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L-GUILDS AND BINARY L-MEROTOPIES

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Abstract. The present paper is primarily concerned with the study of L-guilds in an L-merotopic space. It is shown that every L-cluster is an L-guild; however the converse is not true. For contigual and regular L-merotopies, where on one side we gave an example of a space, which is neither contigual nor binary, on the other side we constructed L-merotopic spaces that are contigual and binary. It is shown that the category LBIN of binary L-merotopic spaces and L-merotopic maps is bireflective in LMER, the category of L-merotopic spaces and L-merotopic maps.

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1. Introduction

In 1965, Katětov [11] defined merotopic spaces by axiomatizing the concept of collection of sets containing arbitrary small members called a micromeric collection. Nearness spaces are those merotopic spaces for which a relationship exists between the near collection and the closure operator. In 1974, Herrlich [9] introduced the concept of nearness spaces by axiomatizing the concept of nearness of arbitrary collections of subsets of X. It is a generalization of the concept of two sets being near. Nearness spaces have contributed to the understanding of various extension problems. Brandenburg [4] and Carlson [6] solved such type of problem for developable spaces and for complete Moore space. Quite a few problems of this type are also solved in [3, 5, 7]. Nearness spaces have been fruitful for approaching topology from categorical viewpoint. The categories of topological R_0 spaces, uniform spaces, proximity spaces and contiguity spaces are embedded in the category of nearness spaces. The category of uniform spaces and uniformly continuous maps form a full bireflective subcategory of NEAR (the category of nearness spaces and nearness maps), which in turn forms a full bireflective subcategory of MER.

In [2], Bentley introduced the concept of a guild in a merotopic space $(X, \bar{\xi})$ as the grill whose every two elements belong to $\bar{\xi}$. Different additional conditions are imposed on grills to define clans, bunches, etc. (see e.g. [1, 8]). Clusters, an analogue of ultrafilter in proximity spaces, were introduced by Leader [14]. Mrówka [17] gave the axiomatic characterization of the family of all clusters in a proximity space. More theory on clusters in proximity spaces can be found in

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[15, 16, 18]. In the year 1974, Herrlich [9] introduced the concept of a cluster in a merotopic space; 'clusters' which appear with the same name in proximity spaces is a different concept.

Prerequisites for the present paper are collected in Section 2. In Section 3, the notion of an L-guild in an L-merotopic space (X, ξ) is given to be an L-grill having an additional condition of being congenial. It is shown by means of an example that a congenial collection need not be an L-grill and also, it may not belong to ξ, followed by it, there is an example of an L-guild which does not belong to ξ. To each guild in classical merotopic space, there is an L-guild in the corresponding L-merotopic space and vice versa. Similar statements are given for congenial collections also. It is shown that every L-cluster, introduced in this section, is an L-guild and the converse is disproved by an example.

In Section 4, binary, contigual and regular L-merotopic spaces are defined. An indiscrete L-merotopic space is neither contigual nor binary. Corresponding to each L-merotopic space, we have a binary L-merotopic space generating the same L-Čech closure operator. It is shown that every binary space is contigual and a regular contigual L-merotopic space is binary. 'Is a contigual space binary?' it remains open question. Examples of a contigual L-merotopic space and a contigual merotopic space are given. We proved that the category LBIN of binary L-merotopic spaces and L-merotopic maps is a bireflective subcategory of LMER whose objects are L-merotopic spaces and morphisms are L-merotopic maps. On the same lines, it is shown that the category of binary L-nearness spaces and L-nearness maps is bireflective in the category of L-nearness spaces and L-nearness maps.

Let X be a nonempty ordinary set and L be an F-lattice (i.e. a completely distributive lattice with largest element 1, smallest element 0, and an order reversing involution $\prime : L \to L$). For t in L, a mapping $f : X \to L$ defined by $f(x) = t$, for all $x \in X$ is denoted by $\boldsymbol{t}$ and the family of all functions from X to L is by L^X. We denote by $\aleph_0$ the first infinite cardinal number, by $|\mathcal{A}|$ the cardinality of $\mathcal{A}$, $\mathcal{A} \subseteq L^X$, and by N the set of positive integers.

