [9] used Jenkins method to show that this result holds for complex μ such that $Re\{\frac{\mu}{1-\mu}\} \geq 0$. Keogh and Merks [4] obtained the solution of the Fekete-Szego problem for the class of close-to-convex functions.

2. Definitions

Definition 2.1. Let $\phi(z)$ be a univalent, analytic function with positive real part on Δ with $\phi(0) = 1, \phi'(0) > 0$ where $\phi(z)$ maps Δ onto a region starlike with respect to 1 and is symmetric with respect to the real axis. Such a function ϕ has a series expansion of the form $\phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots$ with $B_1 > 0,\ B_2 \geq 0$ and $B_n' s$ are real.

Ma and Minda [5, 6] gave a complete answer to the Fekete-Szego problem for the classes of strongly close-to-convex functions and strongly starlike functions. V.Ravichandran et al. [10] have further generalized the classes by defining $S_b^\star(\phi)$ to be the class of all functions $f \in \mathbb{S}$ for which

$$1 + \frac{1}{b}[\frac{zf'(z)}{f(z)} - 1] \prec \phi(z),$$

and $C_b(\phi)$ to be the class of functions $f \in \mathbb{S}$ for which

$$1 + \frac{1}{b}[\frac{zf''(z)}{f'(z)}] \prec \phi(z)$$

where b is a non-zero complex number.

Sharma and Ram Reddy [11, 12] have further generalized the classes defined by $S_b^{\star\gamma}(\phi)$ to be the class of all functions $f \in \mathbb{S}$ for which

$$1 + \frac{1}{b}[\frac{zf'(z)}{f(z)} - 1] \prec [\phi(z)]^\gamma,$$

and $C_b^\gamma(\phi)$ to be the class of all functions $f \in \mathbb{S}$ for which

$$1 + \frac{1}{b}[\frac{zf''(z)}{f'(z)}] \prec [\phi(z)]^\gamma,$$

where b is a non-zero complex number and γ is a real number with $0 < \gamma \leq 1$. For any two functions f analytic in $\mid z \mid < R_1$ and g analytic in $\mid z \mid < R_2$ with two power series expansions, $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ and $g(z) = z + \sum_{k=2}^{\infty} b_k z^k$, the convolution or Hadamard product of f and g is defined as

$$(f * g)(z) = \sum_{k=2}^{\infty} a_k b_k z^k \tag{2.1}$$

and $(f * g)$ is analytic in $| z |< R_1 R_2$.

Recently R.M.Ali et al. [1] have considered the following classes of functions viz

$$R_b(\phi) = \{f \in A : 1 + \frac{1}{b}[f'(z) - 1] \prec \phi(z)\}$$

$$S^\star(\alpha, \phi) = \{f \in A : [\frac{zf'(z)}{f(z)} + \alpha\frac{z^2 f''(z)}{f'(z)}] \prec \phi(z)\}$$

$$L(\alpha, \phi) = \{f \in A : [\frac{zf'(z)}{f(z)}]^\alpha[1 + \frac{zf''(z)}{f'(z)}]^{1-\alpha} \prec \phi(z)\}$$

$$M(\alpha, \phi) = \{f \in A : (1 - \alpha)\{\frac{zf'(z)}{f(z)}\} + \alpha\{1 + \frac{zf''(z)}{f'(z)}\} \prec \phi(z)\},$$

where $z \in \Delta, b \in C - \{0\}$ and $\alpha \geq 0$. Functions in the class $L(\alpha, \phi)$ are called logarithmic $\alpha-$ convex functions with respect to ϕ and the functions in the class $M(\alpha, \phi)$ are called $\alpha-$ convex functions with respect to ϕ. They have obtained the sharp upper bounds for the Fekete - Szego coefficient functional associated with the k^{th} root transformation of the function f belonging to the above mentioned classes. They have also investigated a similar problem for the function $\frac{z}{f(z)}$ when the function f belongs to the above mentioned classes.

Motivated by the above mentioned work, in the present paper we define a subclass of analytic functions with complex order and obtain the k^{th} root transformation of the function f in this class. We also obtain a similar result for the inverse function and for the the function $\frac{z}{f(z)}$. The results obtained in this paper will generalize the work of earlier researchers in this direction.

Let $h,\ \varphi,\ \psi$ and χ be the subclasses of $\mathcal{A}$ and represented as

$$h(z) = z + \sum_{n=2}^{\infty} h_n z^n; \tag{2.2}$$

$$\varphi(z) = z + \sum_{n=2}^{\infty} \alpha_n z^n;$$

$$\psi(z) = z + \sum_{n=2}^{\infty} \delta_n z^n;$$

$$\chi(z) = z + \sum_{n=2}^{\infty} \gamma_n z^n,$$

where $h_n > 0,\ \alpha_n > 0,\ \delta_n > 0,\ \gamma_n > 0$.

By $W^\gamma_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi; \phi)$ we denote the class of functions $f \in \mathcal{A}$ such that

$$(\varphi * f)(z)(\chi * f)(z) \neq 0 \ \ (z \in \Delta - \{0\})$$

We now define the class of functions $W^\gamma_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi; \phi)$ as follows:

Definition 2.2. Let b be a non-zero complex number, α be a real parameter with $0 \leq \alpha \leq 1$, γ be a real number with $0 < \gamma \leq 1$, ϕ be a function as defined in (1.1)and $h,\ \varphi,\ \psi$ and χ be the functions as defined in (2.2). Then the class $W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi;\phi)$ consists of all functions $f \in \mathcal{A}$ satisfying the condition

$$1+\frac{1}{b}[(1-\alpha)\{\frac{h*f}{\varphi*f}\}+\alpha\{\frac{\psi*f}{\chi*f}\}-1] \prec [\phi(z)]^{\gamma}, \tag{2.3}$$

where the powers are taken with their principle values.

