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GENERAL AGGREGATION OPERATORS ACTING ON FUZZY NUMBERS INDUCED BY ORDINARY AGGREGATION OPERATORS

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Abstract. Some special general aggregation operators are given with respect to different ordering relations on the set of all fuzzy subsets of a universe X, $\mathcal{F}(\mathcal{X})$. The proof that pointwise extensions of aggregation operators are general aggregation operators with respect to the ordering fuzzy subset ($\subseteq_{\mathcal{F}}$) is given. Also, we have proved that min–extensions of aggregation operators are general aggregation operators. When pointwise extensions of aggregation are viewed with respect to the ordering $\subseteq_{\mathcal{F}}$ we conclude that they are not necessarily general aggregation operators.

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1. Introduction

In order to expand the classical fuzzy arithmetic general aggregation operators are introduced as special functions defined on the space of all fuzzy subsets of some universe X. Classical fuzzy arithmetic is based on the extension principle (see [6], [7]) and operations on the values of membership functions of the arguments (see [10]).

First definition of aggregation operators on $\mathcal{F}(\mathcal{X})$ is given. They are derived from ordinary aggregation operators on the unit interval. In the definition of general aggregation operators the ordering plays an important role. It can happen that some functions are general aggregation operators with respect to one ordering and not when viewed with respect to another ordering. Also, the ordering relation determines which element of $\mathcal{F}(\mathcal{X})$ are boundary elements for that aggregation operator.

The remainder of the paper has the following structure. In Section 2 general aggregation operators are defined and also three types of possible general aggregation operators are introduced. In Section 3 the orderings on the set $\mathcal{F}(\mathcal{X})$, $\preceq_I$ and $\subseteq_{\mathcal{F}}$ are given. In Section 4 general aggregation operators of type 1 and 2 are viewed in respect to the orderings $\preceq_I$ and $\subseteq_{\mathcal{F}}$.

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2. Aggregation Operators

Definition 1. $A : \cup_{n\in\mathbb{N}}[0,1]^n \to [0,1]$ *is an aggregation operator on the unit interval if the following conditions hold:*

(A1) $A(0,\ldots,0) = 0$,

(A2) $A(1,\ldots,1) = 1$,

(A3) *if* $(\forall i = 1,2,\ldots,n)(x_i \leq y_i)$ *then* $A(x_1,x_2,\ldots,x_n) \leq A(y_1,y_2,\ldots,y_n)$.

Conditions **(A1)** and **(A2)** are called boundary conditions, and **(A3)** resembles the monotonicity property of the operator A. Let us now briefly define the notion of a fuzzy set. One of the most common aggregation operators are t–norms.

Definition 2. *Associative and symmetric aggregation operators with the unit element 1 are called triangular norms (t–norms).*

A fuzzy subset P *of a universe* X *is described by its membership function* $\mu_P : X \to [0,1]$. *The value of the membership function denoted by* μ_P *at the point* $x \in X$ *is a membership degree of the element* x *in the fuzzy set* P.

We denote by $\mathcal{F}(\mathcal{X})$ set of all fuzzy subsets of the universe X. The main contribution of this paper is the definition of aggregation operators on $\mathcal{F}(\mathcal{X})$ which are derived from ordinary aggregation operators on the unit interval. Fuzzy numbers $\tilde{0}$ and $\tilde{1}$ used in the next definition are defined depending on the ordering to resemble the minimal and maximal element in the ordering.

Definition 3. *Let* $\tilde{\mathbf{A}}$ *be a mapping* $\tilde{\mathbf{A}} : \cup_{n\in\mathbb{N}}\mathcal{F}(\mathcal{X}) \to \mathcal{F}(\mathcal{X})$ *and let* $\preceq$ *be an ordering on* $\mathcal{F}(\mathcal{X})$. $\tilde{\mathbf{A}}$ *is called a general aggregation operator on* $\mathcal{F}(\mathcal{X})$ *if the following properties hold:*

($\tilde{\mathbf{A1}}$) $\tilde{\mathbf{A}}(\tilde{0},\ldots,\tilde{0}) = \tilde{0}$,

($\tilde{\mathbf{A2}}$) $\tilde{\mathbf{A}}(\tilde{1},\ldots,\tilde{1}) = \tilde{1}$,

($\tilde{\mathbf{A3}}$) *if* $(\forall i = 1,2,\ldots,n)(P_i \preceq Q_i)$ *then* $\tilde{\mathbf{A}}(P_1,\ldots,P_n) \preceq \tilde{\mathbf{A}}(Q_1,\ldots,Q_n)$, *where* $P_1,P_2,\ldots,P_n,Q_1,Q_2,\ldots,Q_n,\tilde{0},\tilde{1} \in \mathcal{F}(\mathcal{X})$.

