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SOME PROPERTIES OF H–PROJECTING

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Abstract. This paper shows several properties of projecting defined in [4]. The defined subject represents passing of the line through coordinate subspaces.

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1. Introduction

The set of points $\{\mathrm{O}, \mathrm{X}_1, \mathrm{X}_2, \dots, \mathrm{X}_n\}$ represents the simplex of Euclidean space E^n. Let $x_1 = \mathrm{OX}_1, x_2 = \mathrm{OX}_2, \dots, x_n = \mathrm{OX}_n$. Let $\mathrm{O}x_1x_2\dots x_n$ be a coordinate system of the Euclidean space E^n.

Let us complement the Euclidean space E^n with one hyperplane up to the projective space P^n.

Let $\mathrm{X}_{i\infty} \in x_i$ be an infinitely distant point of a straight line x_i, $i \in \{1, 2, \dots, n\}$. Let a pair of points $\mathrm{H}_{ik\infty}, \mathrm{H}_{ki\infty}$ be a harmonic conjugate of $\mathrm{X}_{i\infty}$, $\mathrm{X}_{k\infty}$. Let $\mathrm{E}^k_{1,\dots,k}$ be a coordinate subspace in E^k, defined by axes $x_1, x_2, \dots, x_n$ Let A be a point of the space E^n and let

$$\mathrm{A}^{12\dots(n-1)} = \mathrm{X}_{n\infty}\mathrm{A} \cap E^{n-1}_{12\dots(n-1)}$$

$$\mathrm{A}^n = \mathrm{H}_{(n-1)\infty}\mathrm{A} \cap \mathrm{E}^{(n-1)}_{12\dots(n-1)}$$

be projections of the point A onto the coordinate hyperplane $\mathrm{E}^{n-1}_{12\dots(n-1)}$. Points

$$(\mathrm{A}^{12\dots(n-1)})^{12\dots(n-2)} = \mathrm{A}^{12\dots(n-2)} = \mathrm{X}_{(n-1)\infty}\mathrm{A}^{12\dots(n-1)} \cap \mathrm{E}^{(n-2)}_{12\dots(n-2)}$$

$$(\mathrm{A}^n)^{12\dots(n-2)} = \mathrm{X}_{(n-1)\infty}\mathrm{A}^n \cap \mathrm{E}^{n-2}_{12\dots(n-2)}$$

$$(\mathrm{A}^{12\dots(n-1)})^{(n-1)} = \mathrm{A}^{(n-1)} = \mathrm{H}_{(n-1)1\infty}\mathrm{A}^{12\dots(n-1)} \cap \mathrm{E}^{(n-2)}_{12\dots(n-2)}$$

represent a projection of the points $\mathrm{A}^{12\dots(n-1)}$ and A^n onto the coordinate subspace $\mathrm{E}^{(n-2)}_{12\dots(n-2)}$ from the center of projection $\mathrm{X}_{(n-1)\infty}$ i $\mathrm{H}_{(n-1)1\infty}$. Points

$$(\dots((\mathrm{A}^{12\dots(n-1)})^{12\dots(n-2)})\dots)^{12\dots k} = \mathrm{A}^{12\dots k} = \mathrm{X}_{(n-(k+1))\infty}\mathrm{A}^{12\dots(k+1)} \cap \mathrm{E}^k_{12\dots k}$$

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$$(A^n)^{12\dots k} = X_{(n-(k+1))\infty}(A^n)^{12\dots(k+1)} \cap E^k_{12\dots k}$$

$$(A^{(n-1)})^{12\dots k} = X_{(n-(k+1))\infty}(A^{(n-1)})^{12\dots(k+1)} \cap E^k_{12\dots k}$$

$$(A^{(n-(k+1))})^{12\dots k} = X_{(n-(k+1))\infty}(A^{(n-(k+1))})^{12\dots(k+1)} \cap E^k_{12\dots k}$$

$$A^{(n-k)} = (A^{12\dots(n-k)})^{(n-k)} = H_{(n-(k+1))1\infty}((A^{12\dots(n-1)})^{\cdots})^{12\dots(n-k)} \cap E^k_{12\dots k}$$

represent projections onto the coordinate plane $E^k_{12\dots k}$ from the center of projecting $X_{(n-(k+1))\infty}$ and $H_{(n-(k+1))\infty}$.

Points $H_{n1\infty}, H_{(n-1)1\infty}, \dots, H_{3\infty}$ represent a harmonic conjugate of points $X_{1\infty}, X_{n\infty}; X_{1\infty}, X_{(n-1)\infty}; \dots; X_{1\infty}, X_{3\infty}$.