It can be seen that

1. $W^{1}_{\alpha,1}(h,\ \varphi,\ \psi,\ \chi;\phi) = W_{\alpha}(h,\ \varphi,\ \psi,\ \chi;p)$ defined and studied by Jacek Dziok [2]
2. $W^{\gamma}_{0,b}((h,\varphi),(\psi,\chi);\phi) = C^{\gamma}_{g,h,b}(\phi)$ the class studied by R.B.Sharma and T.Ram Reddy [12]
3. $W^{1}_{0,1}((h,\varphi),(\psi,\chi);\phi) = M_{g,h}(\phi)$ defined and studied by G.Murugusundaramoorthy, S.Kavitha and Thomas Rosy [8].
4. $W^{\gamma}_{0,b}[(\frac{z}{(1-z)^2},\frac{z}{1-z}),(\psi,\chi);\phi] = S^{\star\ \gamma}_{b}(\phi)$ defined and studied by T.Ram Reddy and R.B.Sharma [11].
5. $W^{1}_{0,b}[(\frac{z}{(1-z)^2},\frac{z}{1-z}),(\psi,\chi);\phi] = S^{\star}_{b}(\phi)$ defined and studied by V.Ravichandran, M.Bolcal, Y.Polatoglu and A.Sen [10].
6. $W^{1}_{1,b}[(h,\varphi),(\frac{z+z^2}{(1-z)^3},\frac{z}{(1-z)^2});\phi] = C_{b}(\phi)$ defined and studied by V.Ravichandran, M.Bolcal, Y.Polatoglu and A.Sen [10].
7. $W^{1}_{0,1}[(\frac{z}{(1-z)^2},\frac{z}{1-z}),(\psi,\chi);\phi] = S^{\star}(\phi)$ defined and studied by Ma and Minda [5].
8. $W^{1}_{1,b}[(h,\varphi),(\frac{z+z^2}{(1-z)^3},\frac{z}{(1-z)^2});\phi] = C(\phi)$ defined and studied by Ma and Minda [5].
9. $W^{1}_{0,1}[(\frac{z}{(z-1)^2},z)(\psi,\chi);(\frac{1+z}{1-z})] = Re[f'(z)] > 0 = \Re$ defined and studied by Macgregor [7].

Moreover

1. $W(\psi,\chi,\phi) = W^{1}_{0,1}(h,\ \varphi,\ \psi,\ \chi;\phi)$.
2. $M_{\alpha}(\varphi,\phi) = W_{\alpha}[(z\varphi'(z),\varphi),(z(z\varphi'(z))',z\varphi'(z));\phi]$.
3. $S^{\star}(\varphi,\phi) = M_{0}(\varphi,\phi)$.
4. $S^{\star}(\phi) = S^{\star}(\frac{z}{1-z};\phi)$.

To prove our result we require the following two Lemmas

Lemma 2.3 ([10]). *If $P(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \dots$ is an analytic function with positive real part in Δ then for any complex number μ*

$$| c_2 - \mu c_1^2 | \leq 2\, max\{1, | 2\mu - 1 |\}$$

The result is sharp for the functions defined by $P(z) = \frac{1+z^2}{1-z^2}$ or $P(z) = \frac{1+z}{1-z}$.

Lemma 2.4 ([5]). *If $P(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \dots$ is an analytic function with positive real part in Δ, then for any real number ν we have*

$$| c_2 - \mu c_1^2 | \leq \begin{cases} -4\nu + 2, & \text{if } \nu \leq 0; \\ 2, & \text{if } 0 \leq \nu \leq 1; \\ 4\nu - 2, & \text{if } \nu \geq 1. \end{cases}$$

When $\nu < 0$ or $\nu > 1$ the equality holds if and only if $P(z)$ is $\frac{1+z}{1-z}$ or one of its rotations. If $0 < \nu < 1$ then the equality holds if and only if $P(z)$ is $\frac{1+z^2}{1-z^2}$ or one of its rotations. If $\nu = 0$ then the equality holds if and only if $P(z) = [\frac{1+\lambda}{2}][\frac{1+z}{1-z}] + [\frac{1-\lambda}{2}][\frac{1+z}{1-z}] (0 \leq \lambda \leq 1)$ or one of its rotations. If $\nu = 1$ the equality holds only for the reciprocal of $P(z)$ for the case $\nu = 0$. Also the above upper bound is sharp and it can be further improved as follows when $0 < \nu < 1$.

$$| c_2 - \mu c_1^2 | + \nu | c_1 |^2 \leq 2 \ (0 \leq \nu \leq \frac{1}{2})$$

$$| c_2 - \mu c_1^2 | + (1 - \nu) | c_1 |^2 \leq 2 \ (\frac{1}{2} \leq \nu \leq 1)$$

3. Main results

We now derive our main result for the function in the class $W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi; \phi)$

Theorem 3.1. *Let $f \in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi; \phi)$, $\phi(z)$ be a function as defined in (1.1), $h,\ \varphi,\ \psi\ \ \chi$ be the functions as defined in (2.2) and F be the k^{th} root transformation of f given by (1.2) then for any complex number μ*

