Zbornik radova Prirodno-matematičkog fakulteta — Univerzitet u Novom Sadu, knjiga 6 (1976)
Review of Research of Science — University of Novi Sad, Volume 6 (1976)

Olga Hadžić and *Đura Paunić*

THEOREMS ON THE FIXED POINT FOR SOME CLASSES OF MAPPINGS IN LOCALLY CONVEX SPACES

In [2] we have proved a theorem on the fixed point of the mapping $T:\mathcal{M}\to\mathcal{M}$ where $\mathcal{M}$ is a closed subset of a sequentially complete locally convex space E, in which the topology is defined by a family of seminorms $|\ |_\alpha$, $\alpha\in\mathcal{J}$ and the following inequality holds:

$$(1) \qquad |Tx-Ty|_\alpha \leqslant \sum_{\nu=1}^{n} q(\alpha, \nu)\, |x-y|_{\varphi_\nu(\alpha)}$$

for every $\alpha\in\mathcal{J}$ and every $x, y\in\mathcal{M}$; $q(\alpha, \nu)\geqslant 0$ for every $(\alpha, \nu)\in\mathcal{J}\times\{1, 2, \ldots, n\}$; $\varphi_\nu:\mathcal{J}\to\mathcal{J}$ $\nu=1, 2, \ldots, n$.

The aim of this paper is to generalize some results from [4], [5] and [6] using Theorem 1 from [2]. First, we shall give some notations:

$V(k, n)$ is the set of all variations with repetitions of the numbers $1, 2, \ldots, n$ of the class k; $\pi_k=i_1 i_2\ldots i_k\in V(k, n)$; $\Phi_{\pi_k}(\alpha)=(\varphi_{i_1}\circ\varphi_{i_2}\circ\ldots\circ\varphi_{i_k})(\alpha)$; $\Phi_{\pi_0}(\alpha)=\alpha$ *by* definition, for every $\alpha\in\mathcal{J}$; $\mathcal{J}(\alpha, k)=\{\Phi_{\pi_k}(\alpha)\mid \pi_k\in V(k, n)\}$ for every $\alpha\in\mathcal{J}$; $\mathcal{J}(\alpha, 0)=\{\alpha\}$ by definition for every $\alpha\in\mathcal{J}$; $P(\alpha, k, x)=\underset{\Phi_{\pi_k}(\alpha)\in\mathcal{J}(\alpha, k)}{\mathrm{MAX}}\left\{|Tx-x|_{\Phi_{\pi_k}(\alpha)}\right\}$

$\alpha\in\mathcal{J}$, $x\in\mathcal{M}$, $k=0, 1, 2, \ldots$; $Q(\alpha, k)=\underset{\substack{\Phi_{\pi_k}(\alpha)\in\mathcal{J}(\alpha, k)\\ \nu=1, 2, \ldots, n}}{\mathrm{MAX}}\left\{q(\Phi_{\pi_k}(\alpha), \nu)\right\}$

$\alpha\in\mathcal{J}$, $k=1, 2, \ldots$;

$$S(\alpha, x)=P(\alpha, 0, x)+\sum_{k=2}^{\infty} n^{k-1} P(\alpha, k-1, x)\prod_{\nu=0}^{k-2} Q(\alpha, \nu)$$

and $S_k(\alpha, x)$ is the k-th partial sum of the series $S(\alpha, x)$.

Theorem 1 [2] *Suppose that the following conditions are satisfied:*

1. *For every* $(\alpha, \nu)\in\mathcal{J}\times\{1, 2, \ldots, n\}$ *there exist* $q(\alpha, \nu)\geqslant 0$ *and* $\varphi_\nu:\mathcal{J}\to\mathcal{J}$ *so that the inequality* (1) *holds for every* $\alpha\in\mathcal{J}$ *and* $x, y\in\mathcal{M}$.

