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GAMMA NEARRINGS WITH GENERALIZED GAMMA DERIVATIONS[1]

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Abstract. In this paper, we define the notion of generalized Γ-derivation for Γ-nearring and extend some results of Γ-derivation of Γ-nearrings for generalized Γ-derivation. Also a Posner type result for the composition of generalized Γ-derivations is obtained with some extra condition. Furthermore, examples are given to demonstrate that the restrictions imposed on the hypothesis of the various theorems were not superfluous.

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1. Introduction

In the year 1964, Nabusawa [8] gave a more general concept than a ring, known as Γ-ring. Barnes [1] weakened the condition slightly in the definition of Γ-ring in the sense of Nabusawa. Thereafter, a number of algebraists [1, 6, 7, 9] have studied the structure of Γ-rings and obtained various generalizations analogous to corresponding parts in ring theory. Nearring is a generalization of a ring, as an extension of nearring one can establish Γ-nearrings which is a generalization of Γ-rings. In this context, Satyanarayana [12, 13, 14] introduced Γ-nearrings and studied their properties. Recently, Booth et.al [2, 3] studied various ways to develop Γ-nearrings. Also, Jun (together with Cho and Kim) introduced the notion of Γ-derivations in Γ-nearrings and investigated basic properties (see [4, 5]).

For preliminary definition and results related to nearrings, we refer to Pilz[10]. All nearrings considered in this paper are right distributive. A Γ-nearring is a triple system $(M, +, \Gamma)$, where

1. Γ is a nonempty set of binary operators such that $(M, +, \gamma)$ is a nearring for each $\gamma \in \Gamma$,

2. $(x\gamma y)\mu z = x\gamma(y\mu z)$, for all $x, y, z \in M, \gamma, \mu \in \Gamma$.

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For a Γ-nearring M, the set $M_0 = \{x \in M \setminus x\gamma 0 = 0, \ \forall\ \gamma \in \Gamma\}$, is called the zero-symmetric part of M. A Γ-nearring is said to be zero-symmetric if $M = M_0$. A Γ-nearring is said to be prime if $x\Gamma M\Gamma y = \{0\}$ implies $x = 0$ or $y = 0$, for all $x, y \in M$. For any $x, y \in N$ the symbol $[x, y]_\gamma$ and (xy) will denotes the multiplicative and additive commutators $x\gamma y - y\gamma x$ and $x + y - x - y$. The symbol $Z(N)$ will represent the multiplicative center of N; that is, $Z(N) = \{x \in N : x\gamma y = y\gamma x \ \text{ for all } \ y \in N, \gamma \in \Gamma\}$. A Γ-prime nearring M is said to be 2 torsion free if $(M, +)$ have no element of order 2 (i.e if $a \in M$ and $2a = 0$ then $a = 0$). If M and M' are two Γ-nearrings, then an additive mapping $f : M \longrightarrow M'$ such that $f(x\gamma y) = f(x)\gamma f(y)$ $(f(x\gamma y) = f(y)\gamma f(x))$, for all $x, y \in M$, $\gamma \in \Gamma$ is called a Γ-nearring homomorphism (Γ-nearring anti-homomorphism).

In this note, we investigate the conditions for Γ-nearrings with generalized Γ-derivations to be commutative and an analogous version of Posner theorem is obtained for the case of product of two generalized Γ-derivations on Γ-nearring.

Throughout the paper, M denotes a Γ-nearring unless otherwise specified.

2. Properties of Generalized Γ-Derivations

We start with following definitions and lemmas which will be used extensively.

Definition 2.1. An additive mapping $D : M \longrightarrow M$ is called a Γ-derivation if $D(x\gamma y) = D(x)\gamma y + x\gamma D(y)$ holds for all $x, y \in M$, $\gamma \in \Gamma$.

Definition 2.2. An additive mapping $F : M \longrightarrow M$ is said to be a right generalized Γ-derivation if there exists a Γ-derivation D on M such that

$$F(x\gamma y) = F(x)\gamma y + x\gamma D(y) \ \ \forall \ \ x, y \in M, \ \gamma \in \Gamma,$$

and F is said to be a left generalized Γ-derivation if there exists a Γ-derivation on M such that

$$F(x\gamma y) = x\gamma F(y) + D(x)\gamma y \ \ \forall \ \ x, y \in M, \ \gamma \in \Gamma,$$

Finally, F is said to be a generalized Γ-derivation associated with D if it is both right as well as left generalized Γ-derivation on M. We shall denote generalized Γ-derivation associated with D on M by (F, D).