For $\mathcal{A}$, $\mathcal{B}$ subsets of L^X, we say that $\mathcal{A}$ *corefines* $\mathcal{B}$ if for all $f \in \mathcal{A}$, there exists $g \in \mathcal{B}$ such that $f \geqslant g$; $\mathcal{A} \vee \mathcal{B} \equiv \{f \vee g : f \in \mathcal{A},\, g \in \mathcal{B}\}$; $\mathcal{A} \wedge \mathcal{B} \equiv \{f \wedge g : f \in \mathcal{A},\, g \in \mathcal{B}\}$.

For the theory of grills, filters, ultrafilters, and L-Čech closure spaces we refer the readers to [12, 13, 19, 20, see also 21]. Details on merotopy and nearness, when $L = \mathcal{P}(X)$, can be seen in ([9, 10], where many more references can be found).

2. L-guilds

Definition 2.1. Let X be a nonempty set and $\xi \subseteq \mathcal{P}(L^X)$. Then ξ is called an *L-merotopy* on X provided, for the subsets $\mathcal{A}$, $\mathcal{B}$ of L^X,

(M1) $\mathcal{A}$ corefines $\mathcal{B}$ and $\mathcal{B} \in \xi \Rightarrow \mathcal{A} \in \xi$,

(M2) $\bigwedge \mathcal{A} \neq 0 \Rightarrow \mathcal{A} \in \xi$,

(M3) $\phi \neq \xi \neq \mathcal{P}(L^X)$,

(M4) $\mathcal{A} \vee \mathcal{B} \in \xi \Rightarrow \mathcal{A} \in \xi$ or $\mathcal{B} \in \xi$.

The pair (X, ξ) is called an *L-merotopic space*. For an L-merotopic space (X, ξ), we define:

$$cl_\xi f = \bigvee \{x_p : \{x_p, f\} \in \xi\}, f \in L^X.$$

Here $x_p \in L^X$, $p \in L$, is defined by $x_p(x) = p, x_p(y) = 0, x \neq y$. An L-merotopy ξ on X is called an *L-nearness* if the following additional condition is also satisfied: $\{cl_\xi f, cl_\xi g\} \in \xi \Rightarrow \{f, g\} \in \xi$.

Examples

Let X be a nonempty set. Then:

(i) $\xi = \{\mathcal{A} \subseteq L^X : \mathbf{0} \notin \mathcal{A}\}$ is an L-nearness on X. The pair (X, ξ) is the largest L-nearness space and will be called the *discrete L-space*. Here, $cl_\xi f = \{\mathbf{1}\}, f \in L^X - \{\mathbf{0}\}$.

(ii) $\xi = \{\mathcal{A} \subseteq L^X : \bigwedge \mathcal{A} \neq \mathbf{0}\}$ is an L-nearness on X. The pair (X, ξ) is the smallest L-nearness space (L-merotopic space) and will be called the *indiscrete L-space*. Here, $cl_\xi f = \chi_{\operatorname{supp} f}$, $f \in L^X$; and for $A \subseteq X$, $\chi_A(x) = \mathbf{1}$, if $x \in A$ and $\chi_A(x) = 0$, if $x \notin A$; and $\operatorname{supp} f = \{x \in X : f(x) > 0\}$.

(iii) $\xi = \{\mathcal{A} \subseteq L^X \ : \ |\operatorname{supp} f| \geqslant \aleph_0$ for all $f \in \mathcal{A}\} \cup \{\mathcal{A} \subseteq L^X : \bigwedge \mathcal{A} \neq \mathbf{0}\}$ is an $L-$nearness on an infinite set X. Also, $cl_\xi f = \chi_{\operatorname{supp} f}$.

