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RICCI TYPE IDENTITIES FOR NON-BASIC DIFFERENTIATION IN OTSUKI SPACES[1]

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Abstract. In the Otsuki spaces one uses non-symmetric connections: one for contravariant and other for covariant indices. Also, we have two kinds of covariant differentiation - basic and non-basic. In the present work we investigate the Ricci type identities and curvature tensors for the non-basic differentiation.

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1. Introduction

The Otsuki space O_N is defined (see [3] - [9]) as an N-dimensional differentiable manifold on which, with respect to the local coordinates x^i $(i = 1, \cdots, N)$, is given a tensor field P^i_j $(det(P^i_j) \neq 0)$ and the connection coefficients $'\Gamma^i_{jk}$, $''\Gamma^i_{jk}$, which are generally non-symmetric and in force is the relation

$$P^i_{j,k} +'' \Gamma^i_{pk} P^p_j -' \Gamma^p_{jk} P^i_p = 0, \tag{1}$$

where $P^i_{j,k} = \partial P^i_j / \partial x^k$ and analogously in other cases.

In these spaces, the so-called *basic covariant derivative* of a tensor is defined, for example

$$V^i_{j;k} = V^i_{j,k} +' \Gamma^i_{pk} V^p_j -'' \Gamma^p_{jk} V^i_p, \tag{2}$$

and *non-basic covariant derivative*, for example

$$\nabla_k V^i_j = V^i_{j||k} = P^i_p P^q_j V^p_{q;k}. \tag{3}$$

The relation (1) is equivalent to

$$Q^i_{j||k} = 0, \tag{4}$$

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where $(Q^i_j) = (P^i_j)^{-1}$, i.e.

$$P^i_s Q^s_j = P^s_j Q^i_s = \delta^i_j. \tag{5}$$

2. Ricci type identities for non-basic differentiation of the first and the second kind

2.0. Because the connection coefficients $'\Gamma^i_{jk}$, $''\Gamma^i_{jk}$ are in general non-symmetric with respect to j, k, one can define two kinds of basic (see [3]) and non-basic differentiation. We designate the basic derivative of the kind α ($\alpha \in \{1,2\}$) by $\underset{\alpha}{|}$, and non-basic $\underset{\alpha}{\|}$. So, for a tensor of the type (u, v) we have

$$V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{1}{|} m} = V^{i_1\cdots i_u}_{j_1\cdots j_v , m} + \sum_{\alpha=1}^{u} {'\Gamma}^{i_\alpha}_{pm} \binom{p}{i_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} {''\Gamma}^{p}_{j_\beta m} \binom{j_\beta}{p} V^{\cdots}_{\cdots}, \tag{6}$$

$$V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{2}{|} m} = V^{i_1\cdots i_u}_{j_1\cdots j_v , m} + \sum_{\alpha=1}^{u} {'\Gamma}^{i_\alpha}_{mp} \binom{p}{i_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} {''\Gamma}^{p}_{m j_\beta} \binom{j_\beta}{p} V^{\cdots}_{\cdots}, \tag{7}$$

$$V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{1}{\|} m} = P^{i_1}_{a_1} \cdots P^{i_u}_{a_u} P^{b_1}_{j_1} \cdots P^{b_v}_{j_v} V^{a_1\cdots a_u}_{b_1\cdots b_v \underset{1}{|} m}, \tag{8}$$

$$V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{2}{\|} m} = P^{i_1}_{a_1} \cdots P^{i_u}_{a_u} P^{b_1}_{j_1} \cdots P^{b_v}_{j_v} V^{a_1\cdots a_u}_{b_1\cdots b_v \underset{2}{|} m}, \tag{9}$$

where we used the designations

$$\binom{p}{i_\alpha} V^{\cdots}_{\cdots} = V^{i_1\cdots i_{\alpha-1} p i_{\alpha+1}\cdots i_u}_{j_1\cdots j_v}, \qquad \binom{j_\beta}{p} V^{\cdots}_{\cdots} = V^{i_1\cdots i_u}_{j_1\cdots j_{\beta-1} p j_{\beta+1}\cdots j_v}. \tag{10}$$

From (6,7) for the Kronecker symbols we have

$$\delta^i_{j\underset{1}{|} m} = {'\Gamma}^i_{jm} - {''\Gamma}^i_{jm} \quad \delta^i_{j\underset{2}{|} m} = {'\Gamma}^i_{mj} - {''\Gamma}^i_{mj}. \tag{11}$$

The Ricci type identities for basic differentiation we obtained in [3].

