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BEST APPROXIMATION IN PROBABILISTIC 2-NORMED SPACES

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Abstract. In this article, we studied the best approximation in probabilistic 2-normed spaces. We defined the best approximation on these spaces and generalized some definitions such as set of best approximation, P_b-proximinal set and P_b-approximately compact and orthogonality relative to any set and proved some theorems about them.

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1. Introduction

In [5], K. Menger introduced the notion of probabilistic metric spaces. The idea of K. Menger was to use distribution function instead of non-negative real numbers as values of the metric. The concept of probabilistic normed spaces (briefly, PN-spaces) was introduced by A. N. Sertnev in 1963, [7].

The main aim of this paper is to develop best approximation theory in 2-normed spaces. In [8], M. Shams and S. M. Vaezpour get some results in probabilistic normed spaces. We want to extend those in 2-normed linear space.

In the sequel, after an introduction to probabilistic 2-normed spaces, we define the concept of best approximation in probabilistic 2-normed space and generalize some definitions such as set of best approximation, proximinal set and approximatively compact set [1, 2, 4, 6, 8].

A distance distribution function (briefly, $d.d.f.$), is a function F defined from the extended interval $[0, +\infty]$ into the unit interval $I = [0, 1]$, that is nondecreasing and left-continuous on $(0, +\infty)$ such that $F(0) = 0$ and $F(+\infty) = 1$. The family of all $d.d.f{'}s$ will be denoted by Δ^+, and we denote

$$\mathcal{D}^+ = \{F \in \Delta^+ \mid lim_{t\to\infty}F(t) = 1\}.$$

By setting $F \leq G$ whenever $F(t) \leq G(t)$, for all $t \in \mathbb{R}^+$, one introduces a natural ordering in $\mathcal{D}^+$. If $a \in \mathbb{R}^+$ then H will be an element of $\mathcal{D}^+$, defined by $H(t) = 0$ if $t \leq 0$ and $H(t) = 1$ if $t > 0$. It is obvious that $H \geq F$ if $t > 0$ for all $F \in \mathcal{D}^+$.

A t-norm T is a two-place function $T : I \times I \longrightarrow I$, which is associative, commutative, non-decreasing in each place, and such that $T(a, 1) = a$, for all

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$a \in [0, 1]$.

Let T be a t-norm and T^* is the function given by

$$T^*(x, y) = 1 - T(1 - x, 1 - y)$$

for all $x, y \in I$. Then T^* the t-conorm of T.

A triangle function is a mapping $\tau : \Delta^+ \times \Delta^+ \longrightarrow \Delta^+$, which is associative, commutative, non-decreasing, and for which H is an identity, that is, $\tau(H, F) = F$, for every $F \in \mathcal{D}^+$.

Definition 1.1. *Let V be a linear space of dimension greater than 1 over the field $\mathbb{R}$ of real numbers. Suppose $\|.,.\|$ is a real-valued function on $V \times V$ satisfying the following conditions:*
a) $\|x, y\| = 0$ *if and only if x and y are linearly dependent vectors.*
b) $\|x, y\| = \|y, x\|$ *for all $x, y \in V$.*
c) $\|\lambda x, y\| = |\lambda| \|x, y\|$ *for all $\lambda \in \mathbb{R}$ and $x, y \in V$.*
d) $\|x + y, z\| \leq \|x, z\| + \|y, z\|$ *for all $x, y, z \in V$.*

Then $\|.,.\|$ is called a 2-norm on V and $(V, \|.,.\|)$ is called a 2-normed linear space.

Definition 1.2. *Let V be a linear space of dimension greater than 1 over filed $\mathbb{R}$ of real numbers, τ a triangle function, and let $\mathcal{F}$ be a mapping from $V \times V$ into $\mathcal{D}^+$ satisfying the following conditions:*
a) $F_{x,y} = H$ *if and only if x and y are linearly dependent vectors.*
b) $F_{x,y} \neq H$ *if and only if x and y are linearly independent vectors.*
c) $F_{x,y} = F_{y,x}$, *for all $x, y \in V$.*
d) $F_{\alpha x,y} = F_{x,y}(\frac{t}{|\alpha|})$, *for every $t > 0$, $\alpha \neq 0$, $\alpha \in \mathbb{R}$ and $x, y \in V$.*
e) $F_{x+y,z} \geq \tau(F_{x,z}, F_{y,z})$ *for all $x, y, z \in V$.*

Then, $\mathcal{F}$ is called a probabilistic 2-norm on V and $(V, \mathcal{F}, \tau)$ is called a probabilistic 2-normed linear space (briefly P-2NL space), and $\mathcal{F}$ is a strong probabilistic 2-norm if $b \in V$ and $t > 0$, $x \longrightarrow F_{x,b}(t)$ is a continuous map on V.