Definition 2.3. Let A be a nonempty subset of M and F be a generalized Γ-derivation on M with associated derivation D. A generalized Γ-derivation F of M is said to act as a Γ-homomorphism on A if $F(x\gamma y) = F(x)\gamma y + x\gamma D(y) = F(x)\gamma F(y)$ for all $x, y \in A$, $\gamma \in \Gamma$.

Definition 2.4. Let A be a nonempty subset of M and F be a generalized Γ-derivation on M with associated derivation D. A generalized Γ-derivation F of M is said to act as a Γ-anti-homomorphism on A if $F(x\gamma y) = F(x)\gamma y + x\gamma D(y) = F(y)\gamma F(x)$ for all $x, y \in A$, $\gamma \in \Gamma$.

Now, we try of construct some examples of this type of derivations that would make sense of the theory we are dealing with.

Example 2.5. Assume that R is a Γ_1-ring with a nonzero generalized derivation (f,δ). Now, taking $M = N \bigoplus R$, where N is a Γ_2-nearring which is not a ring. Observe that M is not a Γ-ring and Γ is a direct product of Γ_1 & Γ_2. Define a map $F : M \longrightarrow M$ as $F(n,r) = (0, f(r))$ for all $n \in N,\ r \in R$, is a generalized Γ-derivation associated with D on M, where $D : M \longrightarrow M$ is a Γ-derivation on M define by $D(n,r) = (0,\delta(r))$ for all $n \in N,\ r \in R$.

Example 2.6. In Example 2.5, if R admits a generalized derivation f associated with δ and also acts as a Γ_1-homomorphism, then it is straight to see that F is a Γ-homomorphism on M.

Example 2.7. In Example 2.5, if R admits a generalized derivation f associated with δ and an anti-Γ_1-homomorphism, then it is straight to see that F is an anti-Γ-homomorphism on M.

In general, the additive commutativity is not necessary in a Γ-nearring. The following results (i.e. Lemma 2.8 & 2.9) are significant in their own right.

Lemma 2.8. *Let (F,D) be a right generalized Γ-derivation of M. Then $F(x\gamma y) = x\gamma D(y) + F(x)\gamma y,\ \forall\ x,y \in M\ \gamma \in \Gamma$.*

Proof. For any $x,y \in M,\ \gamma \in \Gamma$, we get

$$\begin{aligned} F((x+x)\gamma y) &= F(x+x)\gamma y + (x+x)\gamma D(y) \\ &= F(x)\gamma y + F(x)\gamma y + x\gamma D(y) + x\gamma D(y) \end{aligned}$$

and

$$\begin{aligned} F(x\gamma y + x\gamma y) &= F(x\gamma y) + F(x\gamma y) \\ &= F(x)\gamma y + x\gamma D(y) + F(x)\gamma y + x\gamma D(y). \end{aligned}$$

Comparing these equations, one can obtain

$$F(x)\gamma y + x\gamma D(y) = x\gamma D(y) + F(x)\gamma y.$$

Hence, $F(x\gamma y) = x\gamma D(y) + F(x)\gamma y$. □

Lemma 2.9. *Let (F,D) be a left generalized Γ-derivation of M. Then*

$$F(x\gamma y) = D(x)\gamma y + x\gamma F(y),\ \forall\ x,y \in M\ \gamma \in \Gamma.$$

Proof. Arguing in the similar manner as we have done in the proof of Lemma 2.8, we get the required result. □

The crucial fact is that the definition of generalized Γ-derivation implies partial distributive law.

Lemma 2.10. *Let (F,D) be a generalized Γ-derivation of M. Then, for all $a,x,y \in M\ \gamma,\eta \in \Gamma$*

$$a\eta F(x\gamma y) = a\eta x\gamma D(y) + a\eta F(x)\gamma y.$$

Proof. By calculating $F(a\gamma x\eta y)$ in two different ways, we obtain the required result easily. □

3. Main results

Our best results are extension of some theorems of [5, 15]. In this section we investigate possible analogues of these results, where D is replaced by a generalized Γ-derivation F.

