Novi Sad J. Math.
Vol. 45, No. 2, 2015, 85-95

ON TOPOLOGICAL NUMBERS OF GRAPHS

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Abstract. This paper introduces the notion of discrete t-set graceful graphs and obtains some of their properties. It also examines the interrelations among different types of set-indexers, namely, *set-graceful, set-semigraceful, topologically set-graceful (t-set graceful), strongly t-set graceful* and *discrete t-set graceful* and establishes how all these notions are interdependent or not.

AMS Mathematics Subject Classification (2010): 05C78; 05C10

Key words and phrases: Set-graceful; set-semigraceful; t-set indexer; t-set graceful

1. Introduction

Acharya introduced in [1] the notion of a set-indexer of a graph as follows:
Let G be a graph and X be a nonempty set. A mapping $f : V \cup E \to 2^X$ is a *set-indexer* of G if

(i) $f(u,v) = f(u) \oplus f(v)$, for all $(u,v) \in E$, where '$\oplus$' denotes the symmetric difference of the sets in 2^X, that is, $f(u) \oplus f(v) = (f(u) \setminus f(v)) \cup (f(v) \setminus f(u))$ and

(ii) the restriction maps $f|_V$ and $f|_E$ are both injective.

In this case, X is called an *indexing set* of G. Clearly a graph can have many indexing sets and the minimum of the cardinalities of the indexing sets is said to be the *set-indexing number* of G, denoted by $\gamma(G)$. The set-indexing number of the trivial graph K_1 is defined to be zero.

He also introduced the following notions:
A graph G is *set-graceful* if $\gamma(G) = \log_2(|E|+1)$ and the corresponding set-indexer is called a *set-graceful labeling* of G.
A graph G is said to be *set-semigraceful* if $\gamma(G) = \lceil \log_2(|E|+1) \rceil$ where $\lceil\ \rceil$ is the ceiling function.

Further, Acharya and Hegde [5] obtained some noteworthy results studying set-sequential labeling as a set analogue of the sequential graphs.
A graph G is said to be *set-sequential* if there exists a nonempty set X and a

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bijective set-valued function $f\colon V \cup E \to 2^X \setminus \{\emptyset\}$ such that $f(u,v) = f(u) \oplus f(v)$ for every $(u,v) \in E$.

Later, Mollard and Payan [11] settled two conjectures about set-graceful graphs suggested by Acharya in [1]. Hegde [8] obtained certain necessary conditions for a graph to have set-graceful and set-sequential labeling. In 1999 Acharya and Hegde putforward many problems regarding set-valuation of graphs in [6]. A new momentum to this area of study was triggered by Acharya [3] in 2001. Many authors [4, 9, 13] later investigated various aspects of set-valuation of graphs deriving new properties. Hegde's [9] conjecture that every complete bipartite graph that has a set-graceful labeling is a star, was settled by Vijayakumar [20] in 2011. Motivated by this, the authors of the paper studied set-indexers of graphs in [14], [16] and [19].

Introducing the concept of topological set-indexers (t-set indexers) in [2], Acharya established a link between Graph Theory and Point Set Topology. He also propounded the notion of the topological number (t-number) of a graph as the following:
A set-indexer f of a graph G with indexing set X is said to be a *topological set-indexer (t-set indexer)* if $f(V) = \{f(v) : v \in V\}$ is a topology on X and X is called the *topological indexing set (t-indexing set)* of G. The minimum number among the cardinalities of such topological indexing sets is said to be the *topological number (t-number)* of G, denoted by $\tau(G)$ and the corresponding t-set indexer is called the *optimal t-set indexer* of G.

A graph for which the set-indexing number and the t-number are equal is termed *topologically set graceful* or *t-set graceful* by Acharya in [3].

K. L. Princy [12] contributed certain results about topological set-indexers of graphs and obtained some classes of topologically set graceful graphs in 2007. The authors of the paper studied topological set-indexers in [15] and t-set graceful graphs in [18]. Following this the authors introduced the concept of strongly t-set graceful graphs in [10] as follows:
A graph G is said to be *strongly t-set graceful*, if every spanning subgraph of G is t-set graceful.