(iv) For $\phi \neq \mathcal{C} \subseteq L^X$, $\xi = \{\mathcal{A} \subseteq L^X : \mathcal{C} \cap \sec \mathcal{A} \neq \phi\} \cup \{\mathcal{A} \subseteq L^X : \bigwedge \mathcal{A} \neq \mathbf{0}\}$ is an $L-$merotopy on X, where $\sec \mathcal{A} = \{f \in L^X : \ f \wedge g \neq \mathbf{0}$ for all $g \in \mathcal{A}\}$.

(v) Let (X, cl) be an $L-$Čech closure space. Then

(a) $\xi = \{\mathcal{A} \subseteq L^X : \bigwedge \{cl f : f \in \mathcal{A}\} \neq \mathbf{0}\}$ is an $L-$merotopy on X.

(b) $\xi = \{\mathcal{A} \subseteq L^X$: for all $\mathcal{B} \subseteq \mathcal{A}$ with $|\mathcal{B}| < \aleph_0$, $\bigwedge\limits_{g \in B} cl g \neq \mathbf{0}\}$ is an $L-$merotopy on X.

Definition 2.2. Let (X, ξ) be an L-merotopic space and let $\mathcal{A}$ be a subset of L^X. Then $\mathcal{A}$ is said to be *congenial* iff $f, g \in \mathcal{A} \Rightarrow \{f, g\} \in \xi$. A congenial L-grill on X is called an *L-guild* in X. A congenial collection (L-guild) $\mathcal{A}$ is called a *maximal* congenial collection (*maximal* L-guild) if it is not properly contained in any other congenial collection (L-guild). An *L-cluster* in an L-merotopic space (X, ξ) is a maximal element of ξ.

By Zorn's lemma, for any congenial subset $\mathcal{A}$ of an L-merotopic space, there exists a maximal congenial collection containing $\mathcal{A}$. Similarly, every L-guild in an L-merotopic space is contained in a maximal L-guild. Also, it may be verified that for any L-guild $\mathcal{G}$ in X and $\mathcal{A}, \mathcal{B} \in P(L^X)$, if $\mathcal{A} \vee \mathcal{B} \subseteq \mathcal{G}$, then either $\mathcal{A} \subseteq \mathcal{G}$, or $\mathcal{B} \subseteq \mathcal{G}$.

Example 2.3. (i) Let X be a set containing at least three points and ξ be the indiscrete L-merotopy on X. Choose three distinct points, say x, y, z of X. Then $Q = (\mathcal{U}[x] \cap \mathcal{U}[y]) \cup (\mathcal{U}[y] \cap \mathcal{U}[z]) \cup (\mathcal{U}[x] \cap \mathcal{U}[z])$ is a *congenial collection in* (X, ξ) *but is not an L-grill*, because $x_1 \vee y_1 \in Q$ while $x_1 \notin Q$ and $y_1 \notin Q$. Here $\mathcal{U}[x] = \{f : f(x) > 0\}$ is the principal L-ultrafilter on X. Since $Q \notin \xi$, this example also shows that a *congenial collection in* (X, ξ) *may not belong to* ξ.

Let $\mathcal{G}$ be a congenial collection with $Q \subseteq \mathcal{G}$. Then $x_1 \vee y_1 \in \mathcal{G}$. Since $\chi_{\{y,z\}} \in \mathcal{G}$ and $\mathcal{G}$ is congenial, we have $x_1 \notin \mathcal{G}$ as $x_1 \wedge \chi_{\{y,z\}} = 0$. Similarly, $y_1 \notin \mathcal{G}$. Hence a *maximal congenial collection may fail to be an L-grill.*

(ii) Let (X, ξ) be an L-merotopic space. Let $x \in X$ and $t \in (0, 1]$. Then ${}_x\mathcal{A}_t = \{f \in I^X : f(x) \geqslant t\}$ is an L-guild in X. Also, $\mathcal{U}[x]$ is an L-guild in an L-merotopic space (X, ξ).

