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LIGHTLIKE HYPERSURFACES OF INDEFINITE COSYMPLECTIC MANIFOLDS WITH PARALLEL SYMMETRIC BILINEAR FORMS[1]

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Abstract

In this paper we study the lightlike hypersurface of an indefinite cosymplectic manifold with parallel symmetric bilinear forms which are tangent to the structure vector field.

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1 Introduction

In the theory of submanifolds of semi-Riemannian manifolds it is interesting to study the geometry of lightlike submanifolds due to the fact that the intersection of a normal vector bundle and the tangent bundle is non-trivial making it more interesting and remarkably different from the study of non-degenerate submanifolds. The geometry of lightlike hypersurfaces and submanifolds of indefinite Kaehler manifolds was studied by Duggal and Bejancu [4]. On the other hand, lightlike hypersurfaces of indefinite Sasakian manifolds was studied in [2, 5], whereas lightlike hypersurfaces in indefinite cosymplectic space form was studied in [6]. In this paper we study lightlike hypersurface of an indefinite cosymplectic manifold with parallel symmetric bilinear forms.

2 Preliminaries

An odd-dimensional semi-Riemannian manifold $\overline{M}$ is said to be an indefinite almost contact metric manifold if there exist structure tensors $\{\overline{\phi}, \xi, \eta, \overline{g}\}$, where $\overline{\phi}$ is a (1,1) tensor field, ξ a vector field, η a 1-form and $\overline{g}$ is the semi-Riemannian metric on $\overline{M}$ satisfying

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$$
(2.1)\qquad \begin{cases} \overline{\phi}^2\,\overline{X} = -\overline{X} + \eta(\overline{X})\xi, \quad \eta\circ\overline{\phi} = 0, \quad \overline{\phi}\xi = 0, \quad \eta(\xi) = 1 \\ \overline{g}(\overline{\phi X}, \overline{\phi Y}) = \overline{g}(\overline{X}, \overline{Y}) - \varepsilon\eta(\overline{X})\eta(\overline{Y}), \\ \eta(\overline{X}) = \varepsilon\overline{g}(\overline{X}, \xi), \overline{g}(\xi, \xi) = \varepsilon, \varepsilon = \pm 1 \end{cases}
$$

for any $\overline{X}, \overline{Y} \in \Gamma(T\overline{M})$, where $\Gamma(T\overline{M})$ denotes the Lie algebra of vector fields on $\overline{M}$.

An indefinite almost contact metric manifold $\overline{M}$ is called an indefinite cosymplectic manifold if [6]

$$
(2.2)\qquad (\overline{\nabla}_{\overline{X}}\overline{\phi})\overline{Y} = 0, \text{ and } \overline{\nabla}_{\overline{X}}\xi = 0
$$

for any $\overline{X}, \overline{Y} \in T\overline{M}$, where $\overline{\nabla}$ denote the Levi-Civita connection on $\overline{M}$.

A plane section $\prod$ in $T_x\overline{M}$ of a cosymplectic manifold $\overline{M}$ is called a $\overline{\phi}$-section if it is spanned by a unit vector $\overline{X}$ orthogonal to ξ and $\overline{\phi X}$, where $\overline{X}$ is a non null vector field on $\overline{M}$. The sectional curvature $\mathrm{K}(\prod)$ with respect to $\prod$ determined by $\overline{X}$ is called a ϕ-sectional curvature. If $\overline{M}$ has a $\overline{\phi}$-sectional curvature c which does not depend on the $\overline{\phi}$-section at each point then c is a constant in $\overline{M}$ and $\overline{M}$ is called an indefinite cosymplectic space form which is denoted by $\overline{M}(c)$. The curvature tensor $\overline{R}$ of $\overline{M}(c)$ is given by [6]

$$
(2.3)\qquad \begin{aligned} \overline{R}(\overline{X}, \overline{Y})\overline{Z} = \tfrac{c}{4}\{&\overline{g}(\overline{Y}, \overline{Z})\overline{X} - \overline{g}(\overline{X}, \overline{Z})\overline{Y} + \eta(\overline{X})\eta(\overline{Z})\overline{Y} - \eta(\overline{Y})\eta(\overline{Z})\overline{X} \\ &+\overline{g}(\overline{X}, \overline{Z})\eta(\overline{Y})\xi - \overline{g}(\overline{Y}, \overline{Z})\eta(\overline{X})\xi + \overline{g}(\overline{\phi Y}, \overline{Z})\overline{\phi X} - \overline{g}(\overline{\phi X}, \overline{Z})\overline{\phi Y} \\ &-2\overline{g}(\overline{\phi X}, \overline{Y})\overline{\phi Z})\} \end{aligned}
$$

for any $\overline{X}, \overline{Y}, \overline{Z} \in \Gamma(T\overline{M})$.