2. *There exists* $x_0\in\mathcal{M}$ *such that:*

$$R=\sup_{\alpha\in\mathcal{J}}\ \overline{\lim_{k\in N}}\sqrt[k]{P(\alpha, k, x_0)\prod_{\nu=1}^{k-1} Q(\alpha, \nu)}<\frac{1}{n}.$$

Then there exists at least one solution x^* of the equation $x=Tx$. Also, the following relations hold:

$$\text{(2)}\qquad \lim_{k\to\infty} n^k \underset{\Phi_{\pi_k}(\alpha)\in\mathcal{J}(\alpha,k)}{\mathrm{MAX}} \{|x^*-x_0|_{\Phi_{\pi_k}(\alpha)}\} \prod_{\nu=0}^{k-1} Q(\alpha,\nu) = 0, \text{ for every } \alpha\in\mathcal{J};$$

$$\text{(3)}\qquad |T^m x_0 - x_0|_{\Phi_{\pi_k}(\alpha)} \leqslant \frac{S(\alpha, x_0)-S_k(\alpha, x_0)}{n^k \prod_{\nu=0}^{k-1} Q(\alpha,\nu)}$$

$$m=1, 2, \ldots, k=1, 2, \ldots, \quad \alpha\in\mathcal{J};$$

(4) $|x^*-x_m|_\alpha \leqslant S(\alpha, x_0)-S_m(\alpha, x_0)$ *for every* $\alpha\in\mathcal{J}$, $m=1, 2, \ldots$ *where* $x_m = T^m x_0$, $m=1, 2, \ldots$.

Every other solution of the equation $x=Tx$ which also satisfies the relation (2) *is identical to the solution* $x^*=\lim\limits_{m\to\infty} x_m$.

Definition. *If we have a family of mappings $\{G_\lambda\}_{\lambda\in\Lambda}$ from $\mathcal{M}$ into $\mathcal{M}$ and a family of mappings $\{\varphi_\lambda\}_{\lambda\in\Lambda}$ from $\mathcal{J}$ into $\mathcal{J}$ then:*

$\Phi_{\lambda,\pi_k}(\alpha)=(\varphi_{\lambda,i_1}\circ\varphi_{\lambda,i_2}\circ\ldots\circ\varphi_{\lambda,i_k})(\alpha)$, $\lambda\in\Lambda$;

$\Phi_{\lambda,\pi_0}(\alpha)=\alpha$ *for every* $\alpha\in\mathcal{J}$, $\lambda\in\Lambda$;

$\mathcal{J}_\lambda(\alpha, k)=\{\Phi_{\lambda,\pi_k}(\alpha) \mid \pi_k\in V(k, n)\}$, *for every* $\lambda\in\Lambda$, $\alpha\in\mathcal{J}$;

$\mathcal{J}_\lambda(\alpha, 0)=\{\alpha\}$ *for every* $\alpha\in\mathcal{J}$, $\lambda\in\Lambda$;

$$P_\lambda(\alpha, k, x)=\underset{\Phi_{\lambda,\pi_k}(\alpha)\in\mathcal{J}_\lambda(\alpha,k)}{\mathrm{MAX}} \{|G_\lambda x - x|_{\Phi_{\lambda,\pi_k}(\alpha)}\}$$

for every $\lambda\in\Lambda$, $\alpha\in\mathcal{J}$, $x\in\mathcal{M}$.

Theorem 2. *Let E be a sequentially complete locally convex space, $\mathcal{M}$ be a closed subset of E, Λ be a topological space and G be a continuous mapping from $\mathcal{M}\times\Lambda$ into $\mathcal{M}$. Further, suppose that the following conditions are satisfied:*

1. *For every* $(\alpha,\nu,\lambda)\in\mathcal{J}\times\{1,\ldots,n\}\times\Lambda$ *there exist* $q(\alpha,\nu,\lambda)\geqslant 0$ *and* $\varphi_{\nu,\lambda}:\mathcal{J}\to\mathcal{J}$ *so that:*

$|G(x_1, \lambda)-G(x_2, \lambda)|_\alpha \leqslant \sum\limits_{\nu=1}^{n} q(\alpha,\nu,\lambda)|x_1-x_2|_{\varphi_{\nu,\lambda}(\alpha)}$ *for every* $(\alpha, x_1, x_2, \lambda)\in\mathcal{J}\times\mathcal{M}^2\times\Lambda$.