(3.1) $$| b_{k+1} | \leq \frac{| b | \gamma B_1}{2k | \tau_{1|}}$$

(3.2) $$| b_{2k+1} | \leq \frac{| b | \gamma B_1}{k | \tau_2 |} max\{1, | 2\beta - 1 |\}$$

(3.3) $$| b_{2k+1} - \mu b_{k+1}^2 | \leq \frac{| b | \gamma B_1}{k | \tau_2 |} max\{1, | 2t - 1 |\},$$

where

(3.4) $$\beta = \frac{1}{2}\{1 - \frac{B_2}{B_1} - \frac{(\gamma - 1)}{2} - \frac{b\gamma B_1}{\tau_2^2}[\tau_3 - \frac{(k-1)}{2k}\tau_1]\}$$

$$t = \beta - \frac{b\gamma\mu B_1\tau_2}{k} \tag{3.5}$$

$$\begin{aligned}\tau_1 &= [(h_2 - \alpha_2)(1-\alpha) + \alpha(\delta_2 - \gamma_2)]\\ \tau_2 &= [(h_3 - \alpha_3)(1-\alpha) + \alpha(\delta_3 - \gamma_3)]\\ \tau_3 &= [(1-\alpha)\alpha_2(h_2 - \alpha_2) + \alpha\gamma_2(\delta_2 - \gamma_2)],\end{aligned} \tag{3.6}$$

where $h_2,\ h_3,\ \alpha_2,$ *and* α_3 *are as defined in* (2.2).

Proof. If $f \in W^{\gamma}_{\alpha,b}(\Phi,\Psi;\phi)$ then there exists a Schwartz function $w(z)$ in B_0 with $w(0) = 0$ and $\mid w(z) \mid \leq 1$ such that

$$1 + \frac{1}{b}\{\{(1-\alpha)\{\frac{h*f}{\varphi*f}\} + \alpha\{\frac{\psi*f}{\chi*f}\} - 1\}\} = [\phi(w(z))]^{\gamma}. \tag{3.7}$$

Consider

$$1+\frac{1}{b}\{(1-\alpha)\{\frac{h*f}{\varphi*f}\}+\alpha\{\frac{\psi*f}{\chi*f}\}-1\} = 1+[\frac{a_2\tau_1}{b}]z+[\frac{a_3\tau_2+a_2^2\tau_3}{b}]z^2+\ldots \tag{3.8}$$

where τ_1, τ_2 and τ_3 are as in (3.6). Define a function $P(z)$ such that

$$P(z) = \frac{1+w(z)}{1-w(z)} = 1 + w_1 z + w_2 z^2 + w_3 z^3 + \ldots$$

By substituting $w(z)$ in $\phi(z)$ and by increasing the power to γ, we have

$$[\phi(w(z))]^{\gamma} =$$

$$1 + \{\frac{\gamma B_1 w_1}{2}\}z + \{\frac{\gamma B_1}{2}[w_2 - \frac{w_1^2}{2}] + [\frac{\gamma B_2 w_1^2}{4}] + [\frac{\gamma(\gamma-1)w_1^2}{8}B_1^2]\}z^2 + \ldots \tag{3.9}$$

From equations (3.7),(3.8) and (3.9) and upon equating the coefficients of z and z^2, we have

$$a_2 = \frac{b\gamma B_1 w_1}{2\tau_1} \tag{3.10}$$

$$a_3 = \frac{b\gamma B_1 w_1}{2\tau_2}\{w_2 - \frac{w_1^2}{2}[1 - \frac{B_2}{B_1} - \frac{(\gamma-1)}{2}B_1 - \frac{b\gamma B_1\tau_3}{\tau_1^2}]\} \tag{3.11}$$

If $F(z)$ is the k^{th} root transformation of $f(z)$ then

$$\begin{aligned}F(z) &= \{f(z^k)\}^{\frac{1}{k}}\\ &= z + (\frac{a_2}{k})z^{k+1} + [\frac{a_3}{k} - \frac{(k-1)}{2k^2}a_2^2]z^{2k+1} + \ldots\\ &= z + \sum_{n=1}^{\infty} b_{nk+1} z^{nk+1}\end{aligned}$$

Upon equating the coefficients of z^{k+1}, z^{2k+1} and from equations (3.10) and (3.11), we have

$$b_{k+1} = \frac{b\gamma B_1 w_1}{2k\tau_1} \tag{3.12}$$

(3.13)

$$b_{2k+1} = \frac{b\gamma B_1}{2k\tau_2}\{w_2 - \frac{w_1^2}{2}[1 - \frac{B_2}{B_1} - \{\frac{\gamma-1}{2}\}B_1 - \frac{b\gamma B_1}{\tau_1^2}(\tau_3 - \frac{(k-1)+2\mu}{2k}\tau_2)]\}$$

