Novi Sad J. Math.
Vol. 43, No. 1, 2013, 89-98

ON ϕ-PSEUDO SYMMETRIC KENMOTSU MANIFOLDS

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Abstract. The object of the present paper is to study ϕ-pseudo symmetric and ϕ-pseudo Ricci symmetric Kenmotsu manifolds. We also studied ϕ-pseudo concircularly symmetric Kenmotsu manifolds and obtained a number of results.

AMS Mathematics Subject Classification (2010): 53C15, 53C25, 53D15

Key words and phrases: ϕ-pseudo symmetric, ϕ-pseudo Ricci symmetric, Kenmotsu manifold, Einstein manifold, concircular curvature tensor.

1. Introduction

In [30], Tanno classified connected almost-contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. The author proved that they could be divided into three classes: (i) homogeneous normal contact Riemannian manifolds with $c > 0$, (ii) global Riemannian products of a line or a circle with a Kähler manifold of constant holomorphic sectional curvature if $c = 0$, and (iii) warped product space $\mathbb{R} \times_f \mathbb{C}^n$ if $c < 0$. It is known that the manifolds of class (i) are characterized by admitting a Sasakian structure. The manifolds of class (ii) are characterized by a tensorial relation admitting a cosymplectic structure. Kenmotsu [19] characterized the differential geometric properties of the manifolds of class (iii), which are nowadays called Kenmotsu manifolds and later studied by several authors.

As a generalization of both Sasakian and Kenmotsu manifolds, Oubiña [20] introduced the notion of trans-Sasakian manifolds, which are closely related to the locally conformal Kähler manifolds. A trans-Sasakian manifolds of type (0, 0), $(\alpha, 0)$ and $(0, \beta)$ are called the cosympletic, α-Sasakian and β-Kenmotsu manifolds respectively, α, β being scalar functions. In particular, if $\alpha = 0, \beta = 1$; and $\alpha = 1, \beta = 0$ then, a trans-Sasakian manifold will be a Kenmotsu and Sasakian manifold respectively.

The study of Riemann symmetric manifolds began with the work of Cartan [3]. A Riemannian manifold (M^n, g) is said to be locally symmetric due to Cartan [3] if its curvature tensor R satisfies the relation $\nabla R = 0$, where ∇ denotes the operator of covariant differentiation with respect to the metric tensor g.

During the last five decades, the notion of locally symmetric manifolds has been weakened by many authors in several ways to a different extent such as

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recurrent manifold by Walker [35], semisymmetric manifold by Szabó [29], pseudosymmetric manifold in the sense of Deszcz [17], pseudosymmetric manifold in the sense of Chaki [4].

A non-flat Riemannian manifold $(M^n, g)(n > 2)$ is said to be pseudosymmetric in the sense of Chaki [4] if it satisfies the relation

$$(1.1)\quad \begin{aligned}(\nabla_W R)(X,Y,Z,U) &= 2A(W)R(X,Y,Z,U) + A(X)R(W,Y,Z,U)\\ &+ A(Y)R(X,W,Z,U) + A(Z)R(X,Y,W,U)\\ &+ A(U)R(X,Y,Z,W),\end{aligned}$$

i.e.,

$$(1.2)\quad \begin{aligned}(\nabla_W R)(X,Y)Z &= 2A(W)R(X,Y)Z + A(X)R(W,Y)Z\\ &+ A(Y)R(X,W)Z + A(Z)R(X,Y)W\\ &+ g(R(X,Y)Z,W)\rho\end{aligned}$$

for any vector field X, Y, Z, U and W, where R is the Riemannian curvature tensor of the manifold, A is a non-zero 1-form such that $g(X,\rho) = A(X)$ for every vector field X. Such an n-dimensional manifold is denoted by $(PS)_n$.

Every recurrent manifold is pseudosymmetric in the sense of Chaki [4] but not conversely. Also, the pseudosymmetry in the sense of Chaki is not equivalent to that in the sense of Deszcz [17]. However, pseudosymmetry by Chaki will be the pseudosymmetry by Deszcz if and only if the non-zero 1-form associated with $(PS)_n$, is closed. Pseudosymmetric manifolds in the sense of Chaki have been studied by Chaki and Chaki [6], Chaki and De [7], De [9], De and Biswas [11], De, Murathan and Özgür [14], Özen and Altay ([22], [23]), Tarafder ([32], [33]), Tarafder and De, [34] and others.