2. *For every* $(k, \alpha)\in[N\cup\{0\}]\times\mathcal{J}$:

$$\underset{\substack{\nu=1,\ldots,n\\ \Phi_{\lambda,\pi_k}(\alpha)\in\mathcal{J}_\lambda(\alpha,k),\,\lambda\in\Lambda}}{\sup} \{q(\Phi_{\pi_k}(\alpha),\nu,\lambda)\}=Q(\alpha, k)<\infty.$$

3. *For every* $(x, k, \alpha, \lambda) \in E \times [N \cup \{0\}] \times \mathcal{J} \times \Lambda$:

$$|x|_{\Phi_{\lambda, \pi_k}(\alpha)} \leqslant a_k(\alpha) |x|_{\beta(\alpha)}$$

for every $\Phi_{\lambda, \pi_k}(\alpha) \in \mathcal{J}_\lambda(\alpha, k)$ *and*

$$R = \sup_{\alpha \in \mathcal{J}} \overline{\lim_{k \in N}} \sqrt[k]{a_k(\alpha) \left(\prod_{\nu=0}^{k-1} Q(\alpha, \nu) \right)} < \frac{1}{n}.$$

Then there exists the mapping $x(\lambda)$ *from* Λ *into* $\mathcal{M}$ *which is continuous and such that:* $x(\lambda) = G[x(\lambda), \lambda]$ *for every* $\lambda \in \Lambda$.

Proof: We shall define the mappings $G_\lambda : \mathcal{M} \to \mathcal{M}$ $(\lambda \in \Lambda)$ in the following way: $G_\lambda x = G(x, \lambda)$. It is easy to see that all the mappings G_λ, $\lambda \in \Lambda$ satisfy the conditions of the Theorem 1 and so there exists the mapping $x(\lambda) : \Lambda \to \mathcal{M}$ such that: $x(\lambda) = G[x(\lambda), \lambda]$, $\lambda \in \Lambda$. We also have:

$$x(\lambda) = \lim_{m \to \infty} x_{m, \lambda} \quad \text{where} \quad x_{m, \lambda} = G(x_{m-1, \lambda}, \lambda).$$

From condition 3. it follows:

$$P_\lambda(\alpha, k, x) \leqslant a_k(\alpha) \mid G_\lambda x - x|_{\beta(\alpha)}$$

and because of $R < \frac{1}{n}$ it does not matter which is the first element $x_{0, \lambda} \in \mathcal{M}$. Let $x_{0, \lambda}$ be the element $x(\lambda_0)$, $\lambda_0 \in \Lambda$ Then we have:

$$|x(\lambda) - x(\lambda_0)|_\alpha \leqslant P_\lambda(\alpha, 0, x(\lambda_0)) + \sum_{\nu=1}^{\infty} P_\lambda(\alpha, \nu, x(\lambda_0)) \times n^\nu \prod_{\mu=0}^{\nu-1} Q(\alpha, \mu) \leqslant | x_{1, \lambda} - x(\lambda_0) |_{\beta(\alpha)} \left[a_0(\alpha) + \sum_{\nu=1}^{\infty} a_\nu(\alpha) n^\nu \prod_{\mu=1}^{\nu-1} Q(\alpha, \mu) \right].$$

Because of $x_{1, \lambda} = G(x(\lambda_0), \lambda)$ we have:

$$|x(\lambda) - x(\lambda_0)|_\alpha \leqslant | G(x(\lambda_0), \lambda) - G(x(\lambda_0), \lambda_0)|_{\beta(\alpha)} \times \left[a_0(\alpha) + \sum_{\nu=1}^{\infty} a_\nu(\alpha) n^\nu \prod_{\mu=0}^{\nu-1} Q(\alpha, \mu) \right]$$

and the mapping $x(\lambda)$ is continuous at the point λ_0 for every $\lambda_0 \in \Lambda$.