Taking modulus on both sides of the equations (3.12) and (3.13) and by applying Lemma 2.3, we obtain the results defined as in (3.1) and (3.2). For any complex number μ, we have

$$[b_{2k+1} - \mu b_{k+1}^2] = \frac{b\gamma B_1}{2k\tau_2}\{w_2 - tw_1^2\}, \tag{3.14}$$

where t is defined by (3.5). Taking modulus on both sides of the equation (3.14) and applying Lemma 2.3 on the right hand side we get the result as (3.3). This proves the result of the Theorem 3.1 and the sharpness of the result follows from

$$\mid b_{2k+1} - \mu b_{k+1}^2 \mid = \begin{cases} \frac{|b|\gamma B_1}{k|\tau_2|}, & \text{if } P(z) = [\frac{1+z^2}{1-z^2}]^\gamma; \\ \frac{|b|\gamma B_1}{k|\tau_2|} \mid \{\frac{B_2}{B_1} + \frac{(\gamma-1)}{2} & \\ +\frac{b\gamma B_1}{\tau_2^2}[\tau_3 - \frac{(k-1)}{2k}\tau_1]\}, & \text{if } P(z) = [\frac{1+z}{1-z}]^\gamma. \end{cases}$$

□

Theorem 3.2. *Let $f \in W_{\alpha,b}^{\gamma}(h,\ \varphi,\ \psi,\ \chi;\phi)$, $\phi(z)$ be a function as defined in (1.1) $h,\ \varphi,\ \psi\ \ \chi$ be the functions as defined in (2.2) and F be the k^{th} root transformation of f given by (1.2) then for any real number μ and for*

$$\sigma_1 = \frac{k\tau_1^2}{\gamma B_1 \tau_2}\{-1 + \frac{B_2}{B_1} + [\frac{\gamma-1}{2}]B_1 + \frac{\gamma B_1}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}$$

$$\sigma_2 = \frac{k\tau_1^2}{\gamma B_1 \tau_2}\{1 + \frac{B_2}{B_1} + [\frac{\gamma-1}{2}]B_1 + \frac{\gamma B_1}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}$$

$$\sigma_3 = \frac{k\tau_1^2}{\gamma B_1 \tau_2}\{\frac{B_2}{B_1} + [\frac{\gamma-1}{2}]B_1 + \frac{\gamma B_1}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\},$$

where τ_1, τ_2 and τ_3 are as in (3.6) and we have

(3.15)

$$\mid b_{2k+1} - \mu b_{k+1}^2 \mid \leq \begin{cases} \frac{\gamma B_1}{k|\tau_2|}\{\frac{B_2}{B_1} + [\frac{\gamma-1}{2}]B_1 & \\ +\frac{\gamma B_1}{\tau_1^2}[\tau_3 - \frac{\tau_2}{2k}[(k-1)+2\mu]]\}, & \text{if } \mu \leq \sigma_1; \\ \frac{\gamma B_1}{k|\tau_2|}, & \text{if } \sigma_1 \leq \mu \leq \sigma_2; \\ \frac{\gamma B_1}{k|\tau_2|}\{-\frac{B_2}{B_1} - [\frac{\gamma-1}{2}]B_1 & \\ -\frac{\gamma B_1}{\tau_1^2}[\tau_3 - \frac{\tau_2}{2k}[(k-1)+2\mu]]\}, & \text{if } \mu \geq \sigma_2. \end{cases}$$

Furthermore, if $\sigma_1 \leq \mu \leq \sigma_3$, then

$$(3.16)\quad \begin{aligned}&[b_{2k+1}-\mu b_{k+1}^2]\\ &+\frac{k\tau_1}{\gamma B_1\tau_2}\{1-\frac{B_2}{B_1}-(\frac{\gamma-1}{2})-\frac{\gamma B_1}{\tau_1^2}[\tau_3-\frac{\tau_2}{2k}[(k-1)+2\mu]]\}\mid b_{k+1}\mid^2\\ &\leq\frac{\gamma B_1}{k\tau_2}\end{aligned}$$

and if $\sigma_3 \leq \mu \leq \sigma_2$, then

$$(3.17)\quad \begin{aligned}&[b_{2k+1}-\mu b_{k+1}^2]\\ &+\frac{k\tau_1}{\gamma B_1\tau_2}\{1+\frac{B_2}{B_1}+(\frac{\gamma-1}{2})B_1+\frac{\gamma B_1}{\tau_1^2}[\tau_3+\frac{\tau_2}{2k}[(k-1)+2\mu]]\}\mid b_{k+1}\mid^2\\ &\leq\frac{\gamma B_1}{k\tau_2}\end{aligned}$$

and the result is sharp.

Proof. Since $f\in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi;\phi)$, for $b=1$ and for any real number μ from equations (3.12) & (3.13) we have

$$(3.18)\qquad [b_{2k+1}-\mu b_{k+1}^2]=\frac{\gamma B_1}{2k\tau_1}\{w_2-tw_1^2\},$$

where $t=\frac{1}{2}\{1-\frac{B_2}{B_1}-[\frac{\gamma-1}{2}]B_1-\frac{\gamma B_1}{\tau_1^2}[\tau_3-\frac{\tau_2}{2k}[(k-1)+2\mu]]\}$. Taking modulus on both sides of (3.18) and applying Lemma 2.4 on the right hand side, we have the following cases

Case(1): If $\mu \leq \sigma_1$ then

$$\Rightarrow \mu\leq\frac{k\tau_1^2}{\gamma B_1\tau_3}\{-1+\frac{B_2}{B_1}+[\frac{\gamma-1}{2}]B_1+\frac{\gamma B_1}{\tau_1^2}[\tau_3-\frac{(k-1)}{2k}\tau_2]\}$$
$$\Rightarrow t\leq 0$$

$$(3.19)\qquad \Rightarrow\mid w_2-tw_1^2\mid\leq\{\frac{2B_2}{B_1}+(\gamma-1)B_1+\frac{2\gamma B_1}{\tau_2}[\tau_3-\frac{\tau_2}{2k}[(k-1)+2\mu]]\}$$