This definition implies that the orderings are important in defining aggregation operators on $\mathcal{F}(\mathcal{X})$. Obviously, different orderings generate different sets of aggregation operators. We will show the ways to derive operators $\tilde{\mathbf{A}}$ from ordinary aggregation operators A. Many ways can be proposed, the three (probably most common ones) will be given in the following definition.

Definition 4. *Let* $P_1,\ldots,P_n \in \mathcal{F}(\mathcal{X})$, $\tilde{\mathbf{A}} : \cup_{n\in\mathbb{N}}\mathcal{F}(\mathcal{X})^n \to \mathcal{F}(\mathcal{X})$ *and* A *be an ordinary aggregation operator on the unit interval.*

1. $\tilde{\mathbf{A}}$ *is a pointwise extension of* A *if the following holds:*

$$(\forall t \in \mathbb{R}\ \mu_{\tilde{\mathbf{A}}(P_1,\ldots,P_n)}(t) = A(\mu_{P_1}(t),\ldots,\mu_{P_n}(t)).$$

$\mu_{\tilde{A}(P_1,\ldots,P_n)}$ *is the membership function of the resulting fuzzy set obtained by applying the operator* $\tilde{\mathbf{A}}$ *to the fuzzy sets* $P_1,\ldots,P_n$.

2. *Let T be a t–norm. $\tilde{\mathbf{A}}$ is defined as a T extension of an aggregation operator A:*

$$\mu_{\tilde{\mathbf{A}}(P_1,\ldots,P_n)}(t) = \sup_{t=A(x_1,\ldots,x_n),x_i\in X} \{T(\mu_{P_1}(x_1), \mu_{P_2}(x_2), \ldots, \mu_{P_n}(x_n))\}.$$

3. *Let $X_1, X_2 \ldots X_n, Y$ be universes and let $P_1, P_2 \ldots P_n$ be fuzzy subsets of those universes, respectively. $\tilde{\mathbf{A}}$ is defined as an A extension of some increasing operator $\varphi : X_1 \times X_2 \times \cdots \times X_n \to Y$:*

$$\mu_{\tilde{\mathbf{A}}(P_1,\ldots,P_n)}(t) = \sup_{t=\varphi(x_1,\ldots,x_n),x_i\in P_i} \{A(\mu_{P_1}(x_1), \ldots, \mu_{P_n}(x_n))\}.$$

The operators that are derived by means of the previous definition we will call operators of type 1,2 and 3 respectively. The question is whether the properties ($\tilde{\mathbf{A1}}$), ($\tilde{\mathbf{A2}}$) and ($\tilde{\mathbf{A3}}$) hold for all three types of extensions $\tilde{\mathbf{A}}$ from Definition 5. Condition ($\tilde{\mathbf{A3}}$) depends on the order on $\mathcal{F}(\mathcal{X})$. In the remainder of the paper we will prove that the properties ($\tilde{\mathbf{A1}}$) and ($\tilde{\mathbf{A2}}$) hold for the extensions of type 1 and 2 and also we will propose fuzzy numbers which will act as numbers $\tilde{0}$ and $\tilde{1}$ depending on the type of general aggregation operator that we are using. Also, we will discus two different orderings on $\mathcal{F}(\mathcal{X})$. As we mentioned earlier the orderings are $\preceq_I$ and fuzzy subset $\subseteq_{\mathcal{F}}$. We will discuss when $\tilde{\mathbf{A}}$ fulfills property ($\tilde{\mathbf{A3}}$) with respect to the orderings and in the next section we will give the definitions of $\preceq_I$ and $\subseteq_{\mathcal{F}}$.

3. Orderings on the set $\mathcal{F}(\mathcal{X})$

3.1. Fuzzy subset ordering $\subseteq_{\mathcal{F}}$

Definition 5. *For any two fuzzy subsets P, Q of the universe X we say that P is a fuzzy subset of Q, denoted by $P \subseteq_{\mathcal{F}} Q$, if the following holds:*

$$(\forall x \in X)(\mu_P(x) \leq \mu_Q(x)),$$

where $\leq$ is an ordering on X.