We will need two easy lemmas.

Lemma 3.1. *[15] Let M be a Γ-prime nearring*

1. *Let $z \in Z \setminus \{0\}$ be an element such that $z + z \in Z$. Then $(M, +)$ is abelian.*
2. *Let $D \neq 0$ be a Γ-derivation on M. Then $x\Gamma D(M) = \{0\}$ implies $x = 0$, and $D(M)\Gamma x = \{0\}$ implies $x = 0$.*
3. *Let M is 2 torsion free and D is a Γ-derivation on M such that $D^2 = 0$, then $D = 0$.*

Lemma 3.2. *Let M be a Γ-prime nearring, (F, D) a nonzero generalized Γ-derivation of M and $a \in M$.*

1. *If $a\Gamma F(M) = 0$, then $a = 0$ or $D = 0$.*
2. *If $F(M)\Gamma a = 0$, then $a = 0$ or $D = 0$.*

Proof. (1) For any $x, y \in M$, $\gamma, \eta \in \Gamma$, we get

$$0 = a\eta F(x\gamma y) = a\eta(x\gamma D(y) + F(x)\gamma y).$$

By using an application of Lemma 2.10, we arrive

$$a\Gamma M\Gamma D(y) = 0.$$

In view of M primeness, we obtain the required result.
(2) A similar argument works if $F(M)\Gamma a = 0$. □

Theorem 3.3. *Let $(F, D \neq 0)$ be generalized Γ-derivation of M. If M is a 2 torsion free Γ-prime nearring and $F^2 = 0$, then $F = 0$.*

Proof. For any arbitrary $x, y \in M$ & $\gamma \in \Gamma$, we have

$$\begin{aligned} 0 = F^2(x\gamma y) &= F(F(x)\gamma y + x\gamma D(y)) \\ &= F^2(x)\gamma y + 2F(x)\gamma D(y) + x\gamma D^2(y). \end{aligned}$$

In view of hypothesis, we have

$$2F(x)\gamma D(y) + x\gamma D^2(y) = 0, \ \forall\, x, y \in M, \ \gamma \in \Gamma. \tag{3.1}$$

Replacing x by $F(x)$ in (3.1) and using the hypothesis, we find that

$$F(x)\gamma D^2(y) = 0, \ \forall\, x, y \in M, \ \gamma \in \Gamma.$$

From Lemma 3.2, we obtain $D^2(M) = 0$ or $F = 0$. If $D^2(M) = 0$, then $D = 0$ from Lemma 3.1(1). It contradicts $D \neq 0$. This completes the proof. □

Remark 3.4. If $F = D$, then we reach the Lemma 3.1(3).

Theorem 3.5. *Let M be a 2 torsion free Γ-prime nearring with a nonzero generalized Γ-derivation $(F, D \neq 0)$. If $F(M) \subset Z$, then $(M, +)$ is abelian. Moreover, if M be a 2 torsion free, then M is commutative.*

Proof. Suppose that $a \in M$ such that $F(a) \neq 0$. So, $F(a) \in Z \setminus \{0\}$ and $F(a) + F(a) \in Z \setminus \{0\}$. It follows from Lemma 3.1 that $(M, +)$ is abelian.

Now, for any $x, y, z \in M,\ \mu, \eta \in \Gamma$, we have

$$\begin{aligned} z\eta F(x\mu y) &= F(x\mu y)\eta z. \\ z\eta(x\mu D(y) + F(x)\mu y) &= (x\mu D(y) + F(x)\mu y)\eta z. \end{aligned}$$

In light of Lemma 2.10, we obtain

$$z\eta x\mu D(y) + z\eta F(x)\mu y = x\mu D(y)\eta z + F(x)\mu y\eta z.$$

Using the fact that $F(x) \in Z$ and $(M, +)$ is abelian, we find that

$$z\eta x\mu D(y) - x\mu D(y)\eta z = [y, z]_\eta \mu F(x),\ \forall\ x, y, z \in M, \mu, \eta \in \Gamma. \tag{3.2}$$