Theorem 3. *Let F be a closed and convex subset of the complete locally convex space E,* Λ *be a topological space, S be a continuous mapping from F into* Λ. *Suppose that G is a continuous mapping from* $F \times \overline{S(F)}$ *into F which satisfies all the conditions of Theorem 2 for* $\mathcal{M} = \mathrm{F}$ *and* $\Lambda = \overline{S(F)}$, *and that the set* $\overline{S(F)}$ *is compact.*

Then there exists at least one solution of the equation $x = G(x, S(x))$.

Proof: The proof is similar to the proof of Theorem 2 in [5]. From Theorem 2 it follows that there exists the mapping $x(\lambda) : \overline{S(F)} \to F$ such that: $x(\lambda) = G[x(\lambda), \lambda]$ for every $\lambda \in \overline{S(F)}$. Let $T(y)$ be, by definition, $T(y) = x[S(y)]$ for every $y \in F$. As we have proved in [5] the mapping T and the set F satisfy all the conditions of Tihonov's fixed point theorem and so there exists $y \in F$ such that:

$$y=T(y)=x(S(y))=G[x(S(y)),S(y)]=G[y,S(y)].$$

In the next theorem we shall use the following notations: E is a complete locally convex space in which the topology is defined by the family of seminorms $|\ |_\alpha$, $\alpha\in\mathcal{J}$; E_ν $(\nu=1,2,\ldots,n)$ is locally convex space in which the topology is defined by the family of seminorms $|\ |_{\nu,\alpha}$, $\alpha\in\mathcal{J}$ $(\nu=1,2,\ldots,n)$; F is a closed and convex subset of E; Λ_ν $(\nu=1,2,\ldots,n)$ are topological spaces; S_ν $(\nu==1,2,\ldots,n)$ are continuous mappings from F into the compact subsets K_ν of Λ_ν; G_ν $(\nu=1,2,\ldots,n)$ are continuous mappings from $F\times K_\nu$ into E; H is a mapping from $\prod_{\nu=1}^{n} E_\nu$ into F; $\mathcal{D}=\{(z_1,z_2,\ldots,z_n)|\ z_\nu=G_\nu(x,S_\nu y),\ \nu=1,\ldots,n;\ x,y\in F\}$.

Theorem 4. *Suppose that the mapping H maps $\mathcal{D}$ into F and that the following conditions are satisfied:*

1. *For every $(\alpha,\nu)\in\mathcal{J}\times\{1,2,\ldots,n\}$ there exists $q_1(\alpha,\nu)\geqslant 0$, $q_2(\alpha,\nu)\geqslant 0$ and the mappings $\psi_\nu:\mathcal{J}\to\mathcal{J}$, $\theta_\nu:\mathcal{J}\to\mathcal{J}$ such that:*

(5) $|H(z_1',\ldots,z_n')-H(z_1'',\ldots,z_n'')|_\alpha\leqslant\sum_{\nu=1}^{n}q_1(\alpha,\nu)\times|z_\nu'-z_\nu''|_{\nu,\ \psi_\nu(\alpha)}$ *for every* $z_\nu', z_\nu''\in E_\nu$, $\nu=1,\ldots,n$

(6) $|G_\nu(x_1,y)-G_\nu(x_2,y)|_{\nu,\alpha}\leqslant q_2(\alpha,\nu)\,|x_1-x_2|_{\theta_\nu(\alpha)}$ *for every* $(\alpha,\nu,x_1,x_2,y)\in\mathcal{J}\times\{1,\ldots,n\}\times F^2\times\overline{S_\nu(F)}$.