Obviously, the relation $\subseteq_{\mathcal{F}}$ is an ordering on $\mathcal{F}(\mathcal{X})$, although it is not a total ordering. It is an ordering that ranks fuzzy numbers depending on their vertical position on the graph, i.e. if the two numbers are comparable the one that is bigger (the superset) is closer to the line $y = 1$ on the graph. It is obvious that the minimal and the maximal element in this order are the lines $y = 1$ (the whole universe) and $y = 0$ (empty set). That is why if we want to study aggregation operators with the ordering $\subseteq_{\mathcal{F}}$ the obvious choice for numbers $\tilde{0}$ and $\tilde{1}$ is :

$$(\forall x \in X)(\mu_{\tilde{0}}(x) = 0) \text{ and } (\forall x \in X)(\mu_{\tilde{1}}(x) = 1), \text{ respectively.}$$

3.2. The $\preceq_I$ ordering

This ordering is introduced in [2].

Definition 6. *Let $\preceq$ be an ordering on X and let $P \in \mathcal{F}(\mathcal{X})$. A fuzzy superset of P, denoted by $LTR(P)$ is defined as:*

$$\mu_{LTR(P)}(x) = \sup\{\mu_P(y) | y \preceq x\}.$$

Similarly, $RTL(P)$ is defined as:

$$\mu_{RTL(P)}(x) = \sup\{\mu_P(y) | x \preceq y\}.$$

$LTR(P)$ is actually the smallest fuzzy superset of P with a non–decreasing membership function.

Likewise, $LTR(P)$ is the smallest fuzzy superset of P with a non–increasing membership function.

Definition 7. *If $P, Q \in \mathcal{F}(\mathcal{X})$ then we can define an ordering $\preceq_I$ on $\mathcal{F}(\mathcal{X})$:*

$$P \preceq_I Q \text{ iff } LTR(P) \supseteq LTR(Q) \ \wedge \ RTL(P) \subseteq RTL(Q).$$

It can easily be seen that two fuzzy sets with different heights cannot be compared. Thus, a new ordering $\preceq_I^{''}$ is defined. First, a new set $\lceil P \rceil$ is defined:

$$\mu_{\lceil P \rceil}(x) = \begin{cases} 1 & \mu_P(x) = \text{ height}(A) \\ \mu_P(x) & \text{otherwise} \end{cases}$$

Definition 8. *For arbitrary $P, Q \in \mathcal{F}(\mathcal{X})$, a new ordering which can be applied to larger number of fuzzy sets is proposed:*

$$P \preceq_I^{''} Q \text{ iff } \lceil P \rceil \preceq_I \lceil Q \rceil$$

The ordering $\preceq_I$ ranks fuzzy numbers depending on their horizontal position on the graph. The more the membership function is to the right on the graph the "bigger" the fuzzy number is.

4. General aggregation operators $\tilde{\mathbf{A}}$

4.1. General aggregation operators of type 1

Definition 9. *General aggregation operators $\tilde{\mathbf{A}}$ of type 1 are derived from the classical aggregation operators A in the following way:*

$$(\forall t \in \mathbb{R}) \ (\mu_{\tilde{A}(P_1,\ldots,P_n)}(t) = A(\mu_{P_1}(t), \ldots, \mu_{P_n}(t)).$$

The question is whether these operators satisfy properties ($\tilde{\mathbf{A1}}$), ($\tilde{\mathbf{A2}}$) and ($\tilde{\mathbf{A3}}$). Since the operators of type 1 are a kind of vertical operators (the result moves vertically on the graph from the arguments) the obvious ordering to use is the fuzzy subset ($\subseteq_{\mathcal{F}}$) since it ranks fuzzy numbers depending on their vertical position on the graph. Now, we will prove that operators type 1 respect to the ordering fuzzy subset are general aggregation operators.

Theorem 1. *Let* $\tilde{\mathbf{A}}$ *be a general aggregation operator of type* 1*, i.e. a pointwise extension of an aggregation operator* A *and the membership functions of fuzzy sets* $\tilde{0}$ *i* $\tilde{1}$ *are respectively defined:*

$$\mu_{\tilde{0}}(x) = 0, \; x \in X, \tag{1}$$

$$\mu_{\tilde{1}}(x) = 1, \; x \in X. \tag{2}$$

Then, for the operator $\tilde{\mathbf{A}}$ *and fuzzy sets* $\tilde{0}$ *i* $\tilde{1}$ *the following conditions hold:*