This paper continues the study of topological numbers of graphs. It is proved that every t-set indexer of the null graph is also a t-set indexer of the star of the same order and vice-versa. A necessary condition for a t-set indexer to be optimal is derived here. A special type of strongly t-set graceful graphs, called discrete t-set graceful has been identified and certain properties of the same are studied in detail. Though the notions "discrete t-set graceful" and "set-graceful" are independent in general, they are identical in the case of a tree. The interrelations among set-semigraceful, set-graceful, t-set graceful, strongly t-set graceful and discrete t-set graceful graphs are brought out by exploring various categories of graphs.

2. Preliminaries

Certain known results needed for the subsequent development of the study are included here. We always denote a graph under consideration by G

and its vertex and edge sets by V and E respectively. By $G' \subseteq G$ we mean G' is a subgraph of G while $G' \subset G$ we mean G' is a proper subgraph of G. The empty graph of order n is denoted by N_n. The basic notations and definitions in graph theory and topology are assumed to be familiar to the reader and can be found in [7] and [21].

Theorem 2.1. ([2]) Every graph has a set-indexer.

Theorem 2.2. ([2]) If X is an indexing set of $G = (V, E)$. Then

(i) $|E| \leq 2^{|X|} - 1$ and

(ii) $\lceil \log_2(|E| + 1) \rceil \leq \gamma(G) \leq |V| - 1$, where $\lceil\ \rceil$ is the ceiling function.

Theorem 2.3. ([10]) For any graph G, $\lceil log_2|V| \rceil \leq \gamma(G)$.

Theorem 2.4. ([2]) If G' is a subgraph of G, then $\gamma(G') \leq \gamma(G)$.

Theorem 2.5. ([2]) $\gamma(K_n) = \begin{cases} n-1 & \textit{if } 1 \leq n \leq 5 \\ n-2 & \textit{if } 6 \leq n \leq 7 \end{cases}$

Theorem 2.6. ([14]) If G is a star graph, then $\gamma(G) = \lceil \log_2 |V| \rceil$.

Theorem 2.7. ([14]) $\gamma(K_{1,n}) = \gamma(N_{n+1})$.

Theorem 2.8. ([16]) For any integer $n \geq 2$, $\gamma(C_{2^n-1} \cup K_1) = n$.

Theorem 2.9. ([16]) $\gamma(P_n) = \begin{cases} n-1 & if \;\; n \leq 2 \\ \lfloor log_2 n \rfloor + 1 & if \;\; n \geq 3 \end{cases}$.

Theorem 2.10. *(*[2]) The star graph $K_{1,2^n-1}$ is set-graceful.

Theorem 2.11. ([11]) For any integer $n \geq 2$, the cycle C_{2^n-1} is set-graceful.

Theorem 2.12. ([16]) $C_{2^n-1} \cup K_1$ is set-graceful.

Theorem 2.13. ([11]) The complete graph K_n is set-graceful if and only if $n \in \{2, 3, 6\}$.

Theorem 2.14. ([2]) For every integer $n \geq 2$, the path P_{2^n} is not set-graceful.

Theorem 2.15. ([12]) If a (p, q)–graph G is set-graceful, then $q = 2^m - 1$ for some positive integer m.

Recall that the *double star graph* $ST(m, n)$ is the graph formed by two stars $K_{1,m}$ and $K_{1,n}$ by joining their centers by an edge.

Theorem 2.16. ([18]) For a double star graph $ST(m, n)$ with $|V| = 2^l$; $l \geq 2$

$$\gamma(ST(m,n)) = \begin{cases} l & \text{if } m \text{ is even,} \\ l+1 & \text{if } m \text{ is odd.} \end{cases}$$

Theorem 2.17. ([17]) The path P_n is set-semigraceful if and only if $n \neq 2^m$; $m > 1$.