(iii) A free L-ultrafilter, say Θ, in the indiscrete L-merotopic space (X, ξ) is an L-guild. Moreover, Θ does not belong to ξ.

Proposition 2.4. *In an L-merotopic space (X, ξ), (i) every L-ultrafilter is an L-guild, and (ii) every L-cluster is an L-guild.*

Proof. (i) Let $\mathcal{U}$ be an L-ultrafilter on X and $f, g \in \mathcal{U}$. Then $f \wedge g \neq 0$, and so $\{f, g\} \in \xi$. Hence $\mathcal{U}$ is congenial. Since every L-ultrafilter is an L-grill, the result follows.

(ii) Let $\mathcal{A}$ be an L-cluster in X. Let $f \vee g \in \mathcal{A}$. Since $(\{f\} \cup \mathcal{A}) \vee (\{g\} \cup \mathcal{A}) \vee (\{g\} \cup \mathcal{A})$ corefines $\mathcal{A}$, we have either $\{f\} \cup \mathcal{A} \in \xi$ or $\{g\} \cup \mathcal{A} \in \xi$. By the maximality of $\mathcal{A}$, $f \in \mathcal{A}$ or $g \in \mathcal{A}$. Let $f \in \mathcal{A}$ and $f \leqslant g$. Then, since $\{g\} \cup \mathcal{A}$ corefines $\mathcal{A}$ and $\mathcal{A}$ is an L-cluster, $g \in \mathcal{A}$. Also, $0 \notin A$.

Remark 2.5. It can be seen from Example 2.3(iii) that the statement (ii) in the above proposition is not reversible.

Proposition 2.6. *Let (X, ξ) be an L-merotopic space and let $\mathcal{A} \subseteq L^X$ be such that every finite subcollection of $\mathcal{A}$ belongs to ξ. Then there exists an L-guild in X containing $\mathcal{A}$.*

Proof. Let $\Omega = \{\mathcal{B} : \mathcal{A} \subset \mathcal{B}$ and every finite subcollection of $\mathcal{B}$ belongs to $\xi\}$. Then $\cup\mathcal{B}_i$ is an upper bound of the chain $\{\mathcal{B}_i\}_{i=1}^{\infty}$. By Zorn's lemma there exists a maximal element, say $\mathcal{G}$, of Ω with $\mathcal{A} \subseteq \mathcal{G}$. Since every finite subcollection of $\mathcal{G}$ belongs to ξ, $\mathcal{G}$ is a congenial collection in (X, ξ). For $\mathcal{G}$ to be an L-grill, let $f \notin \mathcal{G}$ and $g \notin \mathcal{G}$. Then $\{f\} \cup \mathcal{G} \notin \Omega$ follows from the maximality of $\mathcal{G}$. Similarly $\{g\} \cup \mathcal{G} \notin \Omega$. Hence there exist finite subsets $\mathcal{D}$ and $\mathcal{H}$ of $\mathcal{G}$ such that $\{f\} \cup \mathcal{D}$ and $\{g\} \cup \mathcal{H}$ do not belong to ξ. Since $(\{f\} \cup \mathcal{D}) \vee (\{g\} \cup \mathcal{H})$ corefines $\{f \vee g\} \cup \mathcal{D} \cup \mathcal{H}$, we have $\{f \vee g\} \cup \mathcal{D} \cup \mathcal{H} \notin \xi$. Hence $\{f \vee g\} \cup \mathcal{G} \notin \Omega$, or $f \vee g \notin \mathcal{G}$.

