Novi Sad J. Math.
Vol. 38, No. 2, 2008, 145-152

# A COMMON FIXED-POINT THEOREM IN 2 NON-ARCHIMEDEAN MENGER PM-SPACE FOR R-WEAKLY COMMUTING MAPS OF TYPE (P)

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**Abstract.** The present paper deals with the establishment of common fixed-point theorem for R-weakly commuting maps of type $(P)$ in 2 N.A. Menger PM-space.

*AMS Mathematics Subject Classification (2000)*: 47H10, 54H25

*Key words and phrases:* 2 N.A. Menger PM-space, R-weakly commuting maps of type (P) and Fixed points

## 1. Introduction

The notion of probabilistic metric space was introduced in 1942 by K. Menger. The first idea of K. Menger was to use distribution functions instead of non-negative real numbers as values of the metric. Since then the theory of probabilistic metric spaces has been developed in many directions. Renu Chugh and Sumitra [7] introduced the concept of 2 N.A. Menger PM-space in 2001.

In 1994, Pant [8] introduced the concept of R-weakly commuting maps in metric spaces. Later Y.J.Cho et al. [11] generalized this aspect and gave the concept of R-weakly commuting maps of type $(A_g)$ in metric spaces. Vasuki [9] proved some common fixed point theorems for R-weakly commuting maps in fuzzy metric spaces. Quite recently in 2007, Vyomesh Pant and R. P. Pant [10] introduced the concept of R-weakly commuting maps of the type $(A_g)$ in fuzzy metric spaces.

In 2006, Mohd. Imdad and Javid Ali [5] introduced the concept of R-weakly commuting maps of the type (P) in fuzzy metric spaces. The intent of this paper is to define the concept of R-weakly commuting maps of the type (P) in this newly defined space and prove a common fixed theorem for R-weakly commuting three self maps of type (P) along with the example. Hereby we give some preliminary definitions and notations.

**Definition 1.1.** Let $X$ be any non-empty set and $D$ be the set of all left-continuous distribution functions. An ordered pair $(X, F)$ is said to be 2 Non-Archimedean probabilistic metric space (briefly 2 N.A. PM-space) if $F$ is a mapping from $X \times X \times X$ into $D$ satisfying the following conditions where the

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value of $F$ at $x, y, z \in X \times X \times X$ is represented by $F_{x,y,z}$ or $F(x, y, z)$ for each $x, y, z \in X$ such that

i) $F(x, y, z; t) = 1$ for all $t > 0$ if and only if at least two of the three points are equal

ii) $F(x, y, z) = F(x, z, y) = F(z, x, y)$

iii) $F(x, y, z; 0) = 0$

iv) $F(x, y, s; t_1) = F(x, s, z; t_2) = F(s, y, z; t_3) = 1$ then $F(x, y, z; \max\{t_1, t_2, t_3\}) = 1$

**Definition 1.2.** A t-norm is a function $\Delta : [0, 1]\times[0, 1]\times[0, 1] \to [0, 1]$ which is associative, commutative, non-decreasing in each coordinate and $\Delta(a, 1, 1) = a$ for each $a \in [0, 1]$.

**Definition 1.3.** A 2 N.A. Menger PM-space is an ordered triplet $(X, F, \Delta)$, where $\Delta$ is a t-norm and $(X, F)$ is a 2 N.A. PM-space satisfying the following condition

$$F(x, y, z; \max\{t_1, t_2, t_3\}) \geq \Delta\{F(x, y, s; t_1), F(x, s, z; t_2), F(s, y, z, t_3)\}$$

for each $x, y, z \in X, t_1, t_2, t_3 \geq 0$.