Case(2): If $\sigma_1 \leq \mu \leq \sigma_2$ then

$$\begin{aligned}&\Rightarrow\frac{k\tau_1^2}{\gamma B_1\tau_2}\{-1+\frac{B_2}{B_1}+[\frac{\gamma-1}{2}]B_1+\frac{\gamma B_1}{\tau_1^2}[\tau_3-\frac{(k-1)}{2k}\tau_2]\}\leq\mu\\ &\leq\frac{k\tau_1^2}{\gamma B_1\tau_2}\{1+\frac{B_2}{B_1}+[\frac{\gamma-1}{2}]B_1+\frac{\gamma B_1}{\tau_1^2}[\tau_3-\frac{(k-1)}{2k}\tau_2]\}\\ &\Rightarrow 0\leq t\leq 1\end{aligned}$$

$$(3.20)\qquad \Rightarrow\mid w_2-tw_1^2\mid\leq 2$$

Case(3): If $\mu \geq \sigma_2$ then

$$\Rightarrow \mu \geq \frac{k\tau_1^2}{\gamma B_1 \tau_3}\{1 + \frac{B_2}{B_1} + [\frac{\gamma - 1}{2}]B_1 + \frac{\gamma B_1}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}$$
$$\Rightarrow t \geq 1$$

$$\Rightarrow \mid w_2 - t w_1^2 \mid \leq \{-\frac{2B_2}{B_1} - (\gamma - 1)B_1 - \frac{2\gamma B_1}{\tau_2}[\tau_3 - \frac{\tau_2}{2k}[(k-1) + 2\mu]]\} \tag{3.21}$$

From equations (3.18), (3.19), (3.20) and (3.21), we obtain result (3.15).
Case(4): If $\sigma_1 \leq \mu \leq \sigma_3$, then

$$\Rightarrow \frac{k\tau_1^2}{\gamma B_1 \tau_2}\{-1 + \frac{B_2}{B_1} + [\frac{\gamma - 1}{2}]B_1 + \frac{\gamma B_1}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\} \leq \mu$$
$$\leq \frac{k\tau_1^2}{\gamma B_1 \tau_2}\{\frac{B_2}{B_1} + [\frac{\gamma - 1}{2}]B_1 + \frac{\gamma B_1}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}$$
$$\Rightarrow 0 \leq t \leq \frac{1}{2}$$

$$\Rightarrow \mid w_2 - t w_1^2 \mid + t \mid w_1 \mid^2 \leq 2. \tag{3.22}$$

We obtain the result (3.17).
Case(5): If $\sigma_3 \leq \mu \leq \sigma_2$, then

$$\Rightarrow \frac{k\tau_1^2}{\gamma B_1 \tau_2}\{\frac{B_2}{B_1} + [\frac{\gamma - 1}{2}]B_1 + \frac{\gamma B_1}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\} \leq \mu$$
$$\leq \frac{k\tau_1^2}{\gamma B_1 \tau_2}\{1 + \frac{B_2}{B_1} + [\frac{\gamma - 1}{2}]B_1 + \frac{\gamma B_1}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}$$
$$\Rightarrow \frac{1}{2} \leq t \leq 1$$

$$\Rightarrow \mid w_2 - t w_1^2 \mid + (1 - t) \mid w_1 \mid^2 \leq 2$$

We obtain the result (3.18). This completes the proof of the theorem and the sharpness of the result follows from Lemma 2.4. □

4. Coefficient Inequality for the inverse of the function $f(z)$

Theorem 4.1. *If $f \in W_{\alpha,b}^{\gamma}(h,\ \varphi,\ \psi,\ \chi; \phi)$ and $f^{-1}(w) = w + \sum_{n=2}^{\infty} d_n w^n$ is the inverse function of f with $\mid w \mid < r_0$, where r_0 is greater than the radius of the Koebe domain of the class $f \in W_{\alpha,b}^{\gamma}(h,\ \varphi,\ \psi,\ \chi; \phi)$, then for any complex number μ, we have*

$$\mid d_2 \mid \leq \frac{\mid b \mid \gamma B_1}{2k \mid \tau_1 \mid} \tag{4.1}$$

$$|\, d_3 \,| \leq \frac{|\, b \,| \gamma B_1}{|\, \tau_2 \,|} \, max\{1, |\, 2\nu_1 - 1 \,|\} \tag{4.2}$$

$$|\, d_3 - \mu d_2^2 \,| \leq \frac{|\, b \,| \gamma B_1}{|\, \tau_2 \,|} \, max\{1, |\, 2\nu_2 - 1 \,|\}, \tag{4.3}$$

where

$$\nu_1 = \frac{1}{2}\{1 - \frac{B_2}{B_1} - [\frac{\gamma - 1}{2}]B_1 - \frac{b\gamma B_1}{\tau_1^2}[\tau_3 + \tau_2]\} \tag{4.4}$$

$$\nu_2 = \frac{1}{2}\{1 - \frac{B_2}{B_1} - [\frac{\gamma - 1}{2}]B_1 - \frac{b\gamma B_1}{\tau_1^2}[\tau_3 + \frac{\tau_2}{2}(2 + \mu)]\} \tag{4.5}$$

and τ_1, τ_2 *and* τ_3 *are as in* (3.6).