If the triangle inequality (e) is formulated under a t-norm T:
(f) $F_{x+y,z}(t_1 + t_2) \geq T(F_{x,z}(t_1), F_{y,z}(t_2)), for\ all\ x, y, z \in V,\ t_1, t_2 \in \mathbb{R}^+$,
then the triple $(V, \mathcal{F}, T)$ is called a Menger probabilistic 2-normed linear space.

If T is a left-continuous t-norm and τ_T is the associated triangle function, then the inequalities (e) and (f) are equivalent.

Remark 1.3. *It is easy to check that every 2-normed linear space $(V, \|.,.\|)$ can be made a probabilistic 2-normed linear space, in a natural way, by setting $F_{x,y} = H(t - \|x, y\|)$, for every $x, y \in V$, $t \in \mathbb{R}^+$ and $T = Min$.*

Definition 1.4. *Let $G \in \Delta^+$ be different from H, let $(V, \|.,.\|)$ be a 2-normed linear space. Define $\mathcal{F}$ as a mapping from $V \times V$ into Δ^+, by $F_{x,y} = H$ for every $x, y \in V$, if x and y are linearly dependent and*

$$F_{x,y}(t) := G(\frac{t}{\|x, y\|}) \qquad (t > 0)$$

when x and y are linearly independent. The pair $(V, \mathcal{F})$ is called the simple space generated by $(V, \|.,.\|)$ and G.

Let $(V, \|.,.\|)$ be a 2-normed linear space. Define $\tau(F,G)(x) = F(x).G(x)$ for every $F, G \in \Delta^+$ and for each $b \in V$, $F^{\|.,.\|}_{x,b}(t) = \frac{t}{(t+\|x,b\|)}$ for every $x \in V$, then $F^{\|.,.\|}$ is a $P-2$ *norm* which is called the standard $P-2$ *norm* induced by $\|.,.\|$.

I. Golet in [3] proved that if $(V, \mathcal{F}, \tau)$ is a probabilistic 2-normed space and $\mathcal{A}$ is the family of all finite and non-empty subsets of the linear space V, then for every $A \in \mathcal{A}$, $\varepsilon > 0$ and $\lambda \in (0,1)$, $(V, \mathcal{F}, \tau)$ is a Hausdorff topological space in the topology τ induced by the family of (ε, λ)-neighborhoods of x_0 vector:

$$\nu_{x_0} = \{N_{x_0}(\varepsilon, \lambda, A) : \ \varepsilon > 0, \ \lambda \in (0,1), \ A \in \mathcal{A}\}$$

where

$$N_{x_0}(\varepsilon, \lambda, A) = \{x \in V : \ F_{x_0 - x, a}(\varepsilon) > 1 - \lambda, \ a \in A\}$$

under a continuous triangle function τ such that $\tau \geq \tau_{T_m}$, where $T_m(a,b) = max\{a+b-1, 0\}$.

Example 1.5. *Let $(V, \|.,.\|)$ be a 2-normed linear space. Define:*

$$F_{x,y}(t) = \begin{cases} 0 & \text{when} \quad t \leq \|x,y\|, \\ 1 & \text{when} \quad \|x,y\| < t. \end{cases}$$

Then $(V, \mathcal{F}, \tau = min)$ is a P-2NL space.