Substituting $F(y)$ for y in (3.2) and using the hypothesis to find that

$$z\eta x\mu D(F(y)) - x\mu D(F(y))\eta z = 0\ \forall\ x, y, z \in M,\ \mu, \eta \in \Gamma.$$

Since $F(y) \in Z$ it implies that $D(F(y)) \in Z$ and using Lemma 2.10 to get

$$D(F(y))\mu[z, x]_\eta = 0\ \forall\ x, y, z \in M,\ \mu, \eta \in \Gamma.$$

From Lemma 3.1(1), we obtain $D(F(y)) = 0$ for all $y \in M$ or M is commutative. Suppose that $D(F(y)) = 0$, for all $y \in M$. Then

$$0 = D(F(x\gamma y)) = D^2(x)\gamma y + D(x)\gamma D(y) + D(x)\gamma F(y),\ \forall\ x, y \in M,\ \gamma \in \Gamma.$$

Replacing y by $y\eta z$ in last relation and using it, it follow from Lemma 2.10 that

$$2D(x)\gamma y\eta D(z) = 0,\ \forall\ x, y, z \in M,\ \gamma, \eta \in \Gamma.$$

Since M is 2 torsion free, we get

$$D(M)\Gamma M\Gamma D(M) = 0.$$

Thus, we obtain $D = 0$. It contradicts $D \neq 0$. We must have M is commutative. □

Theorem 3.6. *Let M be a Γ-prime nearring and (F, D) a generalized Γ-derivation of M. If F acts as Γ-homomorphism on M, then $D = 0$.*

Proof. By the hypothesis, we have

$$F(x\gamma y) = F(x)\gamma y + x\gamma D(y),\ \forall\ x, y \in M,\ \gamma \in \Gamma. \tag{3.3}$$

Taking $y\eta x$ instead of y in (3.3) we obtain

$$x\gamma D(y\eta x) + F(x)\gamma y\eta x \quad = \quad F(x)\gamma F(y\eta x) = F(x)\gamma(y\eta D(x) + F(y)\eta x).$$

From Lemma 2.10 and using the hypothesis we find that

$$x\gamma y\eta D(x) = F(x)\gamma y\eta D(x),\ \forall\ x, y \in M,\ \gamma, \eta \in \Gamma. \tag{3.4}$$

Replacing y with $F(y)$ in (3.4) and using primeness of M, we have desired conclusion. □

Theorem 3.7. *Let M be a Γ-prime nearring and (F, D) a generalized Γ-derivation of M. If F acts as anti Γ-homomorphism on M, then $D = 0$.*

Proof. Suppose that F acts as anti Γ-homomorphism on M. Then

$$F(x\gamma y) = F(y)\gamma F(x) = x\gamma D(y) + F(x)\gamma y,\ \forall\ x, y \in M,\ \gamma \in \Gamma. \tag{3.5}$$

Replacing x by $x\eta y$ in (3.5) and using Lemma 2.10, we obtain

$$x\gamma y\eta D(y) + F(x\gamma y)\eta y = F(y)\gamma F(x\eta y) = F(y)\gamma x\eta D(y) + F(x\gamma y)\eta y$$

and so,

$$x\gamma y\eta D(y) = F(y)\gamma x\eta D(y),\ \forall\ x, y \in M,\ \gamma, \eta \in \Gamma. \tag{3.6}$$

Take $m\mu x$ instead of x in (3.6) and use it to find that

$$\begin{aligned} F(y)\gamma m\mu x\eta D(y) = m\mu x\gamma y\eta D(y) \quad &= \quad m\mu F(y)\gamma x\eta D(y), \\ &\forall\ x, y, m \in M,\ \gamma, \mu, \eta \in \Gamma. \end{aligned}$$

In particular, if $\mu = \gamma$ and so,

$$[F(y), m]_\mu \gamma x\eta D(y) = 0,\ \forall\ x, y, m \in M,\ \gamma, \mu, \eta \in \Gamma.$$

In view of M primeness, we arrive at $D(y) = 0$ or $F(y) \in Z$ for all $y \in M$. In the latter case, $F(M) \subset Z$, which forces F to act as Γ-homomorphism on M, and so, $D = 0$ by Theorem 3.6. This completes the proof. □

The following examples shows that the restrictions is imposed on the hypotheses of Theorem 3.6 & 3.7 are superfluous.