Proof. As

$$f^{-1}(w) = w + \sum_{n=2}^{\infty} d_n w^n \tag{4.6}$$

is the inverse function of f, we have

$$f^{-1}\{f(z)\} = f\{f^{-1}(z)\} = z \tag{4.7}$$

From equations (4.6) and (4.7) we have

$$f^{-1}\{z + \sum_{n=2}^{\infty} a_n z^n\} = z \tag{4.8}$$

From equations (4.6) and (4.8) and upon equating the coefficient of z and z^2,we get

$$d_2 = -a_2 \tag{4.9}$$

$$d_3 = 2a_2^2 - a_3. \tag{4.10}$$

Proceeding in a way similar to Theorem 3.1 for the function f^{-1} one can obtain the results from (4.1) to (4.3). □

5. Coefficient Inequality for the function $\frac{z}{f(z)}$

Let the function G be defined by

$$G(z) = \frac{z}{f(z)} = 1 + \sum_{n=1}^{\infty} p_n z^n, \tag{5.1}$$

where $f \in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi; \phi)$.

Theorem 5.1. *If $f \in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi;\phi)$, $\phi(z)$ is a function as defined in (1.1) and $G(z) = \frac{z}{f(z)}$ then for any complex number μ, we have*

$$| p_1 |\leq \frac{| b | \gamma B_1}{2 | \tau_1 |} \tag{5.2}$$

$$| p_2 |\leq \frac{| b | \gamma B_1}{| \tau_2 |}\ max\{1, | 2\nu_3 - 1 |\} \tag{5.3}$$

$$| p_2 - \mu p_1^2 |\leq \frac{| b | \gamma B_1}{k | \tau_2 |}\ max\{1, | 2\nu_4 - 1 |\}, \tag{5.4}$$

where

$$\nu_3 = \frac{1}{2}\{1 - \frac{B_2}{B_1} - [\frac{\gamma - 1}{2}]B_1 - \frac{b\gamma B_1}{\tau_1^2}[\tau_3 - \tau_2]\} \tag{5.5}$$

$$\nu_4 = \frac{1}{2}\{1 - \frac{B_2}{B_1} - [\frac{\gamma - 1}{2}]B_1 - \frac{b\gamma B_1}{\tau_1^2}[\tau_3 - \tau_2(1-\mu)]\}. \tag{5.6}$$

Proof. As $f \in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi;\phi); G(z) = \frac{z}{f(z)}$ and by a computation we get

$$\frac{z}{f(z)} = 1 - a_2 z + \{a_2^2 - a_3\}z^2 - \ldots \tag{5.7}$$

From (3.9), (3.10) and (5.7) and upon equating the coefficient of z and z^2, we get

$$p_2 = -a_2 \tag{5.8}$$

$$p_3 = a_2^2 - a_3 \tag{5.9}$$

Proceeding in a way similar to Theorem 3.1 for the function $\frac{z}{f(z)}$ one can obtain the results from (5.3) to (5.6). □

6. Applications

Corollary 6.1. *Let $\alpha \neq -\frac{1}{2}, -1;\ b = 1; \gamma = 1$. If $f \in M^{1}_{\alpha,1}(\varphi, \phi)$ then*

$$| b_{k+1} |\leq \frac{| B_1 |}{2k(1+\alpha) | \alpha_2 |}$$

$$| b_{2k+1} |\leq \frac{| B_1 |}{2k(1+\alpha) | \alpha_3 |}\ max\{1, | 2\beta - 1 |\}$$

$$| b_{2k+1} - \mu b_{k+1}^2 |\leq \frac{| B_1 |}{2k(1+\alpha) | \alpha_3 |}\ max\{1, | 2t - 1 |\},$$

where

$$\beta = \frac{1}{2}\{1 - \frac{B_2}{B_1} - \frac{(1+3\alpha)}{(1+\alpha)^2}B_1 + \frac{\alpha_3(1+2\alpha)(k-1)}{k\alpha_2^2(1+\alpha)^2}B_1\}$$

and $t = \frac{1}{2}\{1 - \frac{B_2}{B_1} - \frac{(1+3\alpha)}{(1+\alpha)^2}B_1 + \frac{\alpha_3(1+2\alpha)B_1}{k\alpha_2^2(1+\alpha)^2}[(k-1) + 2\mu]\}$.

Proof. Let$f \in M^1_{\alpha,1}(\varphi, \phi)$, where $\chi(z) = h(z) = z\varphi'(z)$ and $\psi(z) = z(z\varphi'(z))'$, $\forall z \in \Delta$. Therefore $h_n = \gamma_n = n\alpha_n;\ \ \delta_n = n^2\alpha_n$

Hence the result follows from Theorem 3.1. $\square$

If we take $\alpha = 1,\ \alpha = 0$ in 6.1, then we have the following two corollaries (5.3) & (5.4) respectively.

Corollary 6.2. *Let $\alpha_2\alpha_3 \neq 0$. If $f \in S^c(\varphi, p)$ and $\alpha = 1,\ b = 1$ and $\gamma = 1$, then*

$$| b_{k+1} | \leq \frac{| B_1 |}{4k | \alpha_2 |}$$

$$| b_{2k+1} | \leq \frac{| B_1 |}{6k | \alpha_3 |}\, max\{1, | 2\beta - 1 |\}$$

$$| b_{2k+1} - \mu b^2_{k+1} | \leq \frac{| B_1 |}{6k | \alpha_3 |}\, max\{1, | 2t - 1 |\},$$

where

$$\beta = \frac{1}{2}\{1 - \frac{B_2}{B_1} + \frac{3\alpha_3}{4k\alpha_2^2(k-1) - 1}B_1\}$$

$$t = \frac{1}{2}\{1 - \frac{B_2}{B_1} + \frac{3\alpha_3}{4k\alpha_2^2[(k-1) + 2\mu] - 1}B_1\}.$$

The results are sharp.