Recall also that the *wheel graph* with n spokes, W_n, is the graph that consists of an n-cycle and one additional vertex, say u, that is adjacent to all the vertices of the cycle.

Theorem 2.18. ([17]) The wheel graph W_6 is set-semigraceful with set-indexing number 4.

3. Topological Set-Indexers

This section presents some results on topological set-indexers of graphs subsequently deriving a necessary condition for a t-set indexer to be optimal.

It has been noted by Acharya [2] that every graph with at least two vertices has a t-set indexer.

Since every t-set indexer is also a set-indexer, the next result follows.

Lemma 3.1. ([2]) Let G be any graph with at least two vertices. Then $\gamma(G) \leq \tau(G)$.

Obviously, $\gamma(G_1) \leq \gamma(G_2)$ if $G_1 \subseteq G_2$. But this does not hold in the case of t-numbers. However, for spanning subgraphs, the next result has been proved.

Theorem 3.2. ([15]) If G' is a spanning subgraph of G, then $\tau(G') \leq \tau(G)$.

The following two results on t-numbers of graphs are quoted for later use.

Theorem 3.3. ([15]) Let G be a graph of order n where $3 \cdot 2^{m-2} < n < 2^m$ for $m \geq 3$. Then $\tau(G) \geq m + 1$.

Theorem 3.4. ([10]) $\tau(K_6 \cup K_1) = 4$.

Let G be any graph of order n. Obviously, every t-set indexer of G is also a t-set indexer of N_n. Though the converse is not true in general, it holds good in the case of stars.

Theorem 3.5. Every t-set indexer of N_n; $n \geq 2$ can be extended to a t-set indexer of $K_{1,n-1}$.

Proof. Let $V(N_n) = \{v_1, \ldots, v_n\}$. Let f be any t-set indexer of N_n. Without loss of generality, let $f(v_1) = \emptyset$. Now, drawing the $n-1$ lines (v_1, v_i) for $2 \leq i \leq n$, we get the graph $K_{1,n-1}$. By assigning $f(v_1, v_i) = f(v_i)$, we clearly have $f(v_1, v_i) = f(v_1) \oplus f(v_i)$ for $i = 2, \ldots, n$. Consequently, f is a t-set indexer of $K_{1,n-1}$ also. □

A necessary condition for a t-set indexer to be optimal is given below.

Theorem 3.6. Let f be a t-set indexer of a graph G with indexing set X and τ be a maximal chain topology contained in $f(V)$. If f is optimal, then $|\tau| = |X| + 1$.

Proof. If $|f(V)| = 2$ or 3, then the result is obvious. So we may assume that $|f(V)| \geq 4$. Let $|\tau| = m$ and $\tau = \{A_i \in f(V) : \emptyset = A_1 \subset A_2 \subset \ldots \subset A_m = X\}$. Suppose $|\tau| < |X| + 1$, then there exists an A_k; $2 \leq k \leq m$ in τ such that $|A_k \setminus A_{k-1}| \geq 2$. Let $a, b \in X$ such that $\{a, b\} \subseteq A_k \setminus A_{k-1}$. Since f is optimal, there is an A in $f(V)$, containing exactly one of a, b. Otherwise, every open set containing a also contains b and vice versa. Then, $g(v) = f(v) \setminus \{b\}$; $v \in V$ defines a new t-set indexer of G on $X \setminus \{b\}$, contradicting the optimality of f.