($\tilde{\mathbf{A1}}$) $\tilde{\mathbf{A}}(\tilde{0},\dots,\tilde{0}) = \tilde{0}$,
($\tilde{\mathbf{A2}}$) $\tilde{\mathbf{A}}(\tilde{1},\dots,\tilde{1}) = \tilde{1}$,
($\tilde{\mathbf{A3}}$) *if* $(\forall i = 1,2,\dots,n)(P_i \subseteq_F Q_i)$ *then*

$$\tilde{\mathbf{A}}(P_1, P_2, ..., P_n) \subseteq_F \tilde{\mathbf{A}}(Q_1, Q_2, \dots, Q_n).$$

where $P_1,\dots,P_n, Q1,\dots,Q_n$ *are arbitrary fuzzy sets.*

In other words, $\tilde{\mathbf{A}}$ *is a general aggregation operator in respect to the ordering* $\subseteq_F$ *and fuzzy sets* $\tilde{0}$ *i* $\tilde{1}$ *defined by (1) and (2) respectively.*

Proof. We have:

$$(\tilde{\mathbf{A1}})\ \tilde{\mathbf{A}}(\tilde{0},\dots,\tilde{0})(t) = A(\mu_{\tilde{0}}(t),\dots,\mu_{\tilde{0}}(t)) = A(0,\dots,0) = 0$$

Since the upper presumption holds for every $t \in \mathbb{R}$ we can conclude

$$\tilde{\mathbf{A}}(\tilde{0},\dots,\tilde{0}) = \tilde{0}.$$

($\tilde{\mathbf{A2}}$) is proved analogously as ($\tilde{\mathbf{A1}}$).

Now, let us prove ($\tilde{\mathbf{A3}}$). Let us assume that for some fuzzy sets

$$(P_1,\dots,P_n, Q_1,\dots,Q_n \in \mathcal{F}(\mathcal{X}))(\forall i = 1,2,\dots,n)(P_i \subseteq_F Q_i).$$

Then for any $t \in X$ we have

$$\begin{aligned}\tilde{\mathbf{A}}(P_1, P_2, ..., P_n)(t) &= A(\mu_{P_1}(t),\dots,\mu_{P_n}(t))\\ &\le A(\mu_{Q_1}(t),\dots,\mu_{Q_n}(t))\\ &= \tilde{\mathbf{A}}(Q_1, Q_2,\dots,Q_n)(t).\end{aligned} \qquad \square$$

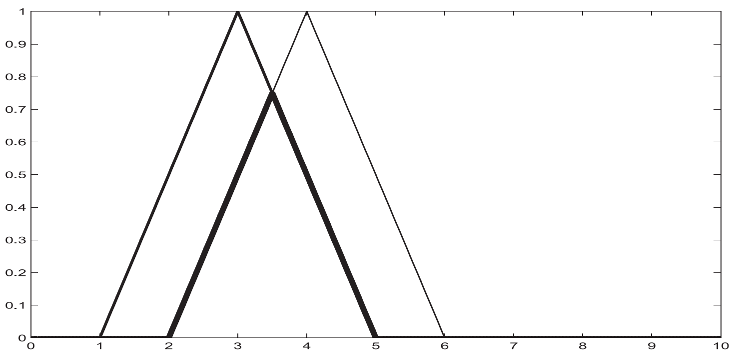

Figure 1. Pointwise minimum of two fuzzy sets

4.2. General aggregation operators of type 1 and the ordering $\preceq_I$

We will now prove that the general aggregation operators of type 1 with respect to the ordering $\preceq_I$ do not fulfill the condition ($\tilde{\mathbf{A}}\mathbf{3}$) in the general case. The proof will be given by a counterexample.

Theorem 2. *Let $\tilde{\mathbf{A}}$ be a general aggregation operator of type 1. The next implication does not hold for arbitrary fuzzy sets K, P i Q and arbitrary operator $\tilde{\mathbf{A}}$:*

$$\text{if } P \preceq_I Q \text{ then } \tilde{\mathbf{A}}(K,P) \preceq_I \tilde{\mathbf{A}}(K,Q)$$

Proof. We will prove the proposition by giving a counterexample. Fuzzy sets K, P and Q will be given so that the following holds $P \preceq_I Q$, but $\tilde{A}(K,P) \preceq_I \tilde{A}(K,Q)$ does not hold. Let μ_K, μ_P and μ_Q be defined by Figure 2:

$$\mu_K(x) = \begin{cases} 2.5x + 0.5, & x \in (0.5, 3); \\ -2x + 5, & x \in [3, 5]; \\ 0 & \text{otherwise}; \end{cases}$$

$$\mu_P(x) = \begin{cases} 0.25x + 1, & x \in [1, 1.25]; \\ 0.3, & x \in [1.25, 5.5]; \\ 0.35x + 2.625, & x \in [5.5, 7.5]; \\ -x + 8.5, & x \in [7.5, 8.5]; \\ 0 & \text{otherwise}; \end{cases}$$