Corollary 6.3. *Let $\alpha_2\alpha_3 \neq 0$,If $f \in S^c(\varphi, p)$ and $\alpha = 0,\ b = 1;\ \gamma = 1$, then*

$$| b_{k+1} | \leq \frac{| B_1 |}{2k | \alpha_2 |}$$

$$| b_{2k+1} | \leq \frac{| B_1 |}{2k | \alpha_3 |}\, max\{1, | 2\beta - 1 |\}$$

$$| b_{2k+1} - \mu b^2_{k+1} | \leq \frac{| B_1 |}{2k | \alpha_3 |}\, max\{1, | 2t - 1 |\},$$

where

$$\beta = \frac{1}{2}\{1 - \frac{B_2}{B_1} + \frac{\alpha_3}{k\alpha_2^2(k-1) - 1}B_1\}$$

$$t = \frac{1}{2}\{1 - \frac{B_2}{B_1} + \frac{\alpha_3}{k\alpha_2^2[(k-1) + 2\mu] - 1}B_1\}.$$

The results are sharp.

Corollary 6.4. *Choose the function $\phi(z) = \frac{1+Az}{1+Bz}(z \in \Delta)$ in Theorem 3.1 and let $(1-\alpha)(h_k - \alpha_k) + \alpha(\delta_k - \gamma_k) \neq 0(k = 2, 3)$; here A and B are complex numbers such that $| B |< 1,\ A \neq B$ and if $f \in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi; \frac{1+Az}{1+Bz})$ and μ is a complex number then*

$$| b_{k+1} | \leq \frac{| A - B |}{k | \tau_1 |}$$

$$| b_{2k+1} |\leq \frac{| A-B |}{k | \tau_2 |} max\{1, | 2\beta - 1 |\}$$

$$| b_{2k+1} - \mu b_{k+1}^2 |\leq \frac{| A-B |}{k | \tau_2 |} max\{1, | 2t - 1 |\},$$

where

$$\beta = \frac{1}{2}\{1 + B - \frac{(\gamma-1)(A-B)}{2} - \frac{b\gamma(A-B)}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}$$

and

$$t = \frac{1}{2}\{1 + B - \frac{(\gamma-1)(A-B)}{2} - \frac{b\gamma(A-B)}{\tau_1^2}[\tau_3 - \frac{\tau_2}{2k}[(k-1) + 2\mu]]\}$$

and τ_1, τ_2 *and* τ_3 *are as in* (3.6).

Corollary 6.5. *Choose the function* $\phi(z) = \frac{1+Az}{1+Bz}(z \in \Delta)$ *in Theorem 3.1 and let* $(1-\alpha)(h_k - \alpha_k) + \alpha(\delta_k - \gamma_k) \neq 0 (k = 2, 3)$. *Here* A *and* B *are complex numbers such that* $| B |< 1$, $A \neq B$ *and if* $f \in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi; \frac{1+Az}{1+Bz})$ *and* μ *is a real number then*

$$\sigma_1 = \frac{k\tau_1^2}{\gamma(A-B)\tau_2}\{-1 - B + [\frac{\gamma-1}{2}](A-B) + \frac{\gamma(A-B)}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}$$

$$\sigma_2 = \frac{k\tau_1^2}{\gamma(A-B)\tau_2}\{1 - B + [\frac{\gamma-1}{2}](A-B) + \frac{\gamma(A-B)}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}$$

$$\sigma_3 = \frac{k\tau_1^2}{\gamma(A-B)\tau_2}\{-B + [\frac{\gamma-1}{2}](A-B) + \frac{\gamma(A-B)}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}.$$

Furthermore if $\sigma_1 \leq \mu \leq \sigma_3$, *then*

$$[b_{2k+1} - \mu b_{k+1}^2] + \frac{k\tau_1}{\gamma(A-B)\tau_2}\{1 + B - (\frac{\gamma-1}{2})(A-B) - \frac{\gamma(A-B)}{\tau_1^2}[\tau_3 - \frac{\tau_2}{2k}[(k-1)+2\mu]]\} | b_{k+1} |^2 \leq \frac{\gamma(A-B)}{k\tau_2}$$

and if $\sigma_3 \leq \mu \leq \sigma_2$, *then*

$$[b_{2k+1} - \mu b_{k+1}^2] + \frac{k\tau_1}{\gamma(A-B)\tau_2}\{1 - B + (\frac{\gamma-1}{2})(A-B) + \frac{\gamma(A-B)}{\tau_1^2}[\tau_3 - \frac{\tau_2}{2k}[(k-1)+2\mu]]\} | b_{k+1} |^2 \leq \frac{\gamma(A-B)}{k\tau_2}$$

Let $0 \leq \theta \leq 1$ and $\phi(z) = \{\frac{1+z}{1-z}\}^{\theta}(z \in \Delta)$ and thus by Theorems 3.1, 3.2, 4.1 and Theorem 5.1, we have the following corollaries