Without loss of generality it is assumed that $a \in A$ and $b \notin A$. Let $C = A \cap A_k$ and $B = A_{k-1} \cup C$. Note that $A_{k-1} \subset B \subset A_k$. Consequently, $\tau_1 = \tau \cup \{B\}$ is also a chain topology contained in $f(V)$. This contradicts the maximality of τ and hence $|\tau| = |X| + 1$. □

Remark 3.7. The converse of Theorem 3.6 is not true. For instance a t-set indexer f of the path $P_5 = (v_1, \ldots, v_5)$ can be obtained by assigning the subsets $\emptyset$, $\{a\}$, $\{a, b\}$, $\{a, b, c\}$, $\{a, b, c, d\}$ of $X = \{a, b, c, d\}$ to the vertices $v_1, \ldots, v_5$ in that order. The maximal chain topology contained in $f(V)$ is $f(V)$ itself and $|\tau| = |X| + 1$. But f is not optimal since by assigning the subsets $\{x, y\}$, $\emptyset$, $\{x\}$, $\{x, y, z\}$ and $\{y\}$ of the set $Y = \{x, y, z\}$ to the vertices $v_1, \ldots, v_5$ in that order we get a t-set indexer of P_5 with indexing set Y of cardinality 3.

Recall that a graph G is said to be *topologically set graceful* or *t-set graceful* if $\gamma(G) = \tau(G)$. Some topologically set-graceful graphs are listed below.

Theorem 3.8. ([18]) P_{2^n+2} is t-set graceful.

Theorem 3.9. ([10]) $C_6 \cup K_1$ is t-set graceful with t-number 4.

Theorem 3.10. ([10]) The wheel graph W_6 is t-set graceful with t-number 4.

Theorem 3.11. ([10]) K_n is t-set graceful if and only if $2 \leq n \leq 5$.

The following two theorems identify certain graphs for which every spanning subgraph is topologically set graceful.

Theorem 3.12. ([10]) Every t-set graceful path P_n; $n \neq 2^m$ is strongly t-set graceful.

Theorem 3.13. ([10]) Every graph of order m; $2 \leq m \leq 5$ is strongly t-set graceful.

4. Discrete T-set Graceful Graphs

By Theorem 2.3, every graph G has $|V(G)| \leq 2^{\gamma(G)}$. This section attempts to answer the natural question, what are the graphs for which $|V(G)| = 2^{\gamma(G)}$. Surprisingly, these graphs form a subclass of strongly t-set graceful graphs.

Definition 4.1. A graph G with optimal set-indexer f is said to be discrete topologically set-graceful (discrete t-set graceful) if G is t-set graceful and $f(V)$ is the discrete topology.

Example 4.2. $K_{1,6} \cup K_1$ is discrete t-set graceful. Let $G = K_{1,6} \cup K_1$. By Theorem 2.3 and Theorem 3.1, $\tau(G) \geq \gamma(G) \geq 3$. But by assigning $\emptyset$ to the central vertex of $K_{1,6}$ and the distinct nonempty subsets of $X = \{a, b, c\}$ to the other vertices of G in any order we get an optimal t-set indexer of G. Consequently, $\tau(G) = 3 = \gamma(G)$.

Remark 4.3. Discrete t-set graceful and set-graceful are two independent notions. For instance, K_6 is set-graceful (by Theorem 2.13) but it is not discrete t-set graceful as it is not t-set graceful by Theorem 3.11. On the other hand $K_{1,6} \cup K_1$, according to Example 4.2, is discrete t-set graceful but it is not set graceful (by Theorem 2.15).

Remark 4.4. Let G be any graph. By Theorem 2.3 and Theorem 3.1, $\lceil \log_2 |V| \rceil \leq \gamma(G) \leq \tau(G)$. Thus, $|V| \leq 2^{\gamma(G)} \leq 2^{\tau(G)}$.
$K_{1,6}$ is an example for which these inequalities become strict. Recall that $\gamma(K_{1,6}) = 3$ and $\tau(K_{1,6}) = 4$. Again, there are graphs that make only the first inequality strict. Note that $\gamma(P_6) = \tau(P_6) = 3$. However, if $|V| = 2^{\gamma(G)}$, then the optimal set-indexer f corresponding to $\gamma(G)$ becomes a t-set indexer of G with discrete topology $f(V)$. Consequently, $\gamma(G) = \tau(G)$ so that $|V| = 2^{\gamma(G)} = 2^{\tau(G)}$.