A Riemannian manifold is said to be Ricci symmetric if its Ricci tensor S of type (0,2) satisfies $\nabla S = 0$, where ∇ denotes the Riemannian connection. During the last five decades, the notion of Ricci symmetry has been weakened by many authors in several ways to a different extent such as Ricci recurrent manifold [24], Ricci semisymmetric manifold [29], pseudo Ricci symmetric manifold by Deszcz [18], pseudo Ricci symmetric manifold by Chaki [5].

A non-flat Riemannian manifold (M^n, g) is said to be pseudo-Ricci symmetric [5] if its Ricci tensor S of type (0,2) is not identically zero and satisfies the condition

$$(1.3)\quad (\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + A(Y)S(X,Z) + A(Z)S(Y,X)$$

for any vector field X, Y, Z, where A is a nowhere vanishing 1-form and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Such an n-dimensional manifold is denoted by $(PRS)_n$. The pseudo-Ricci symmetric manifolds have been also studied by Arslan et al. [1], Chaki and Saha [8], De and Mazumder [13], De, Murathan and Özgür [14], Özen [21], and many others.

The relation (1.3) can be written as

$$(1.4)\quad (\nabla_X Q)(Y) = 2A(X)Q(Y) + A(Y)Q(X) + S(Y,X)\rho,$$

where ρ is the vector field associated to the 1-form A such that $A(X) = g(X, \rho)$ and Q is the Ricci operator, i.e., $g(QX, Y) = S(X, Y)$ for all X, Y.

As a weaker version of local symmetry, the notion of locally ϕ-symmetric Sasakian manifolds was introduced by Takahashi [31]. By generalizing the notion of locally ϕ-symmetric Sasakian manifolds, De, Shaikh and Biswas [15] introduced the notion of ϕ-recurrent Sasakian manifolds. In this connection De [10] introduced and studied ϕ-symmetric Kenmotsu manifolds and in [16] De, Yildiz and Yaliniz introduced and studied ϕ-recurrent Kenmotsu manifolds. In this connection it may be mentioned that Shaikh and Hui studied locally ϕ-symmetric β-kenmotsu manifolds [26] and extended generalized ϕ-recurrent β-Kenmotsu manifolds [27], respectively. Also, in [25] Prakash studied concircularly ϕ-recurrent Kenmotsu Manifolds. Recently, Shukla and Shukla [28] introduced and studied ϕ-Ricci symmetric Kenmotsu manifolds.

The object of the present paper is to study ϕ-pseudo symmetric and ϕ-pseudo Ricci symmetric Kenmotsu manifolds. The paper is organized as follows. Section 2 is concerned with preliminaries. Section 3 is devoted to the study of ϕ-pseudo symmetric Kenmotsu manifolds and also ϕ-pseudo concircularly symmetric Kenmotsu manifolds. It is proved that every ϕ-pseudo symmetric Kenmotsu manifold is an η-Einstein manifold. In section 4, we have studied ϕ-pseudo Ricci symmetric symmetric Kenmotsu manifolds.

2. Preliminaries

A smooth manifold (M^n, g) $(n = 2m+1 > 3)$ is said to be an almost contact metric manifold [2] if it admits a (1,1) tensor field ϕ, a vector field ξ, an 1-form η and a Riemannian metric g which satisfy

$$\phi\xi = 0, \qquad \eta(\phi X) = 0, \qquad \phi^2 X = -X + \eta(X)\xi, \tag{2.1}$$

$$g(\phi X, Y) = -g(X, \phi Y), \qquad \eta(X) = g(X, \xi), \qquad \eta(\xi) = 1, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.3}$$

for all vector fields X, Y on M.