Corollary 6.6. *If $f \in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi;\{\frac{1+z}{1-z}\}^{\theta})$ then from (3.1), we have*

$$| b_{k+1} |\leq \frac{2\theta}{k \mid \tau_1 \mid}$$

$$| b_{2k+1} |\leq \frac{2\theta}{k \mid \tau_2 \mid}\ max\{1, \mid 2\beta - 1 \mid$$

$$| b_{2k+1} - \mu b^2_{k+1} |\leq \frac{2\theta}{k \mid \tau_2 \mid}\ \max\{1, \mid 2t - 1 \mid\} \tag{6.1}$$

where

$$\beta = \frac{1}{2}\{1 - \frac{(1+\theta)}{2} - (\gamma - 1)\theta - \frac{2b\gamma\theta}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\} \tag{6.2}$$

and

$$t = \frac{1}{2}\{1 - \frac{(1+\theta)}{2} - (\gamma - 1)\theta - \frac{2b\gamma\theta}{\tau_1^2}[\tau_3 - \frac{\tau_2}{2k}[(k-1) + 2\mu]]\}.$$

Corollary 6.7. *If $f \in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi;\{\frac{1+z}{1-z}\}^{\theta})$ then from Theorem 3.2, we have*

$$\sigma_1 = \frac{k\tau_1^2}{2\gamma\theta\tau_2}\{-1 + \frac{(\theta+1)}{2} + (\gamma-1)\theta + \frac{\gamma\theta}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}$$
$$\sigma_2 = \frac{k\tau_1^2}{\gamma\theta\tau_2}\{1 + \frac{(\theta+1)}{2} + (\gamma-1)\theta + \frac{\gamma\theta}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}$$
$$\sigma_3 = \frac{k\tau_1^2}{\gamma\theta\tau_2}\{\frac{(\theta+1)}{2} + (\gamma-1)\theta + \frac{\gamma\theta}{\tau_1^2}[\tau_3 - \frac{(k-1)}{2k}\tau_2]\}.$$

Furthermore if $\sigma_1 \leq \mu \leq \sigma_3$, then

$$[b_{2k+1} - \mu b^2_{k+1}] + \frac{k\tau_1}{2\gamma\theta\tau_2}\{1 - \frac{(\theta+1)}{2} - (\gamma-1)\theta - \frac{2\gamma\theta}{\tau_1^2}[\tau_3 - \frac{\tau_2}{2k}[(k-1) + 2\mu]]\} \mid b_{k+1} \mid^2 \leq \frac{2\gamma\theta}{k\tau_2}$$

and if $\sigma_3 \leq \mu \leq \sigma_2$, then

$$[b_{2k+1} - \mu b^2_{k+1}] + \frac{k\tau_1}{2\gamma\theta\tau_2}\{1 + \frac{(\theta+1)}{2} + (\gamma-1)\theta + \frac{\gamma\theta)}{\tau_1^2}[\tau_3 - \frac{\tau_2}{2k}[(k-1) + 2\mu]]\} \mid b_{k+1} \mid^2 \leq \frac{2\gamma\theta}{k\tau_2}.$$

Corollary 6.8. *If* $f \in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi; \{\frac{1+z}{1-z}\}^{\theta})$ *then from Theorem 4.1, we have*

$$
\begin{aligned}
| d_2 | &\leq \frac{2 \mid b \mid \gamma\theta}{2k \mid \tau_1 \mid} \\
| d_3 | &\leq \frac{2 \mid b \mid \gamma\theta}{\mid \tau_2 \mid}\ max\{1, \mid 2\nu_1 - 1 \mid\} \\
| d_3 - \mu d_2^2 | &\leq \frac{2 \mid b \mid \gamma\theta}{\mid \tau_2 \mid}\ max\{1, \mid 2\nu_2 - 1 \mid\}, \\
where\ \nu_1 &= \frac{1}{2}\{1 - \frac{(\theta+1)}{2} - [\gamma-1]\theta - \frac{2b\gamma\theta}{\tau_1^2}[\tau_3 + \tau_2]\} \\
\nu_2 &= \frac{1}{2}\{1 - \frac{(\theta+1)}{2} - [\gamma-1]\theta - \frac{2b\gamma\theta}{\tau_1^2}[\tau_3 + \frac{\tau_2}{2}(2+\mu)]\}
\end{aligned}
$$

Corollary 6.9. *If* $f \in W^{\gamma}_{\alpha,b}(h,\ \varphi,\ \psi,\ \chi; \{\frac{1+z}{1-z}\}^{\theta})$ *then from Theorem 5.1, we have*

$$
\begin{aligned}
| p_1 | &\leq \frac{2 \mid b \mid \gamma\ \theta}{2 \mid \tau_1 \mid} \\
| p_2 | &\leq \frac{2 \mid b \mid \gamma\ \theta}{\mid \tau_2 \mid}\ max\{1, 2\nu_3 - 1\} \\
| p_2 - \mu p_1^2 | &\leq \frac{2 \mid b \mid \gamma\ \theta}{k \mid \tau_2 \mid}\ max\{1, 2\nu_4 - 1\}, \\
where\ \nu_3 &= \frac{1}{2}\{1 - \frac{(\theta+1)}{2} - (\frac{\gamma-1}{2})\theta - \frac{2b\gamma\ \theta}{\tau_1^2}[\tau_3 - \tau_2]\} \\
\nu_4 &= \frac{1}{2}\{1 - \frac{(\theta+1)}{2} - (\frac{\gamma-1}{2})\theta - \frac{2b\gamma\ \theta}{\tau_1^2}[\tau_3 - \tau_2(1-\mu)]\}
\end{aligned}
$$

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