Example 3.8. In Example 2.5, if R admits a generalized derivation f associated with $\delta \neq 0$ and also acts as a Γ_1-homomorphism or anti Γ-homomorphism. Then it is straight to see that F is a Γ-homomorphism or anti Γ-homomorphism on M. However, D is not equal to zero.

A well known theorem due to Posner[11] state that *if the composition of two derivations of a prime ring of characteristic not equal to two is again a derivation, then at least one of them must be zero.* An analogue of this result in nearrings was obtained by Wang [16]. Jun et.al. [5] generalized this result for Γ-derivations of Γ-nearrings. It is naturally ask question: what can we say about this result if we replace Γ-derivations D with generalized Γ-derivations F. The following theorem gives an answer in the affirmative.

Theorem 3.9. *Let (F, D) and (G, H) be generalized* Γ*-derivations of a 2 torsion free* Γ*-prime nearring M. If (FG, DH) acts as a generalized* Γ*-derivation on M, then $F = 0$ or $G = 0$.*

In order to prove above theorem, we need to prove the following lemmas.

Lemma 3.10. *Let (F, D) and (G, H) be* Γ*-derivations of M. If H is a nonzero* Γ*-derivation on M and $F(x)\gamma H(y) = -G(x)\gamma D(y)$, for all $x, y \in M, \gamma \in \Gamma$, then $(M, +)$ is abelian.*

Proof. Assume that

$$F(x)\gamma H(y) = -G(x)\gamma D(y),\ \forall\ x, y \in M,\ \gamma \in \Gamma.$$

Replacing y with $y + z$ in above relation and using Lemma 2.10 to find that

$$\begin{aligned} F(x)\gamma H(y) + F(x)\gamma H(z) &= -G(x)\gamma D(y) - G(x)\gamma D(z), \\ &\quad \forall\ x, y, z \in M,\ \gamma \in \Gamma. \end{aligned}$$

Using the hypothesis and from Lemma 2.10, we obtain

$$F(x)\gamma H(y, z) = 0,\ \forall\ x, y, z \in M,\ \gamma \in \Gamma.$$

It follows from Lemma 3.2 that $H(y, z) = 0$ for all $y, z \in M$. For any $w \in M, \mu \in \Gamma$, we have

$$H(y\mu w, z\mu w) = H((y, z)\mu w) = H(y, z)\mu w + (y, z)\mu H(w) = 0$$

and so,

$$(y, z)\mu H(w) = 0.$$

From Lemma 3.1(2), we have desired conclusion. □

Lemma 3.11. *Let (F, D) and (G, H) be* Γ*-derivations of M. If M is a 2 torsion free* Γ*-prime nearring and $F(x)\gamma H(y) = -G(x)\gamma D(y)$, for all $x, y \in M, \gamma \in \Gamma$, then $F = 0$ or $G = 0$.*

Proof. The proof is trivial, if $D = 0$ or $H = 0$. So, we may assume that $D \neq 0$ and $H \neq 0$. Therefore we know that $(M, +)$ is abelian by Lemma 3.10.

Now, in view of hypothesis, we have

$$F(x)\gamma H(y) + G(x)\gamma D(y) = 0,\ \forall\ x, y \in M,\ \gamma \in \Gamma.$$

Taking $x\mu z$ instead of x in last relation we get

$$\begin{aligned} 0 &= F(x\mu z)\gamma H(y) + G(x\mu z)\gamma D(y) \\ &= x\mu F(z)\gamma H(y) + D(x)\mu z\gamma H(y) + x\mu G(z)\gamma D(y) + H(x)\mu z\gamma D(y), \\ &\qquad \forall\ x, y \in M,\ \gamma \in \Gamma. \end{aligned}$$

In light of hypothesis and from Lemma 2.10, one can find that

$$D(x)\mu z\gamma H(y) = -H(x)\mu z\gamma D(y)\ \forall\ x, y \in M,\ \gamma \in \Gamma. \tag{3.7}$$

Replacing $y\eta m$ with y in (3.7) and using Lemma 2.10, we obtain

$$\begin{aligned} & D(x)\mu z\gamma H(y)\eta m + D(x)\mu z\gamma y\eta H(m) \\ &= \ -H(x)\mu z\gamma D(y)\eta m - H(x)\mu z\gamma y\eta D(m). \end{aligned}$$