Let (M, g) be a hypersurface of a $(2n+1)$-dimensional semi-Riemannian manifold $(\overline{M}, \overline{g})$ with index s, $0 < s < 2n+1$ and $g = \overline{g}_{|M}$. Then M is a lightlike hypersurface of $\overline{M}$ if g is of constant rank $(2n-1)$ and the normal bundle $TM^{\perp}$ is a distribution of rank 1 on M [4]. A non-degenerate complementary distribution $S(TM)$ of rank $(2n-1)$ to $TM^{\perp}$ in TM, that is, $TM = TM^{\perp} \perp S(TM)$, is called screen distribution. The following result (cf. [4], Theorem 1.1, page 79) has an important role in studying the geometry of lightlike hypersurfaces.

Theorem A. *Let $(M, g, S(TM))$ be a lightlike hypersurface of $\overline{M}$. Then, there exists a unique vector bundle $tr(TM)$ of rank 1 over M such that for any non-zero section E of $TM^{\perp}$ on a coordinate neighbourhood $U \subset M$, there exists a unique section N of $tr(TM)$ on U satisfying $\overline{g}(N, E) = 1$ and $\overline{g}(N, N) = \overline{g}(N, W) = 0$, $\forall\ W \in \Gamma(S(TM)|_U)$.*

Then, we have the following decomposition:

$$
(2.4)\qquad TM = S(TM)\perp TM^{\perp}, \quad T\overline{M} = S(TM)\perp(TM^{\perp} \oplus tr(TM)).
$$

Throughout this paper, all manifolds are supposed to be paracompact and smooth. We denote by $\Gamma(E)$ the smooth sections of the vector bundle E, and by $\perp$ and $\oplus$ the orthogonal and the non-orthogonal direct sum of two vector bundles, respectively.

Let $\overline{\nabla}$, ∇ and ∇^t denote the linear connections on $\overline{M}$, M and vector bundle $tr(TM)$, respectively. Then, the Gauss and Weingarten formulae are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y), \ \forall X, Y \in \Gamma(TM), \tag{2.5}$$

$$\overline{\nabla}_X V = -A_V X + \nabla^t_X V, \ \forall V \in \Gamma(tr(TM)), \tag{2.6}$$

where $\{\nabla_X Y, A_V X\}$ and $\{h(X,Y), \nabla^t_X V\}$ belong to $\Gamma(TM)$ and $\Gamma(tr(TM))$, respectively and A_V is the shape operator of M with respect to V. Moreover, in view of decomposition (2.4), equations (2.5) and (2.6) take the form

$$\overline{\nabla}_X Y = \nabla_X Y + B(X,Y)N \tag{2.7}$$

$$\overline{\nabla}_X N = -A_N X + \tau(X)N \tag{2.8}$$

for any $X, Y \in \Gamma(TM)$ and $N \in \Gamma(tr(TM))$, where $B(X,Y)$ and $\tau(X)$ are local second fundamental form and a 1-form on U, respectively. It follows that

$$B(X,Y) = \overline{g}(\overline{\nabla}_X Y, E) = \overline{g}(h(X,Y), E), B(X,E) = 0, \text{ and}$$

$$\tau(X) = \overline{g}(\nabla^t_X N, E).$$

Let P denote the projection of TM on $S(TM)$ and ∇^*, ∇^{*t} denote the linear connections on $S(TM)$ and $TM^\perp$, respectively. Then from the decomposition of tangent bundle of lightlike hypersurface, we have

$$\nabla_X PY = \nabla^*_X PY + h^*(X, PY) \tag{2.9}$$

$$\nabla_X E = -A^*_E X + \nabla^{*t}_X E \tag{2.10}$$

for any $X, Y \in \Gamma(TM)$ and $E \in \Gamma(TM^\perp)$, where h^*, A^* are the second fundamental form and the shape operator of distribution $S(TM)$ respectively.