In order to form the Ricci type identities for non-basic differentiation, e.g. for the tensor $V^{i_1\cdots i_u}_{j_1\cdots j_v}$, we consider the differences

$$V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{\lambda}{\|} m \underset{\mu}{\|} n} - V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{\nu}{\|} n \underset{\omega}{\|} m}, \quad \lambda,\ \mu,\ \nu,\ \omega \in \{1,2\}, \tag{12}$$

having 10 cases:

$$
\begin{aligned}
(\lambda,\mu;\nu,\omega)\in \quad &\{\ (1,1;1,1),(2,2;2,2),(1,2;1,2),(2,1;2,1),(1,1;2,2),\\
&(\ 1,1;1,2),(1,1;2,1),(2,2;1,2),(2,2;2,1),(1,2;2,1)\},
\end{aligned}
\tag{13}
$$

which we are to study.

By virtue of (8,9), we get

$$
\begin{aligned}
&V^{\ i_1\cdots i_u}_{\ j_1\cdots j_v\underset{\lambda}{||}m\underset{\mu}{||}n}=(V^{\cdots}_{\cdots\underset{\lambda}{||}m})_{\underset{\mu}{||}|n}=P^{i_1}_{a_1}\cdots P^{i_u}_{a_u}P^{b_1}_{j_1}\cdots P^{b_v}_{j_v}P^f_m(V^{a_1\cdots a_u}_{b_1\cdots b_v\underset{\lambda}{||}f})_{\underset{\mu}{|}n}=\\
&=P^{i_1}_{a_1}\cdots P^{i_u}_{a_u}P^{b_1}_{j_1}\cdots P^{b_v}_{j_v}P^f_m(P^{a_1}_{p_1}\cdots P^{a_u}_{p_u}P^{q_1}_{b_1}\cdots P^{q_v}_{b_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{\lambda}{|}f})_{\underset{\mu}{|}n}=\\
&=P^{i_1}_{a_1}\cdots P^{i_u}_{a_u}P^{b_1}_{j_1}\cdots P^{b_v}_{j_v}P^f_m(P^{a_1}_{p_1\underset{\mu}{|}n}P^{a_2}_{p_2}\cdots P^{a_u}_{p_u}P^{q_1}_{b_1}\cdots P^{q_v}_{b_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{\lambda}{|}f}+\\
&+P^{a_1}_{p_1}P^{a_2}_{p_2\underset{\mu}{|}}P^{a_3}_{p_3}\cdots P^{a_u}_{p_u}P^{q_1}_{b_1}\cdots P^{q_v}_{b_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{\lambda}{|}f}+\\
&+\cdots+P^{a_1}_{p_1}\cdots P^{a_{u-1}}_{p_{u-1}}P^{a_u}_{p_u\underset{\mu}{|}m}P^{q_1}_{b_1}\cdots P^{q_v}_{b_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{\lambda}{|}f}+\\
&+P^{a_1}_{p_1}\cdots P^{a_u}_{p_u}P^{q_1}_{b_1\underset{\mu}{|}n}P^{q_2}_{b_2}\cdots P^{q_v}_{b_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{\lambda}{|}f}+\\
&+\cdots+P^{a_1}_{p_1}\cdots P^{a_u}_{p_u}P^{q_1}_{b_1}\cdots P^{q_{v-1}}_{b_{v-1}}P^{q_v}_{b_v\underset{\mu}{|}n}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{\lambda}{|}f}+\\
&+P^{a_1}_{p_1}\cdots P^{a_u}_{p_u}P^{q_1}_{b_1}\cdots P^{q_v}_{b_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{\lambda}{|}f\underset{\mu}{|}n}),
\end{aligned}
$$

where we used the fact that for basic derivative the Leibniz rule for the product is valid. Introducing the designation

$$
P^i_aP^a_j=\mathcal{M}^i_j,
\tag{14}
$$

the previous equation becomes

$$
\begin{aligned}
V^{i_1\cdots i_u}_{j_1\cdots j_v\underset{\lambda}{||}m\underset{\mu}{||}n}=&\sum_{\alpha=1}^{u}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}}P^{i_\alpha}_{a_\alpha}\mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}}\cdots\mathcal{M}^{i_u}_{p_u}\cdot\\
&\cdot\ \mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}P^f_mV^{p_1\cdots p_u}_{q_1\cdots q_v\underset{\lambda}{|}f}P^{a_\alpha}_{p_\alpha\underset{\mu}{|}n}+\\
&+\sum_{\beta=1}^{v}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}}P^{b_\beta}_{j_\beta}\cdot\\
&\cdot\ \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}}\cdots\mathcal{M}^{q_v}_{j_v}P^f_mV^{p_1\cdots p_u}_{q_1\cdots q_v\underset{\lambda}{|}f}P^{q_\beta}_{b_\beta\underset{\mu}{|}n}+\\
&+\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\cdots\mathcal{M}^{q_v}_{j_v}P^f_mV^{p_1\cdots p_u}_{q_1\cdots q_v\underset{\lambda}{|}f\underset{\mu}{|}n}.
\end{aligned}
$$