**Definition 1.4.** Let $(X, F, \Delta)$ be 2 N.A. Menger PM-space and $\Delta$ a continuous t-norm, then $(X, F, \Delta)$ is Hausdorff in the topology induced by the family of neighborhoods; $U_x(\varepsilon, \lambda, a_1, a_2, \ldots, a_n)$; $x, a_i \in X$, $\varepsilon > 0$, $i = 1, 2, \ldots, n \in Z^+$, where $Z^+$ is the set of all positive integers and

$$\begin{aligned} U_x(\varepsilon, \lambda, a_1, a_2, \ldots, a_n) &= \{y \in X; F(x, y, a_i; \varepsilon) > 1 - \lambda, 1 \leq i \leq n\} \\ &= \bigcap_{i=1}^{n} \{y \in X; F(x, y, a_i; \varepsilon) > 1 - \lambda, 1 \leq i \leq n\}. \end{aligned}$$

**Definition 1.5.** A 2 N.A. Menger PM-space $(X, F, \Delta)$ is said to be of type $(C)_g$ if there exists a $g \in \Omega$ such that

$$g(F(x, y, z; t)) \leq g(F(x, y, a; t)) + g(F(x, a, z; t)) + g(F(a, y, z; t))$$

for each $x, y, z \in X$, $t \geq 0$, where

$$\Omega = \{g | g : [0, 1] \to [0, \infty) \text{ is continuous, strictly decreasing, } g(1) = 0 \text{ and } g(0) < \infty\}.$$

**Definition 1.6.** A 2 N.A. Menger PM-space $(X, F, \Delta)$ is said to be of type $(D)_g$ if there exists a $g \in \Omega$ such that $g(\Delta(t_1, t_2, t_3)) \leq g(t_1) + g(t_2) + g(t_3)$ for each $t_1, t_2, t_3 \in [0, 1]$.

**Remark 1.**

(i) If 2 N.A. Menger PM-space is of the type $(D)_g$, then $(X, F, \Delta)$ is of the type $(C)_g$.

(ii) If $(X, F, \Delta)$ is 2 N.A. Menger PM-space and $\Delta \geq \Delta(r, s, t) = \min(r, s, t)$, then $(X, F, \Delta)$ is of the type $(D)_g$ , for $g \in \Omega$ and $g(t) = 1 - t$.

## 2. Results

Throughout this paper, let $(X, F, \Delta)$ be a complete 2 N.A. Menger PM-space with a continuous strictly increasing t-norm $\Delta$. Let $\phi : [0, \infty) \to [0, \infty)$ be a function satisfying the condition $(\Phi)$;

$(\Phi)$ $\phi$ is the upper semi-continuous from the right and $\phi(t) < t$ for $t > 0$.

**Lemma 1.** *If a function $\phi : [0, \infty) \to [0, \infty)$ satisfies the condition $(\Phi)$ then we get*

i) *For all $t \geq 0$, $\lim_{n\to\infty} \phi^n(t) = 0$ where $\phi^n(t)$ is the $n^{th}$ iteration of $\phi(t)$.*

ii) *If $\{t_n\}$ is a non-decreasing sequence of real numbers and $t_{n+1} \leq \phi(t_n)$, $n = 1, 2, \ldots$ then $\lim_{n\to\infty} t_n = 0$. In particular, if $t \leq \phi(t)$, for each $t \geq 0$, then $t = 0$.*

**Lemma 2.** *Let $\{y_n\}$ be a sequence in $X$ such that $\lim_{n\to\infty} F(y_n, y_{n+1}, a; t) = 1$ for each $t > 0$. If the sequence $\{y_n\}$ is not a Cauchy sequence in $X$, then there exist $\varepsilon_0 > 0$, $t_0 > 0$, and two sequences $\{m_i\}$ and $\{n_i\}$ of positive integers such that*

i) *$m_i > n_i + 1$ and $n_i \to \infty$ as $i \to \infty$.*

ii) *$F(y_{m_i}, y_{n_i}, a; t_0) < 1 - \varepsilon_0$ and $F(y_{m_i-1}, y_{n_i}, a; t_0) \geq 1 - \varepsilon_0$, $i = 1, 2, \ldots$*

**Definition 2.1.** Two maps $f$ and $g$ of a 2 N.A. Menger PM-space $(X, F, \Delta)$ into itself are said to be R-weakly commuting of the type (P) if there exists some $R > 0$ such that $g(F(ggx, ffx, a; t)) \leq g(F(fx, gx, a; t/R))$ for every $x \in X$ and $t > 0$.