An almost-contact metric manifold $M^n(\phi, \xi, \eta, g)$ is said to be Kenmotsu manifold if the following condition holds [19]:

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.4}$$

$$(\nabla_X \phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.5}$$

where ∇ denotes the Riemannian connection of g.
In a Kenmotsu manifold, the following relations hold [19]:

$$(\nabla_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.6}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.7}$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \tag{2.8}$$

$$\eta(R(X,Y)Z) = \eta(Y)g(X,Z) - \eta(X)g(Y,Z), \tag{2.9}$$
$$S(X,\xi) = -(n-1)\eta(X), \tag{2.10}$$
$$S(\xi,\xi) = -(n-1), \quad \text{i.e.,} \quad Q\xi = -(n-1)\xi, \tag{2.11}$$
$$S(\phi X,\phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{2.12}$$
$$(\nabla_W R)(X,Y)\xi = g(X,W)Y - g(Y,W)X - R(X,Y)W \tag{2.13}$$

for any vector field X, Y, Z on M and R is the Riemannian curvature tensor and S is the Ricci tensor of type (0,2) such that $g(QX,Y) = S(X,Y)$.

Definition 2.1. A Kenmotsu manifold M is said to be η-Einstein if its Ricci tensor S of type (0,2) is of the form

$$S = ag + b\eta\otimes\eta, \tag{2.14}$$

where a, b are smooth functions on M.

3. ϕ-pseudo symmetric Kenmotsu manifolds

Definition 3.1. A Kenmotsu manifold $M^n(\phi,\xi,\eta,g)$ $(n = 2m+1 > 3)$ is said to be *ϕ-pseudo symmetric* if the curvature tensor R satisfies

$$\begin{aligned}\phi^2((\nabla_W R)(X,Y)Z) &= 2A(W)R(X,Y)Z + A(X)R(W,Y)Z \\ &+ A(Y)R(X,W)Z + A(Z)R(X,Y)W \\ &+ g(R(X,Y)Z,W)\rho\end{aligned} \tag{3.1}$$

for any vector field X, Y, Z and W, where A is a non-zero 1-form.
If, in particular, $A = 0$, then the manifold is said to be ϕ-symmetric [10].

We now consider a Kenmotsu manifold $M^n(\phi,\xi,\eta,g)$ $(n = 2m+1 > 3)$, which is ϕ-pseudo symmetric. Then, by virtue of (2.1), it follows from (3.1) that

$$\begin{aligned}&-(\nabla_W R)(X,Y)Z + \eta((\nabla_W R)(X,Y)Z)\xi \\ =\ & 2A(W)R(X,Y)Z + A(X)R(W,Y)Z + A(Y)R(X,W)Z \\ +\ & A(Z)R(X,Y)W + g(R(X,Y)Z,W)\rho\end{aligned} \tag{3.2}$$

from which it follows that

$$\begin{aligned}&-g((\nabla_W R)(X,Y)Z,U) + \eta((\nabla_W R)(X,Y)Z)\eta(U) \\ =\ & 2A(W)g(R(X,Y)Z,U) + A(X)g(R(W,Y)Z,U) + A(Y)g(R(X,W)Z,U) \\ +\ & A(Z)g(R(X,Y)W,U) + g(R(X,Y)Z,W)A(U).\end{aligned} \tag{3.3}$$

Taking an orthonormal frame field and then contracting (3.3) over X and U, and then using (2.2), we get

$$\begin{aligned}&-(\nabla_W S)(Y,Z) + g((\nabla_W R)(\xi,Y)Z,\xi) \\ =\ & 2A(W)S(Y,Z) + A(Y)S(W,Z) + A(Z)S(Y,W) \\ +\ & A(R(W,Y)Z) + A(R(W,Z)Y).\end{aligned} \tag{3.4}$$

Using (2.8), (2.13) and the relation

$$g((\nabla_W R)(X,Y)Z,U) = -g((\nabla_W R)(X,Y)U,Z),$$

we have

$$g((\nabla_W R)(\xi,Y)Z,\xi) = 0. \tag{3.5}$$

By virtue of (3.5) it follows from (3.4) that

$$\begin{aligned}(\nabla_W S)(Y,Z) &= -2A(W)S(Y,Z) - A(Y)S(W,Z) - A(Z)S(Y,W)\\ &- A(R(W,Y)Z) - A(R(W,Z)Y).\end{aligned} \tag{3.6}$$

This leads to the following:

Theorem 3.2. *A ϕ-pseudo symmetric Kenmotsu manifold is pseudo-Ricci symmetric if and only if $A(R(W,Y)Z) + A(R(W,Z)Y) = 0$.*

Setting $Z = \xi$ in (3.2) and using (2.7), (2.9) and (2.13), we get

$$\begin{aligned}&[1 - A(\xi)]R(X,Y)W\\ =\ & g(X,W)Y - g(Y,W)X + 2A(W)[\eta(X)Y - \eta(Y)X]\\ +\ & A(X)[\eta(W)Y - \eta(Y)W] + A(Y)[\eta(X)W - \eta(W)X]\\ +\ & [\eta(X)g(Y,W) - \eta(Y)g(X,W)]\rho.\end{aligned} \tag{3.7}$$

This leads to the following:

Theorem 3.3. *In a ϕ-pseudo symmetric Sasakian manifold, the curvature tensor satisfies the relation* (3.7)*.*

From (3.7), we get

$$\begin{aligned}[1 - A(\xi)]S(Y,W) &= [A(\xi) - (n-1)]g(Y,W)\\ &- 2nA(W)\eta(Y) - (n-2)A(Y)\eta(W).\end{aligned} \tag{3.8}$$

Replacing Y by ϕY and W by ϕW in (3.8), we get

$$[1 - A(\xi)]S(\phi Y,\phi W) = [A(\xi) - (n-1)]g(\phi Y,\phi W). \tag{3.9}$$

By virtue of (2.3) and (2.12), we have from (3.9) that

$$S(Y,W) = \alpha g(Y,W) + \beta\eta(Y)\eta(W), \tag{3.10}$$

where $\alpha = \frac{A(\xi)-(n-1)}{1-A(\xi)}$ and $\beta = \frac{(n-2)A(\xi)}{1-A(\xi)}$, provided $1 - A(\xi) \neq 0$.
This leads to the following:

Theorem 3.4. *A ϕ-pseudo symmetric Kenmotsu manifold is an η-Einstein manifold.*

In particular, if $A = 0$, then from (3.10), we get

$$S(Y,W) = -(n-1)g(Y,W). \tag{3.11}$$

This leads to the following:

Corollary 3.5. [10] *A ϕ-symmetric Kenmotsu manifold is an Einstein manifold.*

4. ϕ-pseudo concircularly symmetric Kenmotsu manifolds

Definition 4.1. A Kenmotsu manifold $M^n(\phi, \xi, \eta, g)(n = 2m + 1 > 3)$ is said to be a *ϕ-pseudo concircularly symmetric* if its concircular curvature tensor $\tilde{C}$, given by [36]

$$\tilde{C}(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y] \tag{4.1}$$

satisfies the relation

$$\begin{aligned} \phi^2((\nabla_W \tilde{C})(X, Y)Z) &= 2A(W)\tilde{C}(X, Y)Z + A(X)\tilde{C}(W, Y)Z \\ &+ A(Y)\tilde{C}(X, W)Z + A(Z)\tilde{C}(X, Y)W \\ &+ g(\tilde{C}(X, Y)Z, W)\rho \end{aligned} \tag{4.2}$$

for any vector field X, Y, Z and W, where A is a non-zero 1-form and r is the scalar curvature of the manifold.

Let us consider a Kenmotsu manifold $M^n(\phi, \xi, \eta, g)$ $(n = 2m + 1 > 3)$, which is ϕ-pseudo concircularly symmetric. Then by virtue of (2.1), it follows from (4.2) that

$$\begin{aligned} &-(\nabla_W \tilde{C})(X, Y)Z + \eta((\nabla_W \tilde{C})(X, Y)Z)\xi \\ = \; &2A(W)\tilde{C}(X, Y)Z + A(X)\tilde{C}(W, Y)Z + A(Y)\tilde{C}(X, W)Z \\ + \; &A(Z)\tilde{C}(X, Y)W + g(\tilde{C}(X, Y)Z, W)\rho \end{aligned} \tag{4.3}$$

from which it follows that

$$\begin{aligned} &-g((\nabla_W \tilde{C})(X, Y)Z, U) + \eta((\nabla_W \tilde{C})(X, Y)Z)\eta(U) \\ = \; &2A(W)g(\tilde{C}(X, Y)Z, U) + A(X)g(\tilde{C}(W, Y)Z, U) + A(Y)g(\tilde{C}(X, W)Z, U) \\ + \; &A(Z)g(\tilde{C}(X, Y)W, U) + g(\tilde{C}(X, Y)Z, W)A(U). \end{aligned} \tag{4.4}$$