2. P_b-best approximation in probabilistic 2-normed space

Definition 2.1. *Let A be a nonempty subset of P-2NL space $(V, \mathcal{F}, \tau)$. For $x, b \in V$, $t > 0$, let*

$$F_{x-A,b}(t) = sup\{F_{x-y,b}(t) : \ y \in A\}.$$

An element $y_0 \in A$ is said to be a P_b-best approximation to x from A if

$$F_{x-y_0,b}(t) = F_{x-A,b}(t).$$

We shall denote by $P^t_{A,b}(x)$ the set of elements of P_b-best approximation of x by elements of the set A, i.e.

$$P^t_{A,b}(x) = \{y \in A : F_{x-y,b}(t) = F_{x-A,b}(t)\}.$$

Also we introduce

$$e^t_{A,b}(x) = 1 - F_{x-A,b}(t).$$

Definition 2.2. *Let $(V, \mathcal{F}, \tau)$ be a P-2NL space. For $b \in V$, $t > 0$, the nonempty subset $A \subset V$ is called P_b-proximinal set if $P^t_{A,b}(x)$ is non-void for every $x \in V \backslash (A + \langle b \rangle)$ and A is called P_b-Chebyshev set if for every $x \in V$ the set $P^t_{A,b}(x)$ contains exactly one element. Also A is called P_b-quasi Chebyshev set if $P^t_{A,b}(x)$ is a compact set.*

Definition 2.3. *Let $(V, \mathcal{F}, \tau)$ be a P-2NL space, and $\{x_n\}$ be a sequence of V. Then the sequence $\{x_n\}$ is said to be P_b-convergent to $x_0 \in V$ and denoted by $x_n \xrightarrow{P_b} x$, if $\lim_{n\to\infty} F_{x_n - x_0, b}(t) = 1$, for all $x \in V$ and $t > 0$.*

Theorem 2.4. *Let A be a nonempty subset of a P-2NL space $(V, \mathcal{F}, \tau)$ and $x, b \in V$. Then, $x \in \overline{A}$ if and only if $F_{x-A,b}(t) = 1$, for $t > 0$.*

Proof. Suppose $x \in \overline{A}$ and $b \in V$. As V is first countable, there exists a sequence $\{x_n\}$ in A, such that $x_n \to x$ as $n \to \infty$. Then for every $t > 0$ and $0 < \lambda < 1$ there exists N, such that for $n \geq N$, $F_{x-x_n,b}(t) > 1 - \lambda$. Hence for all $n \geq N$, we have

$$1 - \lambda < F_{x-x_n,b}(t) \leq F_{x-A,b}(t) \leq 1$$

for all $0 < \lambda < 1$. Thus $F_{x-A,b}(t) = 1$.

Conversely, suppose for $t > 0$, $F_{x-A,b}(t) = 1$. We know $\{N_x(t, \lambda, b) : t > 0, 0 < \lambda < 1\}$ is a local base at x. Now for each $n \in \mathbb{N}$, $F_{x-A,b}(1/n) = 1$, then by definition, there exists $x_n \in A$ such that $F_{x-x_n,b}(1/n) > 1 - 1/n$ and so $x_n \in N_x(1/n, 1/n, b)$.

For the given $t > 0$ and $0 < \lambda < 1$ choose $n \in \mathbb{N}$ such that $t, \lambda > 1/n$, then $N_x(1/n, 1/n, b) \subset N_x(t, \lambda, b)$. So $N_x(t, \lambda, b) \cap A \neq \emptyset$. Thus, $x \in \overline{A}$. □

Theorem 2.5. *Let A be a nonempty subset of a P-2NL space $(V, \mathcal{F}, \tau)$. Then for $b \in V$:*

(i) $P^t_{A+y,b}(x + y) = P^t_{A,b}(x) + y$, for every $x, y \in V$ and $t > 0$.

(ii) $P^{|\alpha|t}_{\alpha A,b}(\alpha x) = \alpha P^t_{A,b}(x)$, for every $x \in V$, $t > 0$ and $\alpha \in \mathbb{R}\backslash\{0\}$.

(iii) A is P_b-proxminal (respectively P_b-Chebyshev) if and only if $A + y$ is P_b-proxminal (respectively P_b-Chebyshev) for any given $y \in V$.

Proof. (i) For any $x, y, b \in V$ and $t > 0$, let $y_0 \in P^t_{A+y,b}(x + y)$. Therefore

$$F_{x-A,b}(t) = F_{A+y-(x+y),b}(t) = F_{x-(y_0-y),b}(t)$$

then $y_0 - y \in P^t_{A,b}(x)$ i.e. $y_0 \in P^t_{A,b}(x) + y$. The converse is obvious.