**Lemma 3.** *Let $A, S : X \to X$ be R-weakly commuting maps of the type (P) and $\{x_n\}$ be a sequence in $X$ such that $\lim_{n\to\infty} Ax_n = z = \lim_{n\to\infty} Sx_n$ for some $z \in X$ , then $\lim_{n\to\infty} ASx_n = Sz$ if $S$ is continuous at $z$.*

*Proof.* Suppose $S$ is continuous and $\{x_n\}$ be a sequence in $X$ such that

$$\lim_{n\to\infty} Ax_n = z = \lim_{n\to\infty} Sx_n$$

for some $z \in X$, so $SSx_n \to Sz$ as $n \to \infty$. Since $A$ and $S$ are R-weakly commuting maps of type (P), so $g(F(ASx_n, Sz, a; t)) = g(F(AAx_n, SSx_n, a; t)) \leq g(F(Ax_n, Sx_n, a; t)) \to 0$ as $n \to \infty$.

Thus $ASx_n \to Sz$ as $n \to \infty$. $\square$

**Example 2.1.** Let $X = [0,1]$ with 2-metric defined as

$$d(x,y,z) = \min[|x-y|, |y-z|, |z-x|] \text{ for each } x,y,z \in X,\ t > 0.$$

Define $F(x,y,z;t) = \frac{t}{t+d(x,y,z)}$, with $\Delta(r,s,t) = \min(r,s,t,)$ or $r \cdot s \cdot t$.

Then (i) $F(x,y,z;0) = \frac{0}{0+d(x,y,z)} = 0$

(ii) and (iii) are trivial;

(iv) Let $F(x,y,s;t_1) = F(x,s,z;t_2) = F(s,y,z;t_3) = 1$, then we need to prove that $F(x,y,z;\max\{t_1,t_2,t_3\}) = 1$.

Now, $F(x,y,s;t_1) = 1$ if and only if $\frac{t_1}{t_1+d(x,y,s)} = 1$, if and only if $d(x,y,s) = 0$.

Similarly, $F(x,s,z;t_2) = 1$ if and only if $\frac{t_2}{t_2+d(x,s,z)} = 1$ if and only if $d(x,s,z) = 0$ and $F(s,y,z;t_3) = 1$ if and only if $\frac{t_3}{t_3+d(s,y,z)} = 1$ if and only if $d(s,y,z) = 0$.

Now, $d(x,y,z) \le d(x,y,s) + d(x,s,z) + d(s,y,z) = 0+0+0 = 0$.

Let $\max\{t_1,t_2,t_3\} = T$, so

$$F(x,y,z;\max\{t_1,t_2,t_3\}) = F(x,y,z;T) = \frac{T}{T+d(x,y,z)} = 1.$$

Also,

$$F(x,y,z;\max\{t_1,t_2,t_3\}) \ge \Delta(F(x,y,s;t_1), F(x,s,z;t_2), F(s,y,z;t_3)).$$

Thus $(X,F,\Delta)$ is 2 N.A. Menger PM-space.

**Theorem 1.** *Let $S$ and $T$ be two continuous self-maps of a complete 2 N.A. Menger PM-space $(X,F,\Delta)$. Let $A$ be self-map of $X$ satisfying*

(i) *$\{A,S\}$ and $\{A,T\}$ are R-weakly commuting of the type (P) and $A(X) \subseteq S(X) \cap T(X)$*

(ii)

$$\begin{aligned} g(F(Ax,Ay,a;t)) \le \phi[\max\{&(Sx,Ty,a;t), g(F(Sx,Ax,a;t)),\\ &g(F(Sx,Ay,a;t)), g(F(Ty,Ay,a;t))\}] \end{aligned}$$

*for every $x,y \in X$, where $\phi : [0,1] \to [0,1]$ is a continuous function such that $\phi(t) < t$ and $\phi(1) = 1$.*

*Then $A$, $S$ and $T$ have a unique common fixed point in $X$.*

*Proof.* Let $x_0 \in X$ such that $A(X) \subseteq S(X)$, there exists $x_1 \in X$ such that $Ax_0 = Sx_1$. Also, since $A(X) \subseteq T(X)$, there is another point $x_2 \in X$ such that $Ax_1 = Tx_2$. Inductively we can choose $x_{2n+1}$ and $x_{2n+2}$ in $X$ such that

$$y_{2n} = Sx_{2n+1} = Ax_{2n};\ Tx_{2n+2} = Ax_{2n+1} = y_{2n+1} \text{ for } n = 0, 1, \ldots \tag{2.1}$$