Taking an orthonormal frame field and contracting (4.4) over X and U, and then using (2.2) and (4.1), we get

$$\begin{aligned} &-(\nabla_W S)(Y, Z) + \frac{dr(W)}{n}g(Y, Z) + g((\nabla_W \tilde{C})(\xi, Y)Z, \xi) \\ = \; &2A(W)S(Y, Z) + A(Y)S(W, Z) + A(Z)S(Y, W) \\ - \; &\frac{r}{n}[2A(W)g(Y, Z) + A(Y)g(W, Z) + A(Z)g(Y, W)] \\ + \; &A(\tilde{C}(W, Y)Z) + A(\tilde{C}(W, Z)Y). \end{aligned} \tag{4.5}$$

By virtue of (3.5), we have from (4.1) that

$$g((\nabla_W \tilde{C})(\xi, Y)Z, \xi) = -\frac{dr(W)}{n(n-1)}[g(Y, Z) - \eta(Y)\eta(Z)]. \tag{4.6}$$

In view of (4.6) it follows from (4.5) that

$$\begin{aligned}(4.7)\quad (\nabla_W S)(Y,Z) &= -2A(W)S(Y,Z) - A(Y)S(W,Z) - A(Z)S(Y,W)\\ &+ \frac{r}{n}[2A(W)g(Y,Z) + A(Y)g(W,Z) + A(Z)g(Y,W)]\\ &+ \frac{dr(W)}{n(n-1)}[(n-2)g(Y,Z) + \eta(Y)\eta(Z)]\\ &- A(\tilde{C}(W,Y)Z) - A(\tilde{C}(W,Z)Y).\end{aligned}$$

This leads to the following:

Theorem 4.2. *A ϕ-pseudo concircularly symmetric Kenmotsu manifold is pseudo-Ricci symmetric if and only if*

$$\begin{aligned}&\frac{r}{n}[2A(W)g(Y,Z) + A(Y)g(W,Z) + A(Z)g(Y,W)]\\ +\ &\frac{dr(W)}{n(n-1)}[(n-2)g(Y,Z) + \eta(Y)\eta(Z)]\\ -\ &A(\tilde{C}(W,Y)Z) - A(\tilde{C}(W,Z)Y) = 0.\end{aligned}$$

Setting $Z = \xi$ in (4.3) and using (2.7), (2.9), (2.13) and (4.1), we get

$$\begin{aligned}(4.8)\qquad & [1 - A(\xi)]R(X,Y)W\\ =\ & \Big[\frac{rA(\xi)}{n(n-1)} + 1\Big]\big[g(X,W)Y - g(Y,W)X\big]\\ -\ & \frac{dr(W)}{n(n-1)}\big[\eta(Y)X - \eta(X)Y\big] - \Big[\frac{r}{n(n-1)} + 1\Big]\big[2A(W)\{\eta(Y)X\\ -\ & \eta(X)Y\} + A(X)\{\eta(Y)W - \eta(W)Y\} + A(Y)\{\eta(W)X - \eta(X)W\}\\ +\ & \{\eta(Y)g(X,W) - \eta(X)g(Y,W)\}\rho\big].\end{aligned}$$

This leads to the following:

Theorem 4.3. *In a ϕ-pseudo concircularly symmetric Kenmotsu manifold, the curvature tensor satisfies the relation* (4.8).