(ii) Let $y_0 \in P^{|\alpha|t}_{\alpha A,b}(\alpha x)$, for any $x \in V$, $t > 0$ and $\alpha \in \mathbb{R}\backslash\{0\}$. Then

$$F_{x-A,b}(t) = F_{\alpha x - \alpha A,b}(|\ \alpha\ |\ t) = F_{\alpha x - y_0,b}(|\ \alpha\ |\ t) = F_{x+y_0/\alpha,b}(t)$$

therefore, $y_0 \in \alpha P^t_{A,b}(x)$. The converse is obvious.

(iii) Is an immediate consequence of (i). □

Theorem 2.6. *Let $(V, \mathcal{F}, \tau)$ be a P-2NL space such that $\tau(F_{p,b}, F_{q,b})(x) = T(F_{p,b}(x), F_{q,b}(x))$, where T is a continuous t-norm. If A is a subspace of a V and $b \in V$. Then,*

(a) $0 \leq e^t_{A,b}(x) \leq 1$,

(b) $e^t_{A,b}(a) = 0$, for all $a \in A$,

(c) *If B is a subspace of A, then we have* $e^t_{B,b}(x) \geq e^t_{A,b}(x)$,
(d) $e^t_{A,b}(x+a) = e^t_{A,b}(x)$, *for* $(x \in V, a \in A)$,
(e) $T^*(e^t_{A,b}(x), e^t_{A,b}(y)) \geq e^t_{A,b}(x+y)$, *where* T^* *is a t-conorm.*

Proof. (a),(b) and (c) are obvious by the definition of $e^t_A(x)$.

(d) Let $x \in V$, $a \in A$ and $\varepsilon > 0$ be arbitrary. By the definition sup, there exits $a_0 \in A$ such that

$$F_{x-A,b}(t) \leq F_{x-a_0,b}(t) + \varepsilon$$

consequently, we have

$$e^t_{A,b}(x) - \varepsilon \geq 1 - F_{x-a_0,b}(t) \geq 1 - F_{x-(-a+A),b}(t) = e^t_{A,b}(x+a).$$

Whence, for $x \in V$ and $a \in A$ we obtain

$$e^t_{A,b}(x) \geq e^t_{A,b}(x+a).$$

Now, since $x + a \in V$ and $-a \in A$ implies $e^t_{A,b}(-a) = 0$ and so, the proof is complete.

(e) Let $x, y \in V$ and $\varepsilon' > 0$ be arbitrary. By the definition for any $\varepsilon > 0$ there exit elements, $a_1, a_2 \in A$ such that

$$F_{x-a_1,b}(t) \geq F_{x-A}(t) - \varepsilon, \qquad F_{y-a_2,b}(t) \geq F_{y-A}(t) - \varepsilon$$

consequently, we have

$$\begin{aligned} F_{x+y-A,b}(t) &\geq \tau\big(F_{x-a_1,b}, F_{y-a_2,b}\big)(t) \\ &= T\big(F_{x-a_1,b}(t), F_{y-a_2,b}(t)\big) \\ &\geq T\big(F_{x-A,b}(t) - \varepsilon, F_{y-A,b}(t) - \varepsilon\big) \end{aligned}$$

by the uniform continuity of T we have:

$$F_{x+y-A,b}(t) \geq T\big(F_{x-A,b}(t), F_{y-A,b}(t)\big) - \varepsilon'$$

therefore

$$e^t_{A,b}(x+y) \leq 1 - T\big(F_{x-A,b}(t), F_{y-A,b}(t)\big) = 1 - T\big(1 - e^t_{A,b}(x), 1 - e^t_{A,b}(y)\big). \qquad \square$$

The following lemma shows that the P_b-best approximation in probabilistic 2-normed spaces is a generalization of the best approximation in 2-normed spaces.