Thus, we obtain the next result.

Theorem 4.5. A graph G is discrete t-set graceful if and only if $|V| = 2^{\gamma(G)}$.

Remark 4.6. From the above theorem it follows that a graph whose order is not a power of 2 is never discrete t-set graceful. For example, K_5 is not discrete t-set graceful even though it is t-set graceful by Theorem 3.11.

Corollary 4.7. If G is discrete t-set graceful, then $|E(G)| < |V(G)|$.

Proof. By Theorem 2.2, $\lceil \log_2(|E| + 1) \rceil \leq \gamma(G)$
$= \log_2 |V|$, by Theorem 4.5.
Hence, $|E| + 1 \leq |V|$ so that $|E(G)| < |V(G)|$. ☐

Remark 4.8. Since $K_{1,5}$ is not discrete t-set graceful, the converse of Corollary 4.7 is not true.

Corollary 4.9. *$C_{2^n - 1} \cup K_1$ is discrete t-set graceful.*

Proof. By Theorem 2.8, $\gamma(C_{2^n-1} \cup K_1) = n$. Now by Theorem 4.5, $C_{2^n-1} \cup K_1$ is discrete t-set graceful. ☐

The following theorem characterizes discrete t-set graceful trees.

Theorem 4.10. A tree is discrete t-set graceful if and only if it is set-graceful.

Proof. Let T be a set-graceful tree. Then $\gamma(T) = \log_2(|E| + 1) = \log_2 |V|$. Therefore, $|V(T)| = 2^{\gamma(T)}$ and T is discrete t-set graceful by Theorem 4.5.

Conversely, let T be discrete t-set graceful. Then by Theorem 4.5,
$\gamma(T) = \tau(T) = \log_2 |V|$
$= \log_2(|E| + 1)$, since T is a tree.
Thus, T is set-graceful. ☐

Corollary 4.11. *$K_{1,2^n-1}$ is discrete t-set graceful.*

Proof. By Theorem 2.10, $K_{1,2^n-1}$ is set-graceful. Now the corollary follows from Theorem 4.10. □

Corollary 4.12. *Let m, n and l be positive integers such that $m+n+2=2^l$ and m is even. Then the double star $ST(m,n)$ is discrete t-set graceful.*

Proof. By Theorem 2.16, $\gamma(ST(m,n)) = l$ so that it is set-graceful. Now, the corollary follows from Theorem 4.10. □

Theorem 4.13. Every spanning subgraph of a discrete t-set graceful graph is discrete t-set graceful.

Proof. Let H be any spanning subgraph of a discrete t-set graceful graph G. By Theorem 2.3,

$$\begin{aligned} \lceil \log_2 |V| \rceil &\leq \gamma(H) \\ &\leq \tau(H), \text{ by Theorem 3.1} \\ &\leq \tau(G), \text{ by Theorem 3.2} \\ &= \log_2 |V|, \text{ by Theorem 4.5.} \end{aligned}$$

Consequently, $\tau(H) = \log_2 |V|$ and H is discrete t-set graceful, by Theorem 4.5. □

Corollary 4.14. *Every discrete t-set graceful graph is strongly t-set graceful.*

Proof. Since every discrete t-set graceful graph is t-set graceful, the corollary follows from Theorem 4.13. □

Remark 4.15. Obviously, all discrete t-set graceful graphs that are set-graceful will also be set-semigraceful, t-set graceful and strongly t-set graceful. By Theorem 2.10 and Corollary 4.11, star graphs of order a power of 2 belong to the above category. However, not all graphs in this category are trees. For example, $C_{2^n-1} \cup K_1$ is both discrete t-set graceful and set-graceful by Corollary 4.9 and Theorem 2.8.

Note 4.16. The next items show a summary of what has been stated in this paper.