Let $M_n = g(F(Ax_n, Ax_{n+1}, a; t))$, $n = 0, 1, 2, \ldots$ then

$$\begin{aligned} M_{2n} &= g(F(Ax_{2n+1}, Ax_{2n}, a; t)) \\ &\leq \phi[\max\{g(F(Sx_{2n+1}, Tx_{2n}, a; t)), g(F(Sx_{2n+1}, Ax_{2n+1}, a; t)), \\ &\quad g(F(Sx_{2n+1}, Ax_{2n}, a; t)), g(F(Tx_{2n}, Ax_{2n}, a; t))\}] \\ &= \phi[\max\{g(F(Sx_{2n+1}, Ax_{2n-1}, a; t)), g(F(Ax_{2n}, Ax_{2n+1}, a; t)), \\ &\quad g(F(Ax_{2n}, Ax_{2n}, a; t)), g(F(Ax_{2n-1}, Ax_{2n}, a; t))\}]. \end{aligned} \tag{2.2}$$

Now, consider

$$\begin{aligned} & g(F(Sx_{2n+1}, Ax_{2n-1}, a; t)) \\ &\quad \leq g(F(Sx_{2n+1}, Ax_{2n-1}, Ax_{2n}; t)) + \\ &\quad\quad g(F(Sx_{2n+1}, Ax_{2n}, a; t)) \\ &\quad\quad +g(F(Ax_{2n}, Ax_{2n-1}, a; t))\}] \\ &\quad = g(F(Ax_{2n}, Ax_{2n-1}, Ax_{2n}; t)) \\ &\quad\quad +g(F(Ax_{2n}, Ax_{2n}, a; t)) + \\ &\quad\quad g(F(Ax_{2n}, Ax_{2n-1}, a; t))\}]. \end{aligned} \tag{2.3}$$

Using (2.3) in (2.2) with $M_{2n} = g(F(Ax_{2n+1}, Ax_{2n}, a; t))$, we get

$$M_{2n} \leq \phi\,[\max\{M_{2n-1}, M_{2n}, 0, M_{2n-1}\}] \tag{2.4}$$

If $M_{2n} > M_{2n-1}$, then by (2.4) $M_{2n} \leq \phi(M_{2n})$, a contradiction.

If $M_{2n-1} > M_{2n}$ then (2.4) gives $M_{2n} \leq \phi(M_{2n-1})$, then by Lemma 1, we get $\lim_{n\to\infty} M_{2n} = 0$ i.e., $\lim_{n\to\infty} g(F(Ax_{2n+1}, Ax_{2n}, a; t)) = 0$.

Similarly, we can show that $\lim_{n\to\infty} g(F(Ax_{2n+2}, Ax_{2n+1}, a; t)) = 0$.

Thus we have

$$\begin{aligned} &\lim\nolimits_{n\to\infty} g(F(Ax_n, Ax_{n+1}, a; t)) = 0 \text{ for every } t > 0, \\ &\text{i.e., } \lim\nolimits_{n\to\infty} g(F(y_n, y_{n+1}, a; t)) = 0 \text{ for every } t > 0. \end{aligned} \tag{2.5}$$

Before proceeding the proof of the theorem, we first prove a Claim.

*Claim:* Let $A, S, T : X \to X$ be maps satisfying (i) and (ii), then the sequence $\{y_n\}$ defined by (2.1) such that $\lim_{n\to\infty} g(F(y_n, y_{n+1}, a; t)) = 0$, $a \in X$, is a Cauchy sequence in $X$.

*Proof of the Claim:* Since $g \in \Omega$ it follows that $\lim_{n\to\infty} F\,(y_n, y_{n+1}, a; t) = 1$ for each $t > 0$, $a \in X$ if and only if $\lim_{n\to\infty} g(F(y_n, y_{n+1}, a; t)) = 0$ for each $t > 0$.