That is,

$$D(x)\mu z\gamma y\eta H(m) = -H(x)\mu z\gamma y\eta D(m),\ \forall\ x, y, z, m \in M,\ \mu, \eta, \gamma \in \Gamma. \tag{3.8}$$

Substituting y by $H(y)$ in (3.8) and from Lemma 2.10, thereafter by using (3.7), we find that

$$H(x)\Gamma M\Gamma(D(y)\eta H(m) - H(y)\eta D(m)) = 0,\ \forall\ x, y, m \in M, \eta \in \Gamma.$$

It follows from Lemma 3.1(2) that

$$D(y)\eta H(m) = H(y)\eta D(m)),\ \forall\ y, m \in M, \eta \in \Gamma. \tag{3.9}$$

Now, taking $x\mu z$ instead of x in the initial hypothesis we obtain

$$F(x)\mu z\gamma H(y) + x\mu D(z)\gamma H(y) + G(x)\mu z\gamma D(y) + x\mu H(z)\gamma D(y) = 0.$$

In view of (3.9), the above expression yields that

$$F(x)\mu z\gamma H(y) + 2x\mu H(z)\gamma D(y) + G(x)\mu z\gamma D(y) = 0.$$

Replace z with $H(z)$ in last expression, we arrive at

$$F(x)\mu H(z)\gamma H(y) + 2x\mu H^2(z)\gamma D(y) + G(x)\mu H(z)\gamma D(y) = 0.$$

In view of hypothesis and from (3.9), one can obtain

$$2x\mu H^2(z)\gamma D(y) = 0,\ \forall\ x, y, z \in M, \mu, \gamma \in \Gamma..$$

Since M is a 2 torsion free Γ-prime nearring, we obtain $H^2(M)\Gamma D(M) = 0$. An appeal of Lemma 3.1(2) & (3) gives that $H = 0$. It contradicts our assumption. This completes the proof. □

Now, we are in position to proof our Theorem 3.9.

Proof. By calculating $FG(x\gamma y)$ in two different ways, we see that

$$G(x)\gamma D(y) + F(x)\gamma H(y) = 0, \ \forall\ x, y \in M, \ \gamma \in \Gamma.$$

The proof is completed by using Lemma 3.11. □

Remark 3.12. If $F = G$, then we find Theorem 3.3.

Using equality $G(x)\gamma D(y) + F(x)\gamma H(y) = 0, \ \forall\ x, y \in M, \ \gamma \in \Gamma$ of Lemma 3.11, we can prove the following interesting result.

Corollary 3.13. *Let M be a Γ-nearring and F and G be generalized Γ-derivations on M such that FG is a Γ-derivation. Then GF is also a Γ-derivation.*

Proof. Obviously GF is an additive endomorphism of M. By equality $G(x)\gamma D(y) + F(x)\gamma H(y) = 0, \ \forall\ x, y \in M, \ \gamma \in \Gamma$, we have

$$\begin{aligned} GF(x\gamma y) &= G(F(x)\gamma y + x\gamma F(y)) = G(F(x)\gamma y) + G(x\gamma F(y)) \\ &= GF(x)\gamma y + (F(x)\gamma G(y) + G(x)\gamma F(y)) + x\gamma GF(y) \end{aligned}$$

Thus, GF is a derivation by Lemma 3.11. This completes the proof. □

The following example demonstrate that the Theorem 3.9 does not hold for arbitrary rings.

Example 3.14. In Example 2.5, define another map $G : M \longrightarrow M$ as $G(n, r) = (g(n), 0)$ for all $n \in N, \ r \in R$, where N is a Γ_2-nearring and admits a nonzero generalized Γ_2-derivation (g, d). Then it is straightforward to see that G is also a generalized Γ-derivation associated with H on M, where $H : M \longrightarrow M$ is defined by $H(n, r) = (d(n), 0)$ for all $n \in N, \ r \in R$ is Γ-derivation on M. We can easily see that (FG, DH) acts as generalized Γ-derivation on M but neither $F = 0$ nor $G = 0$. Hence, in Theorem 3.9 the hypothesis is crucial.

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