2. *For every $(x,m,\alpha)\in E\times[N\cup\{0\}]\times\mathcal{J}$, $|x|_{\Phi_{\pi_m}(\alpha)}\leqslant a_m(\alpha)\,|x|_{\beta(\alpha)}$ for every $\Phi_{\pi_m}\in\mathcal{J}(\alpha,m)$ and*

$$R=\sup_{\alpha\in J}\ \overline{\lim_{m\in N}}\sqrt[m]{a_m(\alpha)\left(\prod_{\nu=0}^{m-1}Q(\alpha,\nu)\right)}<\frac{1}{n}\ \textit{where}\ \varphi_\nu=\theta_\nu\circ\psi_\nu\quad \nu=1,\ldots,n,$$

$$Q(\alpha,m)=\underset{\substack{\nu=1,2,\ldots,n\\ \Phi_{\pi_m}(\alpha)\in J(\alpha,m)}}{\mathrm{MAX}}\{q_1(\Phi_{\pi_m}(\alpha),\nu)\times q_2(\Phi_{\pi_m}(\alpha),\nu)\},\ \alpha\in\mathcal{J};\ m=0,1,\ldots$$

Then there exists at least one solution of the equation:

(7) $x=H[G_1(x,Sx),\ldots,G_n(x,S_n x)]$

Proof: First, we shall define the mappings $G:F\times\prod_{\nu=1}^{n}K_\nu\to E$ and $S:F\to\prod_{\nu=1}^{n}K_\nu$ in the following way: $G(x,Y)=H[G_1(x,y),\ldots,G_n(x,y_n)]$, $x\in F$, $Y\in\prod_{\nu=1}^{n}K_\nu$, $S(x)=(S_1x,\ldots,S_nx)$, $x\in F$.

It is easy to see that the mappings G and S are continuous. We also have:

$$\overline{S(F)}\subset\overline{\prod_{\nu=1}^{n}S_\nu(F)}\subset\prod_{\nu=1}^{n}\overline{S_\nu(F)}\subset\prod_{\nu=1}^{n}K_\nu$$

and from this we conclude that the set $\overline{S(F)}$ is compact. Further from (5) and (6) it follows:

$$| G(x_1, Y)-G(x_2, Y) |_\alpha \leqslant \sum_{\nu=1}^{n} q_1(\alpha, \nu) \mid G_\nu(x_1, y_\nu)-G_\nu(x_2, y_\nu) \mid_{\nu, \psi_\nu(\alpha)} \leqslant$$

$$\leqslant \sum_{\nu=1}^{n} q_1(\alpha, \nu)\, q_2(\alpha, \nu) \mid x_1-x_2 \mid (\theta_\nu \circ \psi_\nu)(\alpha)$$

for every $(x_1, x_2, y) \in F^2 \times \overline{S(F)}$.

Because of $G(F, \overline{S(F)}) \subset F$ we conclude that all the conditions of Theorem 3 are satisfied and so there exists at least one element $x \in F$ such that:

$$x=G(x, Sx) \text{ i. e. } x=H[G_1(x, S x), \ldots, G_n(x, S_n x)]$$

Using this Theorem we can generalize the Theorem in [6]. Here we shall only formulate this theorem, since the proof of it is similar to the proof of the Theorem in [6].

Theorem 5. *Suppose that H and g_ν $(\nu=1, 2, \ldots, n)$ are as in the Theorem in* [6], *and that the conditions* 1. *and* 2. *of this theorem are satisfied. Further, suppose that $f_\nu \in \mathrm{Lip}_x(q_1(\alpha, \nu), \varphi_\nu, G_{\nu+1})$ $\nu=1, \ldots, n$ and for every $(x, m, \alpha) \in E \times [N \cup \{0\}] \times \mathcal{J}$ also holds:*

$$|x|_{\Phi_{\pi_m}}(\alpha) \leqslant a_m(\alpha)\, |x|_{\beta(\alpha)} \ \textit{for every}$$

$$\Phi_{\pi_m}(\alpha) \in \mathcal{J}(\alpha, m) \ \textit{and} \ R=\sup_{\alpha \in J} \lim_{m \in N} \sqrt[m]{q_m(\alpha) \prod_{\nu=0}^{m-1} Q(\alpha, \nu)} < \frac{1}{n(T+1)},$$

where $Q(\alpha, m)= \underset{\substack{\nu=1, 2, \ldots, n \\ \Phi_{\pi_m}(\alpha) \in J(\alpha, m)}}{\mathrm{MAX}} \left\{ q(\Phi_{\pi_m}(\alpha), \nu) \times q_1 \Phi_{\pi_m}(\alpha), \nu) \right\}$ $m=0, 1, 2, \ldots$ $\alpha \in \mathcal{J}$