3. Binary L-merotopic spaces

Definition 3.1. Let (X, ξ) be an L-merotopic space and $\mathcal{A} \subseteq L^X$. Then

(i) (X, ξ) is said to be *binary* iff every L-guild in (X, ξ) belongs to ξ;

(ii) (X, ξ) is said to be *contigual* iff every finite subset of $\mathcal{A}$ belongs to $\xi \Rightarrow \mathcal{A} \in \xi$;

(iii) (X, ξ) is said to be *regular* iff

$$\{f \in I^X : \{g, \mathbf{1} - f\} \notin \xi, \text{ for some } g \in \mathcal{A}\} \in \xi \Rightarrow \mathcal{A} \in \xi.$$

If (X, ξ) is a binary (contigual, regular respectively) L-merotopic space, then ξ is referred to as a binary (contigual, regular respectively) L-merotopy on X; (X, ξ) is called a *binary L-nearness space* provided ξ is an L-nearness on X.

Remark 3.2. An L-guild in (X, ξ) may or may not belong to ξ, e.g. L-guilds ${}_x\mathcal{A}_t$ and $\mathcal{U}[x]$ of Example 2.3(ii) belong to any merotopy ξ on X, however in the indiscrete L-merotopic space (X, ξ) of Example 2.1(ii) the L-guild Θ does not belong to ξ. In the same indiscrete L-merotopic space (X, ξ), Θ also gives an example of a set whose every finite subset belong to ξ but the set itself does not belong to ξ. Thus the indiscrete L-merotopic space is neither contigual nor binary.

Corresponding to each L-merotopic space, the following theorem produces a binary L-merotopic space.

Theorem 3.3. *Let (X, ξ) be an L-merotopic space. Define $\hat{\xi} = \{\mathcal{A} \subseteq L^X : \mathcal{A} \subseteq G$, $\mathcal{G}$ is an L-guild in $(X, \xi)\}$. Then $(X, \hat{\xi})$ is a binary L-merotopic space and, for $f \in L^X$, $cl_{\hat{\xi}} f \leqslant cl_{\xi} f$. Moreover, if ξ is an L-nearness, then $\hat{\xi}$ is also an L-nearness.*

Proof. Let $\mathcal{A}$ corefines $\mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{G}$ for some L-guild $\mathcal{G}$ in (X, ξ). Then for $f \in \mathcal{A}$, there exists $g \in \mathcal{B}$ such that $f \geqslant g \in \mathcal{G}$. Since $\mathcal{G}$ is an L-grill, $f \in \mathcal{G}$. Hence $\mathcal{A} \subseteq \mathcal{G}$. Let $\wedge\mathcal{A} \neq \mathbf{0}$. Then there exists $x \in X$ such that $f(x) \neq 0$ for every $f \in \mathcal{A}$. We have an L-guild $\mathcal{U}[x]$ in (X, ξ) which contains $\mathcal{A}$. If $\mathbf{0} \in \mathcal{A}$, then $\mathcal{A} \notin \hat{\xi}$ and so $\xi \neq P(L^X)$. Finally, observe that every L-guild in $\hat{\xi}$ is an L-guild in ξ also. Let $\{cl_{\hat{\xi}} f, cl_{\hat{\xi}} g\} \subseteq \mathcal{G}$, for some L-guild $\mathcal{G}$ in ξ. Then the family $\mathcal{H} = \{h \in L^X : cl_{\xi} f \in \mathcal{G}\}$ is an L-guild in ξ containing $\{f, g\}$.

Example 3.4. Let X be a nonempty set. Then $\xi = \{\mathcal{A} \subseteq L^X :: \mathcal{A} \subset \mathcal{G}$, $\mathcal{G}$ is an L-grill on $X\}$ is a contigual L-merotopy on X: By the similar arguments as given in the proof of the above theorem, ξ is an L-merotopy on X. Let $\mathcal{A} \subseteq L^X$ and every finite subset $\mathcal{B}$ of $\mathcal{A}$ belongs to ξ. Then there exists an L-grill $\mathcal{G}_B$ in X such that $\mathcal{B} \subseteq \mathcal{G}_B$. Hence $\mathcal{A} \subseteq \cup\{\mathcal{G}_B : \mathcal{B} \subseteq \mathcal{A}, |\mathcal{B}| < \aleph_0\}$.