By Lemma 2, if $\{y_n\}$ is not a Cauchy sequence in $X$, there exist $\varepsilon_0 > 0$, $t_0 > 0$, and two sequences $\{m_i\}$ and $\{n_i\}$ of positive integers such that

A) $m_i > n_i + 1$ and $n_i \to \infty$ as $i \to \infty$

B) $g(F(y_{m_i}, y_{n_i}, a; t_0)) > g(1-\varepsilon_0)$ and $g(F(y_{m_i-1}, y_{n_i}, a; t_0)) \leq g(1-\varepsilon_0)$, $i = 1, 2, ..,$ since $g(t) = 1 - t$. Thus, we have

$$\begin{aligned} g(1-\varepsilon_0) &< g(F(y_{m_i}, y_{n_i}, a; t_0)) \\ &\leq g(F(y_{m_i}, y_{n_i}, y_{m_i-1}; t_0)) + g(F(y_{m_i}, y_{m_i-1}, a; t_0)) \\ &\quad + g(F(y_{m_i-1}, y_{n_i}, a; t_0)) \\ &\leq g(F(y_{m_i}, y_{n_i}, y_{m_i-1}; t_0)) + g(F(y_{m_i}, y_{m_i-1}, a; t_0)) + g(1-\varepsilon_0) \end{aligned} \tag{2.6}$$

as $i \to \infty$ in (2.6) we get

$$\lim_{n\to\infty} g(F(y_{m_i}, y_{n_i}, a; t_0)) = g(1-\varepsilon_0). \tag{2.7}$$

On the other hand, we have

$$\begin{aligned} g(1-\varepsilon_0) &< g(F(y_{m_i}, y_{n_i}, a; t_0)) \\ &\leq g(F(y_{m_i}, y_{n_i}, y_{n_i+1}; t_0)) + g(F(y_{m_i}, y_{n_i+1}, a; t_0)) \\ &\quad + g(F(y_{n_i+1}, y_{n_i}, a; t_0)). \end{aligned} \tag{2.8}$$

Now, consider $g(F(y_{m_i}, y_{n_i+1}, a; t_0))$ in (2.8) and assume that both $m_i$ and $n_i$ are even. Then by (ii), we have

$$\begin{aligned} & g(F(y_{m_i}, y_{n_i+1}, a; t_0)) \\ &= g(F(Ax_{m_i}, Ax_{n_i+1}, a; t_0)) \\ &\leq \phi[\max\{g(F(Sx_{m_i}, Tx_{n_i+1}, a; t_0)), g(F(Sx_{m_i}, Ax_{m_i}, a; t_0)), \\ &\quad g(F(Sx_{m_i}, Ax_{n_i+1}, a; t_0)), g(F(Tx_{n_i+1}, Ax_{n_i+1}, a; t_0))\}] \\ &\leq \phi[\max\{g(F(y_{m_{i-1}}, y_{n_i}, a; t_0)), g(F(y_{m_i-1}, y_{m_i}, a; t_0)), \\ &\quad g(F(y_{m_i-1}, y_{n_i+1}, a; t_0)), g(F(y_{n_i}, y_{n_i+1}, a; t_0))\}]. \end{aligned} \tag{2.9}$$

Now, consider $g(F(y_{m_i-1}, y_{n_i+1}, a; t_0))$ from (2.9).

$$\begin{aligned} g(F(y_{m_i-1}, y_{n_i+1}, a; t_0)) &\leq g(F(y_{m_i-1}, y_{n_i+1}, y_{n_i}; t_0)) \\ &\quad + g(F(y_{m_i-1}, y_{n_i}, a; t_0)) + g(F(y_{n_i}, y_{n_i+1}, a; t_0)). \end{aligned} \tag{2.10}$$

Using (2.10) in (2.9) and letting $i \to \infty$

$$g(1-\varepsilon_0) \leq \phi\,[\max\{g(1-\varepsilon_0), 0, g(1-\varepsilon_0), 0\}] \text{ i.e., } g(1-\varepsilon_0) \leq \phi\,(g(1-\varepsilon_0)$$

which is a contradiction. Hence the sequence $\{y_n = Ax_n\}$ defined by (2.1) is a Cauchy sequence, which completes the proof of Claim.

By the completeness of $X$, $\{Ax_n\}$ converges to a point $z \in X$. Consequently, the subsequences $\{Sx_{2n+1}\}$ and $\{Tx_{2n}\}$ of $\{Ax_n\}$ also converge to $z \in X$. Since $A$ and $S$ are R-weakly commuting of type (P), so $g(F(SSx_{2n+1}, AAx_{2n+1}, a; t)) \leq g(F(Ax_{2n+1}, Sx_{2n+1}, a; t/R))$, which gives (using Lemma 3) $\lim_{n\to\infty} ASx_{2n+1} = \lim_{n\to\infty} SSx_{2n+1} = Sz$ (as $S$ is continuous), now, we prove that $Sz = z$.