From here we have

$$
\begin{aligned}
&V^{\,i_1\cdots i_u}_{\,j_1\cdots j_v \underset{\lambda}{||} m \underset{\mu}{||} n} - V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{\nu}{||} n \underset{\omega}{||} m} = \\
&= \sum_{\alpha=1}^{u} \mathcal{M}^{i_1}_{p_1}\cdots \mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}} P^{i_\alpha}_{a_\alpha} \mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1}\cdots \mathcal{M}^{q_v}_{j_v}\cdot \\
&\cdot\ (P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{\lambda}{|} f} P^{a_\alpha}_{p_\alpha \underset{\mu}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{\nu}{|} f} P^{a_\alpha}_{p_\alpha \underset{\omega}{|} m}) + \\
&+ \sum_{\beta=1}^{v} \mathcal{M}^{i_1}_{p_1}\cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1}\cdots \mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}} P^{b_\beta}_{j_\beta} \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}} \cdots \mathcal{M}^{q_v}_{j_v}\cdot \qquad (15)\\
&\cdot\ (P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{\lambda}{|} f} P^{q_\beta}_{b_\beta \underset{\mu}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{\nu}{|} f} P^{q_\beta}_{b_\beta \underset{\omega}{|} m}) + \\
&+ \mathcal{M}^{i_1}_{p_1}\cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1}\cdots \mathcal{M}^{q_v}_{j_v} (P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{\lambda}{|} f \underset{\mu}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{\nu}{|} f \underset{\omega}{|} m}).
\end{aligned}
$$

2.1. Let us consider the 1^st case according to (12) and (13), that is put into (15) $\lambda = \mu = \nu = \omega = 1$. Then, for the term in the first brackets on the right side one obtains

$$
\begin{aligned}
&P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} P^{a_\alpha}_{p_\alpha \underset{1}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} P^{a_\alpha}_{p_\alpha \underset{1}{|} m} \\
&\qquad = V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} P^{a_\alpha}_{p_\alpha \underset{1}{|} g} (P^f_m \delta^g_n - P^f_n \delta^g_m), \qquad (16)
\end{aligned}
$$

for the term in the second brackets:

$$
\begin{aligned}
&P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} P^{q_\beta}_{b_\beta | n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} P^{q_\beta}_{b_\beta \underset{1}{|} m} = \\
&\qquad = V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} P^{q_\beta}_{b_\beta \underset{1}{|} g} (P^f_m \delta^g_n - P^f_n \delta^g_m), \qquad (17)
\end{aligned}
$$

and for the term in the third brackets:

$$
(18)\quad P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f \underset{1}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f \underset{1}{|} m} = V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f \underset{1}{|} g} (P^f_m \delta^g_n - P^f_n \delta^g_m).
$$

Further we have

$$
2V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f \underset{1}{|} g} = V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} (f \underset{1}{|} g)} + V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} [f \underset{1}{|} g]}, \qquad (19)
$$

where

$$V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_{(f}\underset{1}{|}{}_{g)} = V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_f\underset{1}{|}{}_g + V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_g\underset{1}{|}{}_f, \tag{20}$$

$$V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_{[f}\underset{1}{|}{}_{g]} = V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_f\underset{1}{|}{}_g - V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_g\underset{1}{|}{}_f. \tag{21}$$

In [5], [6] is given:

$$\begin{aligned} V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_{[f}\underset{1}{|}{}_{g]} &= V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{|}{|}{}_f\underset{1}{|}{}_g - V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_g\underset{1}{|}{}_f = \\ &= \sum_{\alpha=1}^{u}{}'\underset{1}{R}{}^{p_\alpha}_{sfg}\binom{s}{p_\alpha}V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}''\underset{1}{R}{}^{s}_{q_\beta fg}\binom{q_\beta}{s}V^{\cdots}_{\cdots} \\ &\quad - {}''\Gamma^s_{[fg]}V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{|}{|}{}_s, \end{aligned} \tag{22}$$

where

$$^{\theta}\underset{1}{R}{}^i_{jfg} = {}^\theta\Gamma^i_{jf,g} - {}^\theta\Gamma^i_{jg,f} + {}^\theta\Gamma^p_{jf}\,{}^\theta\Gamma^i_{pg} - {}^\theta\Gamma^p_{jg}\,{}^\theta\Gamma^i_{pf}, \quad \theta\in\{{}',{}''\}, \tag{23}$$