Theorem 3.5. *Let (X, ξ) be an L-merotopic space. Define $B\xi = \xi \cup \{\mathcal{A} \subseteq L^X : \mathcal{A} \subseteq \mathcal{G}$, $\mathcal{G}$ is an L-guild in $(X, \xi)\}$. Then*

(i) $(BX, B\xi)$ is an L-merotopic space, where BX has the same underlying set of points as X;

(ii) for $f, g \in L^X, \{f, g\} \in \xi$ iff $\{f, g\} \in B\xi$;

(iii) $\mathcal{G}$ is an L-guild in ξ iff $\mathcal{G}$ is an L-guild in $B\xi$;

(iv) the L-merotopic space $(BX, B\xi)$ is binary;

(v) for any $f \in L^X, cl_{\xi} f = cl_{B\xi} f$;

(vi) if (X, ξ) is an L-nearness space, then $(BX, B\xi)$ is an L-nearness space.

Proof. (i) Follows from Theorem 3.3.

(ii) Let $\{f, g\} \in B\xi$ and there exists an L-guild $\mathcal{G}$ in (X, ξ) with $\{f, g\} \subseteq \mathcal{G}$. Then $\{f, g\} \in \xi$.

(v) For $f \in L^X, cl_\xi f = \vee\{x_p : \{x_p, f\} \in \xi\} = \vee\{x_p : \{x_p, f\} \in B\xi\} = cl_{B\xi} f$.

(vi) Let $\{cl_{B\xi} f, cl_{B\xi} g\} \in B\xi$ and there exists an L-guild $\mathcal{G}$ in (X, ξ) such that $\{cl_{B\xi} f, cl_{B\xi} g\} \subseteq \mathcal{G}$. Since $\mathcal{G}$ is congenial, $\{cl_{B\xi} f, cl_{B\xi} g\} \in \xi$. Hence by (v), $\{f, g\} \in \xi$.

Definition 3.6. (i) The family of all L-merotopic spaces (L-nearness spaces) and L-merotopic maps forms a category and is denoted by LMER (LNEAR).

(ii) The category whose objects are binary L-merotopic spaces and morphisms are L-merotopic maps is denoted by LBIN.

Proposition 3.7. *Let T be an onto L-merotopic map from an L-merotopic space (X, ξ) to an L-merotopic space (Y, η). If $\mathcal{G}$ is an L-guild in (X, ξ), then $T(\mathcal{G}) = \{T(f) : f \in \mathcal{G}\}$ is an L-guild in (Y, η).*

Proof. Since T is an L-merotopic map and $\{f_1, f_2\} \in \xi$ for all $f_1, f_2 \in \mathcal{G}$, $\{T(f_1), T(f_2)\} \in \eta$ for all f_1, $f_2 \in \mathcal{G}$. Hence $T(\mathcal{G})$ is a congenial collection in η. Let $g \geqslant T(f), f \in \mathcal{G}$. Then $T^{-1}(g) \geqslant T^{-1}(T(f)) \geqslant f$ and so $T^{-1}(g) \in \mathcal{G}$. Since T is onto $g = T(T^{-1}(g))$ which shows that $g \in T(G)$.

Remark 3.8. Let (X, ξ) and (Y, η) be L-merotopic spaces. Let $T : (X, \xi) \to (Y, \eta)$ be a bijective map such that $T(\mathcal{A}) \in \eta \Rightarrow \mathcal{A} \in \xi$ (that is T^{-1} is an L-merotopic map). Then by the above proposition if $\mathcal{H}$ is an L-guild in (Y, η), then $T^{-1}(\mathcal{H}) = \{T^{-1}(g) : g \in \mathcal{H}\}$ is an L-guild in (X, ξ).