Then there exists at least one solution of the initial value problem

$$\frac{dx}{dt}=H\left[f_1\left(t, x, g_1\left(t, x, \frac{dx}{dt}\right)\right), \ldots, f_n\left(t, x, g_n\left(t, x, \frac{dx}{dt}\right)\right)\right] \quad x(t_0)=x_0.$$

Theorem 6. *Suppose E is a locally convex space, F is a locally convex space, U is a closed subset of E, V is a closed and convex subset of F, H is a mapping of $U \times V$ into U and K is a mapping of $U \times V$ into V. Further, suppose that the following conditions are satisfied:*

1. *The mapping H satisfies all the conditions of Theorem 2 for $\mathcal{M}=U$ and $\Lambda=V$.*

2. *One of the following conditions is satisfied:*

a) *F is semireflexive, V is a bounded subset of F, K is a continuous, limiting compact mapping* [10].

b) *The measure of noncompactness* Ψ [10] *is defined on the set F and K is a continuous* Ψ*-densifying mapping. Also the mapping* Ψ *is monotone and has the following properties: Either*

I) *For every* $x_0 \in V$, $Q \subseteq V$, $Q \neq \varnothing$

$$\Psi(\{x_0\} \cup Q) = \Psi(Q)$$

or II) *For every* $x_0 \in V$, $Q_1 \subseteq V$, $Q_2 \subseteq V$

$$\Psi(x_0 + Q_1) = \Psi(Q_1) \textit{ and } \Psi(Q \cup Q_2) = \mathrm{MAX}\{\Psi(Q_1), \Psi(Q_2)\}.$$

Then there exists at least one $z \in U \times V$ *such that:*

$$z = (Hz, Kz).$$

Proof: The proof is similar (almost identical) to the proof of Theorem 4 in [4] where the existence of the mapping $R: V \to U$ such that $Ry = H(Ry, y)$ follows from Theorem 2. Further the mapping $Ty = K(Ry, y)$ is, by definition, $T: V \to V$ and, as in [4], it can be shown that T is either limiting compact mapping (if 2. I holds) or Ψ is densifying (if 2. II holds). In both cases there exists at least one element $y_0 \in V$ such that $y_0 = Ty_0$, and so $z = (Hz, Kz)$, where $z = (Ry_0, y_0)$.

REFERENCES

[1] Avramescu C., *Asupra unei teoreme de punct fix*, Studii si Cercetari mat. Acad. RSR, No 22, 1970., 215–221.

[2] O. Hadžić, Đ. Paunić, *A theorem on the fixed point in locally convex spaces*, Publ. Inst. Math. (in print).

[3] O. Hadžić, B. Stanković, *Some theorems on the fixed point in locally convex spaces*, Publ. Inst. Math., Beograd, T10 (24), 1970, 9–19.

[4] O. Hadžić, *Existence theorems for the system* $x = H(x, y)$ $y = K(x, y)$ *in locally convex spaces*, Publ. Inst. Math., Beograd, T. 16 (30), 1973., 65–73.

[5] O. Hadžić, *Existence of the solution to the equation* $x = G(x, Sx)$ *in locally convex spaces*, Math. Balk., 3, 1973., 118–123.

[6] O. Hadžić, *Implicit differential equation:* $\dot{x} = H(f_1(t, x, g_1(t, x, \dot{x})), \ldots, f_n(t, x, g_n(t, x, \dot{x})))$ $x(t_0)) = x_0$ *in locally convex spaces*, Zbornik radova Prirodno-matematičkog fakulteta u Novom Sadu, knj. 6, 1976.