Lemma 2.7. *Let $(V, \|.,.\|)$ be a 2-normed space and $F^{\|.,.\|}$ be the induced probabilistic 2-norm. Then for $b \in V$, $y_0 \in A$ is a best approximation to $x \in V$ in the 2-normed linear space if and only if y_0 is a P_b-best approximation to x in the induced probabilistic 2-normed linear space $(V, \mathcal{F}^{\|.,.\|}, \tau)$, for each $t > 0$.*

Proof. For $b \in V$, since y_0 is a best approximation to $x \in V$, we have $\|x - A, b\| = \|x - y_0, b\| = inf\{\|x - y, b\| : \ y \in A\}$ if and only if $F^{\|.,.\|}_{x-A,b}(t) = \frac{t}{(t+\|x-A,b\|)} = \frac{t}{(t+\|x-y_0,b\|)} = F^{\|.,.\|}_{x-y_0,b}(t)$. □

Theorem 2.8. *Let $(V, \mathcal{F}, \tau)$ be a P-2NL space, A be a subset of V, $b \in V$ and $x \in V \backslash (\overline{A} + < b >)$, $t > 0$. Then $P^t_{A,b}(x) = A \cap N_x(t, e^t_{A,b}(x), b)$.*

Proof. It is obvious that

$$P^t_{A,b}(x) \subset A \cap N_x(t, e^t_{A,b}(x), b).$$

Conversely, let $y_0 \in A \cap N_x(t, e^t_{A,b}(x), b)$ therefore, $F_{x-y_0,b}(t) \geq 1 - e^t_{A,b}(x)$ whence, $y_0 \in P^t_{A,b}(x)$. □

Corollary 2.9. *Let $(V, \mathcal{F}, \tau)$ be a P-2NL space, A be a subset of V, $b \in V$. Then*
(a) The set $P^t_{A,b}(x)$ is bounded.
(b) If A is closed, then $P^t_{A,b}(x)$ is closed.

Remark 2.10. *Let $(V, \mathcal{F}, \tau)$ be a P-2NL space, A be a subset of V, $b \in V$ and $x \in V \backslash (\overline{A} + < b >)$, $t > 0$. Then we have $A \cap N_x(t, e^t_{A,b}(x), b) = \emptyset$.*

Proof. Suppose the contrary, so that, there is $y_0 \in A \cap N_x(t, e^t_{A,b}(x), b)$ such that

$$F_{x-y_0,b}(t) > 1 - e^t_{A,b}(x) = F_{x-A,b}(t) \geq F_{x-y_0,b}(t)$$

which is a contradiction. □

We recall that a set A is said to be countably compact if for every decreasing sequence $A_1 \supset A_2 \supset ...$ of nonvoid closed subsets of A we have $\bigcap_{n=1}^{\infty} A_n \neq \emptyset$.

Theorem 2.11. *Let $(V, \mathcal{F}, \tau)$ be a P-2NL space. If $b \in V$ and A is a nonvoid set of V, $0 < \lambda < 1$ and $t > 0$ such that $A \cap N_x(t, \lambda, b)$ is countably compact, then A is P_b-proximinal.*

Proof. For $b \in V$ and every $n \in \mathbb{N}$, $0 < 1 - F_{x-A,b}(t) + F_{x-A,b}(t)/(n+1) < 1$. Put

$$A^t_n = A \bigcap N_x(t, 1 - F_{x-A,b}(t) + F_{x-A,b}(t)/(n+1), b) \quad (n = 1, 2, ...)$$

We have obviously $A^t_1 \supset A^t_2 \supset ...$ and each A^t_n is nonvoid. Since for every $n \in \mathbb{N}$, $F_{x-A,b}(t)(1 - 1/(n+1)) < F_{x-A,b}(t)$ hence there exists $a^t_n \in A$ such that $F_{x-A,b}(t)(1 - 1/(n+1)) < F_{x-a^t_n,b}(t)$. So $a^t_n \in A^t_n$. Since each A^t_n is countably compact and closed, it follows that there exists an $a_0 \in \bigcap_{n=1}^{\infty} A^t_n$. Then we have that

$$F_{x-A,b}(t) \geq F_{x-a_0,b}(t) \geq F_{x-A,b}(t)(1 - 1/(n+1)) \quad (n = 1, 2, ...)$$

implies $F_{x-a_0,b}(t) = F_{x-A,b}(t)$, whence $a_0 \in P^t_{A,b}(x)$. □

Definition 2.12. *Let $b \in V$. A nonempty subset A of a P-2NL space $(V, \mathcal{F}, \tau)$ is said to be P_b-approximatively compact if for each $x \in V$ and each sequence y_n in A with $F_{x-y_n,b}(t) \to F_{x-A,b}(t)$, there exists a subsequence y_{n_k} of y_n converging to an element y_0 in A.*