By direct calculations using the Gauss-Weingarten formulae, (2.9) and (2.10), we find

$$g(A_N Y, PW) = \overline{g}(N, h^*(Y, PW)); \quad \overline{g}(A_N Y, N) = 0 \tag{2.11}$$

$$g(A^*_E X, PY) = \overline{g}(E, h(X, PY)); \quad \overline{g}(A^*_E X, N) = 0 \tag{2.12}$$

for any $X, Y, W \in \Gamma(TM)$, $E \in \Gamma(TM^\perp)$ and $N \in \Gamma(tr(TM))$.

Locally, we define on U

$$C(X, PY) = \overline{g}(h^*(X, PY), N), \text{ and } \lambda(X) = \overline{g}(\nabla^{*t}_X E, N). \tag{2.13}$$

Hence,

$$h^*(X, PY) = C(X, PY)E, \text{ and } \nabla^{*t}_X E = \lambda(X)E. \tag{2.14}$$

On the other hand, by using (2.7), (2.8), (2.10) and (2.13), we obtain

$$\lambda(X) = \overline{g}(\nabla_X E, N) = \overline{g}(\overline{\nabla}_X E, N) = -\overline{g}(E, \overline{\nabla}_X N) = -\tau(X).$$

Thus, locally (2.9) and (2.10) become

$$\nabla_X PY = \nabla^*_X PY + C(X, PY)E, \text{ and } \nabla_X E = -A^*_E X - \tau(X)E. \tag{2.15}$$

Finally, (2.11) and (2.12), locally become

$$g(A_N Y, PW) = C(Y, PW); \quad \overline{g}(A_N Y, N) = 0, \tag{2.16}$$

$$g(A^*_E X, PY) = B(X, PY); \quad \overline{g}(A^*_E X, N) = 0. \tag{2.17}$$

In general, the induced connection ∇ on M is not a metric connection. Since $\overline{\nabla}$ is a metric connection, we have

$$0 = (\overline{\nabla}_X \overline{g})(Y, Z) = X(\overline{g}(Y, Z)) - \overline{g}(\overline{\nabla}_X Y, Z) - \overline{g}(Y, \overline{\nabla}_X Z).$$

By using (2.7) in this equation, we obtain

$$(\nabla_X g)(Y, Z) = B(X, Y)\theta(Z) + B(X, Z)\theta(Y) \ X, Y, Z \in \Gamma(S(TM)|_u), \tag{2.18}$$

where θ is a differential 1-form locally defined on M by $\theta(\cdot) = \overline{g}(N, \cdot)$.

If $\overline{R}$ and R are the curvature tensors of $\overline{M}$ and M, then using (2.7) in the equation

$$\overline{R}(X, Y)Z = \overline{\nabla}_X \overline{\nabla}_Y Z - \overline{\nabla}_Y \overline{\nabla}_X Z - \overline{\nabla}_{[X,Y]} Z,$$

we obtain

$$\begin{aligned} \overline{R}(X, Y)Z &= R(X, Y)Z + B(X, Z)A_N Y - B(Y, Z)A_N X \\ &+\{(\nabla_X B)(Y, Z) - (\nabla_Y B)(X, Z) + \tau(X)B(Y, Z) - \tau(Y)B(X, Z)\}N \end{aligned} \tag{2.19}$$

$$(\nabla_X B)(Y, Z) = XB(Y, Z) - B(\nabla_X Y, Z) - B(Y, \nabla_X Z) \tag{2.20}$$

3 Lightlike hypersurfaces of indefinite cosymplectic manifolds

Let $(\overline{M}, \overline{\phi}, \xi, \eta, \overline{g})$ be an indefinite cosymplectic manifold and (M, g) be its lightlike hypersurface, tangent to the structure vector field ξ with $\overline{g}(\xi, \xi) = \varepsilon = +1$.

If E is a local section of $TM^\perp$, then $\overline{g}(\overline{\phi}E, E) = 0$ implies that $\overline{\phi}E$ is tangent to M. Thus $\overline{\phi}(TM^\perp)$ is a distribution on M of rank 1 such that $\overline{\phi}(TM^\perp) \bigcap TM^\perp = \{0\}$. This enables us to choose a screen distribution $S(TM)$ such that it contains $\phi(TM^\perp)$ as vector subbundle.