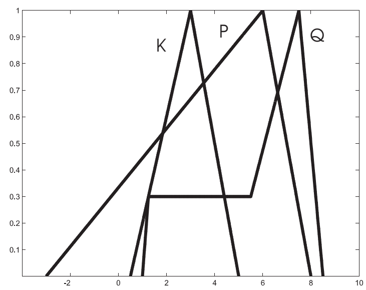

Figure 2. K, P and Q

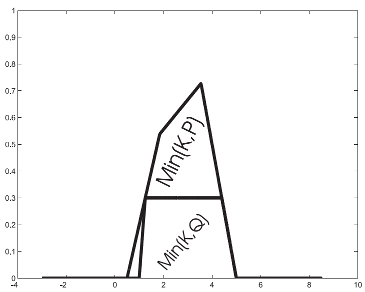

Figure 3. $\min(K, P)$ and $\min(K, Q)$

$$\mu_Q(x) \quad = \quad \begin{cases} 9x - 3, & x \in [-3, 6]; \\ -2x + 8, & x \in [6, 8]; \\ 0 & \text{otherwise.} \end{cases}$$

When we compare the fuzzy numbers P and K and the fuzzy numbers $\min(K, P)$ and $\min(K, Q)$ (where min is the pointwise extension of the classical minimum), we obtain that it holds $P \preceq_I Q$, but $\min(K, P) \preceq_I \min(K, Q)$ does not hold (see Figure 3), i.e. pointwise extensions are not always general aggregation operators with respect to $\preceq_I$, because the monotonicity property does not generally hold. □

Remark 1. *From Figure 3 we can also see that* $\min(K, P) \preceq_{I''} \min(K, Q)$, *which leads to a similar conclusion for the ordering* $\preceq_{I''}$.

5. General aggregation operators of type 2 and the ordering $\preceq_I$

General aggregation operators of type 2 are a min–extension of a arbitrary aggregation operator A on any interval $[a, b]$:

$$\mu_{\tilde{\mathbf{A}}(X_1,\ldots,X_n)}(t) = \sup_{t=A(x_1,\ldots,x_n),x_i \in X_i} \{\min(\mu_{P_1}(x_1), \mu_{P_2}(x_2), \ldots, \mu_{P_n}(x_n))\}.$$

The result that is obtained by applying a type 2 operator on any two fuzzy sets is always a movement horizontally on the graph (see Figure 4). So naturally, we will investigate whether the type 2 operators are aggregation operators with respect to the ordering $\preceq_I$.

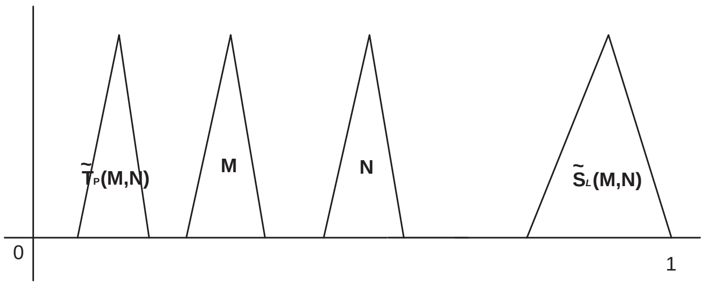

Figure 4. $\tilde{T_P}$ i $\tilde{S_L}$ type 2 operators applied on fuzzy numbers M i N

First we need to determine the values of fuzzy sets $\tilde{0}$ and $\tilde{1}$. As we mentioned earlier, the sets $\tilde{0}$ and $\tilde{1}$ are respectively the minimal and maximal element with respect to the ordering. As is well known, aggregation operators can act on any interval $[a, b]$ of the real line. Then, the obvious choices for $\tilde{0}$ and $\tilde{1}$ are: $\tilde{0} = \{a\}$ i $\tilde{1} = \{b\}$, the most left and the most right fuzzy set we can obtain from the interval $[a, b]$. Furthermore we will consider aggregation operators to act on any interval on the real line $[a, b]$ not only on the unit interval.