Lemma 2.13. *(1) If A is approximatively compact in a 2-normed space $(V, \|.,.\|)$ then for each $t > 0$, $b \in V$, A is P_b-approximatively compact in the induced P-2NL space.*

(2) If A is a compact subset of a P-2NL space $(V, \mathcal{F}, \tau)$ and $b \in V$ then A is P_b-approximatively compact for each $t > 0$.

Theorem 2.14. *For $b \in V$ and $t > 0$, let A be a nonempty P_b-approximatively compact subset of a strong P-2NL space $(V, \mathcal{F}, \tau)$. Then*

(1) A is a P_b-proximinal set.

(2) A is closed in V.

Proof. (1) For $b \in V$ and $x \in V$, there exists $\{y_n\} \subset A$ such that $F_{x-y_n,b}(t) \to F_{x-A,b}(t)$. Since A is a P_b-approximatively compact set, there exists a subsequence y_{n_k} of y_n and y_0 in A such that $y_n \to y_0$, and since $(V, \mathcal{F}, \tau)$ is a strong P-2NL space, we have, $F_{x-y_{n_k},b}(t) \to F_{x-y_0,b}(t)$. Hence $F_{x-y_0,b}(t) = F_{x-A,b}(t)$, then y_0 is a P_b-best approximation to x from A.

(2) Obviously, $A \subseteq \overline{A}$, let $x \in \overline{A}$. Then, $F_{x-A,b}(t) = 1$. Since A is P_b-approximatively compact, there exists $y \in A$ such that $F_{x-y,b}(t) = F_{x-A,b}(t) = 1$, then $F_{x-y,b} = H$, therefore $x \in A$. □

Theorem 2.15. *If A is a P_b-approximatively compact set then A is a P_b-quasi Chebyshev set.*

Proof. Let $\{y_n\}$ be a sequence in $P^t_{A,b}(x)$. Since A is a P_b-approximatively compact so, there exists a subsequence $\{y_{n_k}\}$ of $\{y_n\}$ and $y_0 \in A$ such that $y_n \to y_0$. Then $F_{x-y_{n_k},b}(t) \to F_{x-y_0,b}(t)$. On the other hand, $F_{x-y_{n_k},b}(t) \to F_{x-A,b}(t)$, therefore, $F_{x-y_0,b}(t) = F_{x-A,b}(t)$, and so $y_0 \in P^t_{A,b}(x)$. Thus $P^t_{A,b}(x)$ is compact. □

3. Orthogonality

Definition 3.1. *Let $(V, \mathcal{F}^{\|.,.\|}, \tau)$ be a P-2NL space with $P-2$ norm $F^{\|.,.\|}$ and A be a subset of V and $b \in V$. An element $x \in V$ is said to be orthogonal to an element $y \in V$, and we denote $x \perp^b y$, if $F^{\|.,.\|}_{x+\lambda y,b}(t) \le F^{\|.,.\|}_{x,b}(t)$ for all scalar $\lambda \in \mathbb{R}$, $\lambda \neq 0$ and $t > 0$.*

Also, an element $x \in V$ is said to be orthogonal to E, and we denote $x \perp^b E$, if $x \perp^b y$, for all $y \in E$.

Theorem 3.2. *Let $(V, \mathcal{F}^{\|.,.\|}, \tau)$ be a P-2NL space with $P-2$ norm $F^{\|.,.\|}$ and E be a subset of V and $x, b \in V$. Then $y_0 \in P^t_{E,b}(x)$ if and only if $x - y_0 \perp^b E$.*

Proof. Note that, $F^{\|\cdot,\cdot\|}_{x-y_0+\lambda z,b}(t) \leq F^{\|\cdot,\cdot\|}_{x-y_0,b}(t)$ for all $z \in E$ and all scalar $\lambda \in \mathbb{R}$, $\lambda \neq 0$ and $t > 0$, if and only if $y_0 \in P^t_{E,b}(x)$. □

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