Now, we consider a local section N of $tr(TM)$. Then $\overline{\phi}N$ is tangent to M and belongs to $S(TM)$ as $\overline{g}(\overline{\phi}N, E) = -\overline{g}(N, \overline{\phi}E) = 0$ and $\overline{g}(\overline{\phi}N, N) = 0$.

From (2.1), we have

$$\overline{g}(\overline{\phi}N, \overline{\phi}E) = \overline{g}(N, E) - \eta(N)\eta(E) = \overline{g}(N, E) = 1.$$

Therefore, $\overline{\phi}(TM^{\perp}) \oplus \overline{\phi}(tr(TM))$ is a direct sum but not orthogonal, and is a non-degenerate vector subbundle of $S(TM)$ of rank 2.

It is known [1] that if M is tangent to the structure vector field ξ, then ξ belongs to $S(TM)$. Since $\overline{g}(\overline{\phi}E, \xi) = \overline{g}(\overline{\phi}N, \xi) = 0$, there exists a non-degenerate invariant distribution D_0 of rank $(2n-4)$ on M such that

$$S(TM) = \{\overline{\phi}(TM^{\perp}) \oplus \overline{\phi}(tr(TM))\}\perp D_0 \perp <\xi> \text{ and } \overline{\phi}(D_0) = D_0 \tag{3.1}$$

where $<\xi> =$ span ξ.

Moreover, from (2.4) and (3.1), we obtain

$$TM = \{\overline{\phi}(TM^{\perp}) \oplus \overline{\phi}(tr(TM))\}\perp D_0 \perp <\xi> \perp TM^{\perp}. \tag{3.2}$$

Now, we consider the distributions D and D' on M as follows

$$D = TM^{\perp} \perp \overline{\phi}(TM^{\perp}) \perp D_0, \ D' = \overline{\phi}(tr(TM)).$$

Then D is invariant under $\overline{\phi}$ and

$$TM = D \oplus D' \perp <\xi> . \tag{3.3}$$

If P_1 and Q denote the projection morphisms of TM on D and D' and $U = -\overline{\phi}N, V = -\overline{\phi}E$ are local lightlike vectors, respectively, then we write

$$X = P_1 X + QX + \eta(X)\xi \tag{3.4}$$

for $X \in \Gamma(TM)$, where $QX = u(X)U$, and u is a differential 1-form locally defined on M by $u(\cdot) = g(V, \cdot)$.

From (3.1) and (3.4), we obtain

$$\overline{\phi}X = \phi X + u(X)N \text{ and } \phi^2 X = -X + \eta(X)\xi + u(X)U, \ \forall X \in \Gamma(TM)$$

where ϕ is a tensor field of type (1, 1) defined on M by $\phi X = \overline{\phi}P_1 X$.

Applying ϕ to $\phi^2 X$ and using the fact that $\phi U = 0$, we obtain

$$\phi^3 + \phi = 0$$

which shows that ϕ is an f-structure [3] of constant rank.

Using (2.1), we get

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) - u(Y)\nu(X) - u(X)\nu(Y),$$

where ν is a 1-form locally defined on M by $\nu(\cdot) = g(U, \cdot)$.

From direct calculations, we have

$$\nabla_X \xi = 0 \tag{3.5}$$

$$B(X, \xi) = 0 \text{ for any vector field } X \in \Gamma(TM). \tag{3.6}$$

The Lie derivative with respect to the vector field V is given by

$$(L_V g)(X, Y) = Xu(Y) + Yu(X) + u([X, Y]) - 2u(\nabla_X Y)$$

for any $X, Y \in \Gamma(TM)$.