Suppose that $Sz \neq z$, then using (ii) we get

$$g(F(ASx_{2n+1}, Ax_{2n}, a; t)) \leq \phi[\max\{g(F(SSx_{2n+1}, Tx_{2n}, a; t)), \\ g(F(SSx_{2n+1}, ASx_{2n+1}, a; t)), g(F(SSx_{2n+1}, Ax_{2n}, a; t)), g(F(Ax_{2n}, Tx_{2n}, a; t))\}].$$

Taking $n \to \infty$ we get,

$$g(F(Sz, z, a; t)) \leq \phi[\max\{g(F(Sz, z, a; t)), g(F(Sz, Sz, a; t)), g(F(Sz, z, a; t)), \\ g(F(z, z, a; t))\}] = \phi(g(F(Sz, z, a; t))) < g(F(Sz, z, a; t)),$$

which is a contradiction.

Thus $z$ is a fixed point of $S$. Similarly, we can show that $z$ is a fixed point of $A$.

Now, the pair $\{A, T\}$ is R weakly commuting of the type (P), so

$$g(F(AAx_{2n+1}, TTx_{2n+1}, a; t)) \leq g(F(Ax_{2n+1}, Tx_{2n+1}, a; t/R))$$

which gives $\lim_{n\to\infty} ATx_{2n+1} = \lim_{n\to\infty} TTx_{2n+1} = Tz$ (as T is continuous).

Now, we claim that $z$ is also a fixed point of $T$.

Suppose that $Tz \neq z$, then using (ii) we have

$$\begin{aligned} g(F(Az, ATx_{2n}, a; t)) \leq\ & \phi[\max\{g(F(Sz, T^2x_{2n}, a; t)), g(F(Sz, Az, a; t)), \\ & g(F(Sz, ATx_{2n}, a; t)), g(F(T^2x_{2n}, ATx_{2n}, a; t))\}]. \end{aligned}$$

On taking limit as $n \to \infty$, it yields

$$\begin{aligned} g(F(z, Tz, a; t)) \leq\ & \phi[\max\{g(F(z, Tz, a; t)), g(F(z, z, a; t)), \\ & g(F(z, Tz, a; t)), g(F(Tz, Tz, a; t))\}]. \end{aligned}$$

This gives that $z = Tz$. Thus $z$ is a common fixed point of $A$ , $S$ and $T$.

Uniqueness can be proved by using condition (ii).

Taking $T = S$ in the above theorem we get the following corollary unifying Vasuki's theorem [9], which in turn also generalizes the result of Pant [8].

**Corollary 1.** *Let $(X, F, \Delta)$ be a complete 2 N.A. Menger PM-space and $S$ be a continuous self-mappings of $X$. Let $A$ be another self-mapping of $X$ satisfying that $\{A, S\}$ is R-weakly commuting of the type (P) with $A(X) \subseteq S(X)$ and*

$$\begin{aligned} g(F(Ax, Ay, a; t)) \leq\ & \phi[\max\{g(F(Sx, Ty, a; t)), g(F(Sx, Ax, a; t)), \\ & g(F(Sx, Ay, a; t)), g(F(Ty, Ay, a; t))\}] \end{aligned}$$

*for each $x, y \in X$, where $\phi : [0, 1] \to [0, 1]$ is a continuous function such that $\phi(t) < t$ for each $0 \leq t < 1$ and $\phi(t) = 1$ for $t = 1$, then the maps $A$ and $S$ have a unique common fixed point.*

**Remark 2.** Our results extend, generalize and unify the results of Jungck [3], B. Shweizer and A. Sklar [2], Mohd. Imdad and Javid Ali [5], R. Vasuki [9], R. P. Pant [8] and B. C. Dhage [1] in different spaces like metric space, probabilistic metric space, fuzzy metric space and D metric space in the framework of 2 N.A. Menger PM space.

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*Received by the editors July 16, 2008*