From (4.8), we get

$$\begin{aligned}(4.9)\quad [1 - A(\xi)]S(Y,W) &= \Big[A(\xi) - \frac{(n-2)rA(\xi)}{n(n-1)} - (n-1)\Big]g(Y,W)\\ &- \frac{dr(W)}{n}\eta(Y) - \Big[\frac{r}{n(n-1)} + 1\Big]\big[2nA(W)\eta(Y)\\ &+ (n-2)A(Y)\eta(W)\big].\end{aligned}$$

Replacing Y by ϕY and W by ϕW in (4.9), we get

$$(4.10)\quad [1 - A(\xi)]S(\phi Y,\phi W) = \Big[A(\xi) - \frac{(n-2)rA(\xi)}{n(n-1)} - (n-1)\Big]g(\phi Y,\phi W).$$

By virtue of (2.3) and (2.12), we have from (4.10) that

$$S(Y,W) = \gamma g(Y,W) + \delta\eta(Y)\eta(W), \tag{4.11}$$

where $\gamma = \frac{1}{1-A(\xi)}\Big[A(\xi) - \frac{(n-2)rA(\xi)}{n(n-1)} - (n-1)\Big]$ and $\delta = \frac{(n-2)A(\xi)}{1-A(\xi)}\Big[\frac{r}{n(n-1)} + 1\Big]$, provided $1 - A(\xi) \neq 0$.
This leads to the following:

Theorem 4.4. *A ϕ-pseudo concircularly symmetric Kenmotsu manifold is an η-Einstein manifold.*

5. ϕ-pseudo Ricci symmetric Kenmotsu manifolds

Definition 5.1. A Kenmotsu manifold $M^n(\phi,\xi,\eta,g)$ $(n = 2m+1 > 3)$ is said to be *ϕ-pseudo Ricci symmetric* if the Ricci operator Q satisfies

$$\phi^2((\nabla_X Q)(Y)) = 2A(X)QY + A(Y)QX + S(Y,X)\rho \tag{5.1}$$

for any vector field X, Y, where A is a non-zero 1-form.

If, in particular, $A = 0$, then (5.1) turns into the notion of ϕ-Ricci symmetric Kenmotsu manifold introduced by Shukla and Shukla [28].

Let us take a Kenmotsu manifold $M^n(\phi,\xi,\eta,g)$ $(n = 2m+1 > 3)$, which is ϕ-pseudo Ricci symmetric. Then by virtue of (2.1) it follows from (5.1) that

$$-(\nabla_X Q)(Y) + \eta((\nabla_X Q)(Y))\xi = 2A(X)QY + A(Y)QX + S(Y,X)\rho$$

from which it follows that

$$\begin{aligned} & -g(\nabla_X Q(Y), Z) + S(\nabla_X Y, Z) + \eta((\nabla_X Q)(Y))\eta(Z) \\ = \;& 2A(X)S(Y,Z) + A(Y)S(X,Z) + S(Y,X)A(Z). \end{aligned} \tag{5.2}$$

Putting $Y = \xi$ in (5.2) and using (2.4) and (2.10), we get

$$[A(\xi) - 1]S(X,Z) = (n-1)[g(X,Z) + 2A(X)\eta(Z) + A(Z)\eta(X)]. \tag{5.3}$$

Replacing X by ϕX and Z by ϕZ in (5.3) and using (2.1), we get

$$[A(\xi) - 1]S(\phi X, \phi Z) = (n-1)g(\phi X, \phi Z). \tag{5.4}$$

In view of (2.3) and (2.12), we have from (5.4) that

$$S(X,Z) = \frac{(n-1)}{A(\xi)-1}g(X,Z) + \frac{(n-1)A(\xi)}{1-A(\xi)}\eta(X)\eta(Z),\ 1 - A(\xi) \neq 0, \tag{5.5}$$

which implies that the manifold under consideration is η-Einstein. Thus we can state the following:

Theorem 5.2. *Every ϕ-pseudo Ricci symmetric Kenmotsu manifold is an η-Einstein manifold.*

In particular, if $A = 0$, then from (5.5), we obtain

$$S(X,Z) = -(n-1)g(X,Z). \tag{5.6}$$

This leads to the following:

Corollary 5.3. [28] *A ϕ-Ricci symmetric Kenmotsu manifold is an Einstein manifold.*

Acknowledgement

The author gratefully acknowledges the UGC Minor Research Project [Project No. F. PSW-018/11-12(ERO)], India. He also wishes to express his sincere thanks to an unknown reviewer for his/her valuable suggestions towards to the improvement of the paper.

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Received by the editors January 25, 2012