4 Lightlike real hypersurfaces with parallel symmetric bilinear forms

Let $\overline{M}(c)$ be an indefinite cosymplectic space form and M be a real lightlike hypersurface of $\overline{M}(c)$. Let us consider the pair $\{E, N\}$ on $U \subset M$ as in Theorem A. Then, using (2.19), we obtain

$$\overline{g}(\overline{R}(X,Y)Z,E) = (\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) + \tau(X)B(Y,Z) - \tau(Y)B(X,Z) \tag{4.1}$$

for any $X, Y, Z \in \Gamma(TM|_U)$. From (4.1) and (2.3), we have

$$(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) = \tau(Y)B(X,Z) - \tau(X)B(Y,Z) + \tfrac{c}{4}\{\overline{g}(\overline{\phi}Y,Z)u(X) - \overline{g}(\overline{\phi}X,Z)u(Y) - 2\overline{g}(\overline{\phi}X,Y)u(Z)\}. \tag{4.2}$$

Definition 4.1. [2] (a) A distribution Ξ on M is a Killing distribution (respectively $D\perp\xi$- Killing distribution) if $(L_X g)(Y,Z) = 0$, for any $X \in \Gamma(\Xi)$ and $Y, Z \in \Gamma(TM)$ (respectively $Y, Z \in \Gamma(D\perp <\xi>)$).

(b) A distribution Ξ on M is parallel (respectively $D\perp\xi$-parallel) if $\nabla_X Y \in \Gamma(\Xi)$, for any $X \in \Gamma(TM)$ (respectively $X \in \Gamma(D\perp <\xi>)$) and $Y \in \Gamma(\Xi)$.

We prove the following theorem.

Theorem 4.2. *Let M be a lightlike hypersurface of an indefinite cosymplectic space form $\overline{M}(c)$ of constant curvature c. Then the Lie-derivative of the local second fundamental form B with respect to ξ is given by*

$$(L_\xi B)(X,Y) = -\tau(\xi)B(X,Y), \ \forall X, Y \in \Gamma(TM). \tag{4.3}$$

Proof. Replacing Z with ξ in (2.20) and using (3.5), we obtain

$$(\nabla_X B)(\xi, Y) = 0. \tag{4.4}$$

By direct calculation, we have

$$(\nabla_\xi B)(X,Y) = (L_\xi B)(X,Y). \tag{4.5}$$

From (4.4) and (4.5), we obtain

$$(\nabla_\xi B)(X,Y) - (\nabla_X B)(\xi,Y) = (L_\xi B)(X,Y). \tag{4.6}$$

From (4.2), we obtain

$$(\nabla_\xi B)(X,Y) - (\nabla_X B)(\xi,Y) = -\tau(\xi)B(X,Y). \tag{4.7}$$

Using (4.6) and (4.7), we obtain the result. □

Definition 4.3. [2] A lightlike hypersurface M is said to be totally geodesic (respectively $D\perp\xi$ or D'-totally geodesic) if $B(X,Y) = 0$, for any $X, Y \in \Gamma(TM)$ (respectively $X, Y \in \Gamma(D\perp <\xi>)$ or $\Gamma(D')$).

From Theorem 4.2, we have the following result.

Corollary 4.4. *Let M be a lightlike hypersurface of an indefinite cosymplectic space form $\overline{M}(c)$ of constant curvature c, with $\xi \in TM$. Then ξ is a Killing vector field with respect to the local second fundamental form B if and only if $\tau(\xi) = 0$ or M is totally geodesic.*

The second fundamental form h is said to be parallel if $(\nabla_Z h)(X, Y) = 0$, which implies that

$$(\nabla_Z B)(X, Y) = -\tau(Z)B(X, Y) \; \forall X, Y, Z \in \Gamma(TM). \tag{4.8}$$

Hence, in general, the parallelism of h does not imply the parallelism of B and vice versa. Moreover,

$$(\nabla_Z h)(X, E) = (\nabla_Z B)(X, E)N.$$

Theorem 4.5. *Let M be a lightlike hypersurface of an indefinite cosymplectic space form of constant curvature c. If the local second fundamental form B is parallel on M and $\tau(\xi) \neq 0$, then M is totally geodesic.*

Proof. The result follows from (4.3), (4.4) and the parallelism of local second fundamental form B. □

Proposition 4.6. *There exists no lightlike hypersurface of indefinite cosymplectic space forms $\overline{M}(c)(c \neq 0)$ with parallel second fundamental form.*

Proof. Suppose $c \neq 0$ and the second fundamental form is parallel. Then, if we take $Y = E$ and $Z = U$ in (4.8), we obtain that $\frac{c}{4}u(X) = 0$. Taking $X = U$, we have $c = 0$, which is a contradiction. □

We have the following theorem.