[7] О. Хаџић, *Гранични задатак за диференцијалне једначине у локалнo конвексним просторима*, Мат. весник, 1975, 143—150.

[8] William R. Melvin, *Some extensions of the Krasnoseljskij fixed point theorem*, Journal of Differential Equation, 11 (1972), 335–348.

[9] В. В. Мосягин, А. И. Поволоцкий, *Задача коши для неявного дифференциального уравнения в локально выпуклом пространстве*, Уч. зап. Петрозав. Ун., Т. XVIII, Вып. 2, 1970, 122–127.

[10] Б. Н. Садовский, *Предельно компактные и уплотняющие оператори*, У. М. Н., Т. XXVII, Вып. 1 (163), 1972, 81–146.

Olga Hadžić i Đura Paunić

TEOREME O NEPOKRETNOJ TAČKI ZA NEKE KLASE PRESLIKAVANJA U LOKALNO KONVEKSNIM PROSTORIMA

Rezime

Korišćenjem teoreme o nepokretnoj tački iz rada [2] dokazana je teorema o neprekidnoj zavisnosti nepokretne tačke $x(\lambda)$ preslikavanja $G_\lambda(x)=G(x,\lambda)$ od parametra λ koji pripada topološkom prostoru Λ. Primenom dobivene teoreme uopšteni su neki rezultati radova [5], [4] i [6].

Formulisaćemo Teoremu 2 koja ima osnovnu ulogu u radu.

Teorema 2: *Neka je Λ topološki prostor, $\mathcal{M}$ zatvoren podskup sekvencijalno kompletnog lokalno konveksnog prostora E, G neprekidno preslikavanje proizvoda $\mathcal{M}\times\Lambda$ u skup $\mathcal{M}$ tako da su zadovoljeni sledeći uslovi:*

1. *Za svako $(\alpha, \nu, \lambda)\in J\times\{1, 2, \dots, k\}\times\Lambda$ postoji $q(\alpha, \nu, \lambda)\geqslant 0$ i preslikavanja $\varphi_{\nu,\lambda}$ skupa J u samog sebe tako da je:*

$$|G(x_1,\lambda)-G(x_2,\lambda)|_\alpha\leqslant\sum_{\nu=1}^{n} q(\alpha,\nu,\lambda)\,|x_1-x_2|_{\varphi_{\nu,\lambda}(\alpha)}$$

za svako $(x_1, x_2, \lambda, \alpha)\in M^2\times\Lambda\times J$.

2. *Za svako $(m, \alpha)\in[N\cup\{0\}]\times J$ je:*

$$\sup_{\substack{=1,2,\dots,k\\ \Phi\pi_m(\alpha)\in J_\lambda(\alpha,m),\ \lambda\in\Lambda}}\left\{q(\Phi\pi_m(\alpha),\nu,\lambda)\right\}=Q(\alpha,m)<\infty$$

3. *Za svako $(x, n, \alpha, \lambda)\in E\times[N\cup\{0\}]\times J\times\Lambda$ važi nejednakost: $|x|_{\Phi\pi_n(\alpha)}\leqslant a_n(\alpha)\,|x|_{\beta(\alpha)}$ za svako $\Phi\pi_n(\alpha)\in J(\alpha, n)$ i*

$$R=\sup_{\alpha\in J}\ \overline{\lim_{n\in N}}\ \sqrt[k]{a_n(\alpha)\left(\prod_{\nu=0}^{k-1}Q(\alpha,\nu)\right)}<\frac{1}{k}.$$

Tada za svako $\lambda\in\Lambda$ postoji $x(\lambda)\in\mathcal{M}$ tako da je:

$$x(\lambda)=G(x(\lambda),\lambda)$$

za svako $\lambda\in\Lambda$ i preslikavanje $x(\lambda)$ je neprekidno.