Theorem 3. *Let $\tilde{\mathbf{A}}$ be a general aggregation operator of type 2 which is a* min *extension of a classical aggregation operator $A : [a, b]^n \to [a, b]$. Then, the operator $\tilde{\mathbf{A}}$ is a general aggregation operator of type 2 with respect to $\preceq_I$ and the fuzzy sets $\tilde{0}$ and $\tilde{1}$ are defined by $\tilde{0} = \{a\}$ and $\tilde{1} = \{b\}$ respectively.*

Proof. We have to prove that $\tilde{\mathbf{A}}$ satisfies conditions $(\tilde{\mathbf{A1}}) - (\tilde{\mathbf{A3}})$. First we will prove $(\tilde{\mathbf{A1}})$. Rewritten, the condition $(\tilde{\mathbf{A1}})$ is

$$\tilde{\mathbf{A}}(\{b\}, \dots, \{b\}) = \{b\}.$$

On the other hand, since $\tilde{\mathbf{A}}$ is a min extension of A we know that

$$\tilde{\mathbf{A}}(\{b\}, \dots, \{b\})(t) = \sup_{A(x_1,\dots,x_n)=t} \min\{\mu_{\{b\}}(x_1), \dots, \mu_{\{b\}}(x_n)\}.$$

If at least one of the values $x_i,\ i = 1, 2 \dots, n$, is different from b we have that $\mu_{\{b\}}(x_i) = 0$ and

$$\min\{\mu_{\{b\}}(x_1), \dots, \mu_{\{b\}}(x_i), \dots, \mu_{\{b\}}(x_n)\} = 0.$$

Analogously, if $x_1 = \cdots = x_n = b$ then

$$\min\{\mu_{\{b\}}(b), \dots, \mu_{\{b\}}(b), \dots, \mu_{\{b\}}(b)\} = 1.$$

We can now conclude that only at the point b we have a value of 1 and in every other point the value is 0 which proves condition ($\tilde{\mathbf{A}}\mathbf{1}$).

We can prove ($\tilde{\mathbf{A}}\mathbf{2}$) analogously.

($\tilde{\mathbf{A}}\mathbf{3}$) is proved in [2] by proving the following theorem. □

The following theorem proves that MIN_φ, the min extension of the increasing operation φ is always an aggregation operator on $\mathcal{F}(\mathcal{X})$, i.e. it fulfills property ($\tilde{\mathbf{A}}\mathbf{3}$).

Theorem 4. *Let $\varphi : X_1 \times X_2 \times \cdots X_n \to Y$ be a mapping whose partial mappings are non–decreasing, i.e. for $\forall i = 1, 2 \dots n$, and for $\forall x_i, x_i^{'} \in X_i$ we have*

$$x_i \preceq_i x_i^{'} \Longrightarrow \varphi(x_1, \dots, x_i, \dots, x_n) \preceq_y \varphi(x_1, \dots, x_i^{'}, \dots, x_n),$$

where $\preceq_i$ is an ordering on X_i. Then, all the partial mappings of min extension of φ, denoted by $\hat{\varphi}$ are non–decreasing, i.e. for $\forall i = 1, 2 \dots n$ and for $\forall P_i, P_i^{'} \in \mathcal{F}(\mathcal{X}_i)$ we have:

$$P_i \preceq_{I_i} P_i^{'} \Longrightarrow \varphi(P_1, \dots, P_i, \dots, P_n) \preceq_Y \varphi(P_1, \dots, P_i^{'}, \dots, P_n),$$

where other components P_j, $j \neq i$, are arbitrary but fixed.

6. Conclusion and further work

In conclusion we would like to say that the results that are obtained are natural. The operators that are vertical by nature (general operators of type 1) are aggregation operators with respect to the ordering $\subseteq_{\mathcal{F}}$ which is a vertical ordering. Analogously, horizontal operators work well with horizontal orderings. On the other hand, we cannot mix vertical operators with horizontal orderings. In our further work we will investigate in more detail the set of general aggregation operators on fuzzy numbers.

Many applications of classical aggregation operators can be found (see [1], [5]). A relatively new concept of agent oriented programming seems to be an area that operators $\tilde{\mathbf{A}}$ can be applied, in most cases, agent is embedded in an unpredictable and dynamic environment (see [12], [13]). Therefore, agent needs a capability to adapt itself to new circumstances that emerge during its life. One way to achieve this adaptation is to use machine learning and fuzzy capabilities in which classic and extended aggregation operators have to play a certain role.

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