Theorem 1.1. *In H projecting, projections are collinear points on the support which is parallel with x_1 axis.*

Proof. Planes $\alpha(A, X_{n\infty}, H_{n1\infty})$ and $\beta(O, X_{1\infty}, X_{n\infty})$ are parallel. They contain the line $X_{1\infty}X_{n\infty}$. These planes intersect coordinate hyperplane $E^{(n-1)}_{12\dots(n-1)}$ along parallel lines x_1 and $p(A^{12\dots(n-1)}, A^n)$. The lines x_1 and $p(A^{12\dots(n-2)}, (A^{12\dots(n-1)})^{(n-1)})$ are also parallel, that is, $p(A^{12\dots(n-1)}, A^n) \cap p(O, X_{1\infty}) = X_{1\infty}$. Point $(A^n)^{12\dots(n-1)}$ belongs to the straight line $p(A^{12\dots(n-2)}, (A^{12\dots(n-1)})$ $(n-1))$. The plane $\delta(A^{12\dots(n-1)}, X_{1\infty}, X_{(n-1)\infty}) = \delta(A^n, X_{1\infty}, X_{(n-1)\infty})$ intersects $E^{(n-2)}_{12\dots(n-2)}$ along the line $p(A^{12\dots(n-2)}, (A^n)^{12\dots(n-2)})$. Consequently, points

$$A^{12\dots(n-2)}, (A^{12\dots(n-1)})^{(n-1)}, (A^n)^{12\dots(n-2)}$$

are collinear.

By analogy, by projecting from the points $X_{(n-2)\infty}, \dots, X_{3\infty}$, that is $H_{(n-2)1\infty}, \dots, H_{(3)1\infty}$, we would obtain collinear points in the plane x_1x_2

$$A^{12} = (\dots((A^{12\dots(n-1)})^{12\dots(n-2)})\dots)^{12},$$

$$A^3 = (A^{123})^3,$$

$$A^4 = (((A^{1234})^4)^{123})^{12}\dots$$

$$A^n = ((\dots((A^n)^{12\dots(n-1)})^{12\dots(n-2)}\dots)^{123})^{12}$$

Planes $\alpha(\mathrm{O}, \mathrm{X}_{1\infty}, \mathrm{X}_{n\infty})$ and $\beta(\mathrm{A}^n, \mathrm{A}^{12\ldots(n-1)}, \mathrm{A})$ are parallel. It holds that

$$\alpha \cap \mathrm{E}^{(n-1)} = x_1, \beta \cap \mathrm{E}^{(n-1)} = p(\mathrm{A}^n, \mathrm{A}^{12\ldots(n-1)})$$

and

$$p(\mathrm{A}^n, \mathrm{A}^{12\ldots(n-1)}) \cap x_1 = \mathrm{X}_{1\infty},$$

that is, line

$$p(\mathrm{A}^{12}, \mathrm{A}^3, \mathrm{A}^4, ..., \mathrm{A}^n) \cap x_1 = \mathrm{X}_{1\infty}.$$

□

Theorem 1.2. *Point A^{12} obtained by this method of projecting, and point A^{12}_m obtained by projecting of point A from the subspace defined by points $X_{3\infty}, X_{4\infty}, \ldots, X_{n\infty}$, coincide.*

Proof. Point A^{12} has been obtained as an intersection of a straight line from the subspace defined by points A, O, $\mathrm{X}_{3\infty}, \mathrm{X}_{4\infty}, \ldots \mathrm{X}_{n3\infty}$ and the plane $p1(\mathrm{X}_{1\infty}, \mathrm{X}_{2\infty}, \mathrm{O})$. As the subspace defined by points $\mathrm{A}, \mathrm{X}_{3\infty}, \mathrm{X}_{4\infty}, \ldots, \mathrm{X}_{n\infty}$ and the plane π_1 intersect at a point, it follows that $\mathrm{A}^{12} = \mathrm{A}^{12}_m$. □

Theorem 1.3. *Straight line $p(A, B)$ of Euclidean space En corresponds to the set of $n-1$ straight lines of plane E_{12}.*

Proof. Projection of point A is the set of $n-1$ collinear points $\mathrm{A}^{12}, \mathrm{A}^3, \mathrm{A}^4, \ldots, \mathrm{A}^n$ and by analogy, projection of point B is the set of $n-1$ collinear points $\mathrm{B}^{12}, \mathrm{B}^3, \mathrm{B}^4, \ldots, \mathrm{B}^n$. Straight lines $p^i(\mathrm{A}^i, \mathrm{B}^i)$ $(i = 12, 3, 4, \ldots, n)$ represent projection of the straight line $p(\mathrm{A}, \mathrm{B})$. □

2. Intersection of a straight line and a coordinate subspace

By projecting the straight line $p(\mathrm{A}, \mathrm{B})$ onto the hyperplane $\mathrm{E}_{12\ldots(n-1)}$, as a projection we get the set of two straight lines $p^{12\ldots(n-1)}(\mathrm{A}^{12\ldots(n-1)}; \mathrm{B}^{12\ldots(n-1)})$ and $p^n(\mathrm{A}^n; \mathrm{B}^n)$. The point of intersection of the line p and hyperplane is

$$P = p(\mathrm{A}, \mathrm{B} \cap \mathrm{E}_{12\ldots(n-1)}$$

As incidence is invariant of projecting, that implies $P = P^{12\ldots(n-1)} \in p^{12\ldots(n-1)}$ and $P = P^n \in p^n$, that is $P = p^{12\ldots(n-1)} \cap p^n$.

Straight line $p(P^{12}, P^3, ..., P^4)$ intersects the hyperplane $\mathrm{E}_{12\ldots(i-1)(i+1)\ldots n}$ at the point $P(P^{12}, P^3, \ldots, P^{i-1}, P^{12}, P^{i+1}, \ldots, P^n)$, where $P^{12} = P^i = \mathrm{A}^{12}\mathrm{B}^{12} \cap \mathrm{A}^i\mathrm{B}^i$.

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