(i). There are set-semigraceful graphs which are not set-graceful as well as t-set graceful. For example, P_{2^n-1}; $n \geq 3$ is set-semigraceful (see Theorem 2.17) but not set-graceful (by Theorem 2.15). Again,

$$\begin{aligned} \gamma(P_{2^n-1}) &= n, \text{ by Theorem 2.9} \\ &< \tau(P_{2^n-1}), \text{ by Theorem 3.3} \end{aligned}$$

so that P_{2^n-1}; $n \geq 3$ is not t-set graceful.

(ii). By Theorem 2.11, the cycles C_{2^n-1}; $n \geq 3$ is set-graceful so that

$$\begin{aligned} \gamma(C_{2^n-1}) &= n \\ &< \tau(C_{2^n-1}), \text{ by Theorem 3.3.} \end{aligned}$$

Therefore, the cycles C_{2^n-1}; $n \geq 3$ constitute a class of set-graceful graphs which are not t-set graceful.

(iii). Recall from Theorem 3.4 that,
$\tau(K_6 \cup K_1) = 4$
$\geq \gamma(K_6 \cup K_1)$, by Theorem 3.1
$\geq \gamma(K_6)$, by Theorem 2.4
$= 4$, by Theorem 2.5
$= \log_2(|E(K_6 \cup K_1)| + 1)$.
Thus, $K_6 \cup K_1$ is set-graceful as well as t-set graceful. However, it is not strongly t-set graceful as the spanning subgraph N_7 is not t-set graceful. Note that,
$\gamma(N_7) = \gamma(K_{1,6})$, by Theorem 2.7
$= 3$, by Theorem 2.6
$< \tau(N_7)$, by Theorem 3.3.

(iv). It is known that, K_3 is set-graceful (by Theorem 2.13) and strongly t-set graceful (by Theorem 3.13). But, K_3 is not discrete t-set graceful by Theorem 4.5.

(v). The family of stars $K_{1,2^n-1}$ is set-graceful as well as discrete t-set graceful by Theorem 2.10 and Corollary 4.11.

(vi). There are set-semigraceful graphs which are not set-graceful but discrete t-set graceful. By Corollary 4.11 and Theorem 4.13, $K_{1,2^n-2} \cup K_1$ is discrete t-set graceful. But by Theorem 2.15, it is not set-graceful. Further,
$n = \lceil \log_2 |E(K_{1,2^n-2} \cup K_1)| + 1 \rceil$
$\leq \gamma(K_{1,2^n-2} \cup K_1)$, by Theorem 2.2
$\leq \gamma(K_{1,2^n-1})$, by Theorem 2.4
$= n$, by Theorem 2.6
so that $K_{1,2^n-2} \cup K_1$ is set-semigraceful.

(vii). $K_{1,2^n-1} \cup N_{2^n} = G$ constitutes a family of discrete t-set graceful graphs which are not set-semigraceful. We have,
$\lceil \log_2(|E| + 1) \rceil = n$
$< n + 1$
$= \lceil \log_2 |V| \rceil$
$\leq \gamma(G)$, by Theorem 2.3
$\leq \gamma(K_{1,2^{n+1}-1})$, by Theorem 2.4
$= n + 1$, by Theorem 2.6
so that G is not set-semigraceful and $\gamma(G) = n + 1$. Then by Theorem 4.5, G is discrete t-set graceful.

(viii). Now consider the family of graphs $P_{2^n-1} \cup N_3$; $n \geq 3$. Obviously,
$\lceil \log_2(|E| + 1) \rceil = n$
$< n + 1$
$= \lceil \log_2 |V| \rceil$
$\leq \gamma(P_{2^n-1} \cup N_3)$, by Theorem 2.3
$\leq \gamma(P_{2^n+2})$, by Theorem 2.4
$= n + 1$, by Theorem 2.9.
Thus, $P_{2^n-1} \cup N_3$ is not set-semigraceful and $\gamma(P_{2^n-1} \cup N_3) = n + 1 \neq$

$\log_2 |V|$ so that by Theorem 4.5, $P_{2^n-1} \cup N_3$ is not discrete t-set graceful. Now, by Theorem 3.8, P_{2^n+2} is t-set graceful and hence strongly t-set graceful, by Theorem 3.12. Being a spanning subgraph of a strongly t-set graceful graph, then $P_{2^n-1} \cup N_3$ is strongly t-set graceful. Thus, there are strongly t-set graceful graphs that are neither discrete t-set graceful nor set-semigraceful.