Proposition 4.7. *Let M be the lightlike hypersurface of an indefinite cosymplectic space form $\overline{M}(c)$ of constant curvature c such that its local second fundamental form B is parallel. If $\tau(\xi) \neq 0$, then $c = 0$ if and only if M is D' totally geodesic.*

Proof. Suppose B is parallel. Then, taking $Y = E$ in (4.2), we obtain

$$3\frac{c}{4}u(X)u(Z) = \tau(E)B(X, Z).$$

Taking $X = Z = U$, we have $3\frac{c}{4} = \tau(E)B(U, U)$ and if $\tau(E) \neq 0$, then the equivalence follows. □

Theorem 4.8. *Let M be a lightlike hypersurface of an indefinite cosymplectic manifold $(\overline{M}, \overline{g})$ with $\xi \in \Gamma(TM)$. If the second fundamental form h of M is parallel, then*

$$(L_E B)(X, Y) = -\tau(E)B(X, Y) \; \forall X, Y \in \Gamma(TM). \tag{4.9}$$

Proof. Taking $Z = E$ in $(\nabla_Z h)(X, Y) = 0$, we have

$$(\nabla_E B)(X, Y) = -\tau(E)B(X, Y). \tag{4.10}$$

Now,

$$(L_E B)(X, Y) = (\nabla_E B)(X, Y) - 2B(A^*_E X, Y). \tag{4.11}$$

On the other hand,

$$0 = \overline{g}((\nabla_X h)(Y, E), E) = B(A^*_E X, Y). \tag{4.12}$$

Using (4.11) and (4.9) in (4.10), we have

$$(L_E B)(X, Y) = -\tau(E)B(X, Y).$$

□

The following Corollary follows from Theorem 4.8.

Corollary 4.9. *Let M be a lightlike hypersurface with parallel second fundamental form of an indefinite cosymplectic manifold $(\overline{M}, \overline{g})$ with $\xi \in \Gamma(TM)$. Then, M is totally geodesic or $\tau(E) = 0$ if $(L_E B)(X, Y) = 0, \forall X, Y \in \Gamma(TM)$ and $E \in \Gamma(TM^{\perp})$.*

We now prove the following theorem.

Theorem 4.10. *Let M be the lightlike hypersurface of an indefinite cosymplectic manifold $\overline{M}$ with $\xi \in TM$. Then M is $D\perp < \xi >$-totally geodesic if and only if for any $X \in \Gamma(D\perp < \xi >)$, $A^*_E X = u(A_N X)V$.*

Proof. Since $A^*_E X \in \Gamma(S(TM))$ and

$$S(TM) = \{\overline{\phi}(TM^{\perp}) \oplus \overline{\phi}(tr(TM))\}\perp D_0 \perp < \xi > .$$

We can write

$$A^*_E X = \sum_{i=1}^{2n-4} \frac{B(X, F_i)}{g(F_i, F_i)} F_i + B(X, V)U + B(X, U)V, \text{ where } B(X, \xi) = 0.$$

Now, if M is $D\perp < \xi >$ totally geodesic then $B(X, Y) = 0, \forall X, Y \in D\perp < \xi >$. But, we have $B(X, U) = g(A_N X, V) = u(A_N X)$. So, we have the required result. □

References

[1] Calin, C., Contribution to geometry of CR-submanifold. Ph.D thesis, University of Iasi, Romania, 1998.

[2] Massambae, F., Lightlike hypersurfaces of indefnite Sasakian manifolds with parallel symmetric bilinear forms. Differential Geometry - Dynamical Systems, 10 (2008), 226-234.

[3] Duggal, K. L., Bejancu, A., Light-like CR hypersurfaces of indefinite Kaehler manifolds. Acta Appl. Math. 31(1993), 171-190.

[4] Duggal, K. L., Bejancu, A., Lightlike submanifolds of semi-Riemannian manifolds and applications. Dordrecht: Kluwer Academic Publishers 1996.

[5] Kang, T. H., Jung, S. D., Kim, B. H., Pak, H. K., Pak, J. S., Lightlike hypersurfaces of indefinite Sasakian manifolds. Indian J. Pure and Appl. Math. 34(9) (2003), 1369-1380.

[6] Kang, T. H., Kim, S. K., Lightlike Hypersurfaces of Indefinite Cosymplectic Manifolds. International Mathematical Forum, 2(67) (2007), 3303 - 3316.

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