Let T be an L-merotopic map from an L-merotopic space (X, ξ) to a binary L-merotopic space (Y, η). Let $\mathcal{A} \in B\xi$. If $\mathcal{A} \in \xi$, then $T(\mathcal{A}) \in \eta$, otherwise there exists an L-guild $\mathcal{G}$ in (X, ξ) with $\mathcal{A} \subseteq \mathcal{G}$. Consider $\mathcal{H} = \{h \in L^Y : T(g) \leqslant h$ for some $g \in \mathcal{G}\}$. We assert that $\mathcal{H}$ is an L-guild in (Y, η) containing $T(\mathcal{A})$: Let $h_1, h_2 \in \mathcal{H}$. Then there exist $g_1, g_2 \in \mathcal{G}$ such that $T(g_1) \leqslant h_1$ and $T(g_2) \leqslant h_2$. Since $\{g_1, g_2\} \in \xi, \{T(g_1), T(g_2)\} \in \eta$. Again, since $\{h_1, h_2\}$ corefines $\{T(g_1), T(g_2)\}$, we have $\{h_1, h_2\} \in \eta$ that is $\mathcal{H}$ is congenial. Let $h_1 \vee h_2 \in \mathcal{H}$. Then $T(g) \leqslant h_1 \vee h_2$ for some $g \in \mathcal{G}$. Since $g \leqslant g \leqslant T^{-1}(T(g)) \leqslant T^{-1}(h_1) \vee T^{-1}(h_2)$, we get either $T^{-1}(h_1) \in \mathcal{G}$ or $T^{-1}(h_2) \in \mathcal{G}$. Using $T(T^{-1}(h)) \leqslant h$ for all $h \in I^Y$, we get $h_1 \in \mathcal{H}$ or $h_2 \in \mathcal{H}$. Hence $T : (BX, B\xi) \to (Y, \eta)$ is an L-merotopic map and so the identity mapping $I_X : (X, \xi) \to (BX, B\xi)$ is a binary reflection for (X, ξ). Thus we obtain the following :

Theorem 3.9. *LBIN is bireflective in LMER.*

Theorem 3.10. *The category of binary L-nearness spaces and L-nearness maps is bireflective in LNEAR.*

Proposition 3.11. *Every binary L-merotopic space is contigual.*

Proof. Let (X, ξ) be a binary L-merotopic space and $\mathcal{A} \subseteq L^X$ be such that every finite subcollection of $\mathcal{A}$ belongs to ξ. By Proposition 2.6, there exists an L-guild $\mathcal{G}$ in X with $\mathcal{A} \subseteq \mathcal{G}$. Since X is a binary space, $\mathcal{G} \in \xi$. Hence $\mathcal{A} \in \xi$.

Theorem 3.12. *Every regular contigual L-merotopic space is binary.*

Proof. Let (X, ξ) be a regular contigual L-merotopic space and let $\mathcal{G} \neq \phi$ be an L-guild in X. Suppose that $\mathcal{G} \notin \xi$. Since X is regular, $B = \{f \in L^X : \{g, \mathbf{1} - f\} \notin \xi$ for some $g \in \mathcal{G}\} \notin \xi$. Since X is contigual, there exists a finite collection $\mathcal{A}$, with $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{A} \notin \xi$. Therefore $\bigwedge \mathcal{A} = \mathbf{0}$ and so $(\bigvee_{f \in \mathcal{A}} (\mathbf{1} - f)) \in \mathcal{G}$. Hence there exists $f \in \mathcal{A}$ such that $\mathbf{1} - f \in \mathcal{G}$. Since $f \in \mathcal{B}$, there exists $g \in \mathcal{G}$ such that $\{g, \mathbf{1} - f\} \notin \xi$, contradicting the fact that $\mathcal{G}$ is congenial.

Following questions remain open in this section:

(i) Is there any example of a contigual L-merotopic space that is not binary?

(ii) Is there any example of a regular L-merotopic space that is not binary?

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