(ix). Further, there are t-set graceful graphs that are neither strongly t-set graceful nor set-semigraceful. For example $C_6 \cup K_1$ is one of such graphs as shown in Theorem 3.9.

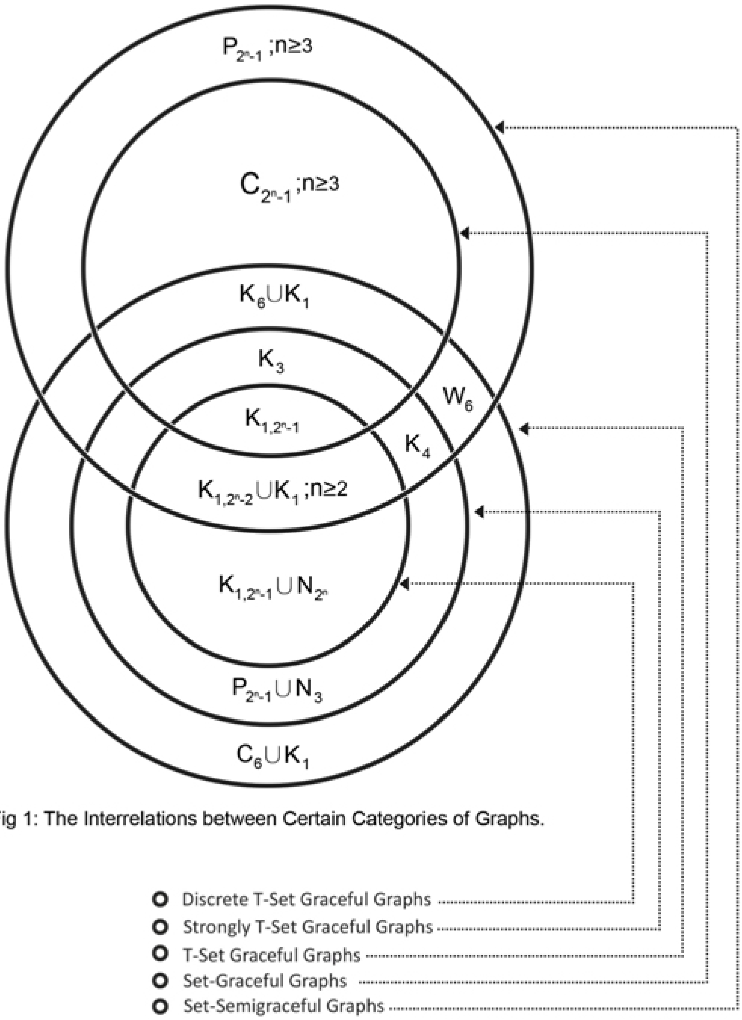

Fig 1: The Interrelations between Certain Categories of Graphs.

(x). We know that W_6 is set-semigraceful (by Theorem 2.18) and t-set graceful (by Theorem 3.10). However, W_6 is not strongly t-set graceful as the spanning subgraph $C_6 \cup K_1$ is not strongly t-set graceful. Again, by Theorem 2.15, W_6 is not set-graceful.

(xi). By Theorem 3.13 and Theorem 2.17, K_4 is strongly t-set graceful and set-semigraceful. But, K_4 is not set-graceful by Theorem 2.13. Finally, by Theorem 2.5 and Theorem 4.5, K_4 is not discrete t-set graceful.

We summarize these discussions in the diagram given in Figure 1.

Acknowledgement

The authors are thankful to a referee for his valuable comments and suggestions which made the presentation of this paper much better.

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Received by the editors June 21, 2014