is the *curvature tensor of the* 1^{st} *kind* in O_N, obtained by means of the connection ${}^\theta\Gamma$. The equation (22) is *the* 1^{st} *Ricci type identity for basic differentiation* in O_N. In view of (16)-(23) the eq. (15) for $\lambda=\mu=\nu=\omega=1$ becomes

$$\begin{aligned} V^{i_1\cdots i_u}_{j_1\cdots j_v}\underset{1}{\|}{}_m\underset{1}{\|}{}_n - V^{i_1\cdots i_u}_{j_1\cdots j_v}\underset{1}{\|}{}_n\underset{1}{\|}{}_m &\equiv V^{i_1\cdots i_u}_{j_1\cdots j_v}\underset{1}{\|}{}_{[m}\underset{1}{\|}{}_{n]} = \\ &= P^f_{[m}\delta^g_{n]}\Big\{\sum_{\alpha=1}^{u}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}}P^{i_\alpha}_{a_\alpha}\mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}}\cdots\mathcal{M}^{i_u}_{p_u}\,\cdot \\ &\cdot\ \mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_f P^{a_\alpha}_{p_\alpha}\underset{1}{|}{}_g + \\ &+ \sum_{\beta=1}^{v}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}}\,\cdot \\ &\cdot\ P^{b_\beta}_{j_\beta}\mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_f P^{q_\beta}_{b_\beta}\underset{1}{|}{}_g + \\ &+\ \frac{1}{2}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_{(f}\ \underset{1}{|}{}_{g)} + \\ &+ \sum_{\alpha=1}^{u}{}'\underset{1}{R}{}^{p_\alpha}_{sfg}\binom{s}{p_\alpha}V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}''\underset{1}{R}{}^{s}_{q_\beta fg}\binom{q_\beta}{s}V^{\cdots}_{\cdots} - \\ &- {}''\Gamma^s_{[fg]}V^{p_1\cdots p_u}_{q_1\cdots q_v}\underset{1}{|}{}_s\Big]\Big\}, \end{aligned} \tag{24}$$

which is *the* 1^{st} *Ricci type identity for non-basic differentiation in* O_N. So, we have

Theorem 1. *The first Ricci type identity for non-basic differentiation* $(8,9)$ *in* O_N *is the equation* (24), *where* $[mn]$, $[fg]$ *mean antisymmetrisation and* (fg) *the symmetrisation, without division with* 2, $\mathcal{M}^i_j$ *being given by* (14), *and* $'\underset{1}{R}$, $''\underset{2}{R}$ *according to* (23).

2.2. For $\lambda=\mu=\nu=\omega=2$, by the same procedure as in the previous case, according to (15), using the 2^{nd} Ricci identity for basic differentiation for $V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{2}{|}[f\underset{2}{|}g]}$ (eq. (18) in [3]), we get

Theorem 2. *In* O_N *is valid the* 2^{nd} *Ricci type identity for non-basic differentiation*

$$
\begin{aligned}
&V^{i_1\cdots i_u}_{j_1\cdots j_v\underset{2}{\|}m\underset{2}{\|}n}-V^{i_1\cdots i_u}_{j_1\cdots j_v\underset{2}{\|}n\underset{2}{\|}m}=\\
&\quad=P^f_{[m}\delta^g_{n]}\Big\{\sum_{\alpha=1}^{u}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}}P^{i_\alpha}_{a_\alpha}\mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}}\cdots\mathcal{M}^{i_u}_{p_u}\cdot\\
&\quad\cdot\ \mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{2}{|}f}P^{a_\alpha}_{p_\alpha\underset{2}{|}g}+\\
&\quad+\sum_{\beta=1}^{v}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}}P^{b_\beta}_{j_\beta}\cdot\\
&\quad\cdot\ \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{2}{|}f}P^{q_\beta}_{b_\beta\underset{2}{|}g}+\\
&\quad+\frac{1}{2}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{2}{|}(f\underset{2}{|}g)}+\\
&\quad+\sum_{\alpha=1}^{u}{}'\underset{2}{R}{}^{p_\alpha}_{sfg}\binom{s}{p_\alpha}V^{\cdots}_{\cdots}-\sum_{\beta=1}^{v}{}''\underset{2}{R}{}^{s}_{q_\beta fg}\binom{q_\beta}{s}V^{\cdots}_{\cdots}-\\
&\quad-{}''\Gamma^s_{[fg]}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{2}{|}s}\Big]\Big\},
\end{aligned}
\tag{25}
$$

where

$$
{}^\theta\underset{2}{R}{}^i_{jfg}={}^\theta\Gamma^i_{fj,g}-{}^\theta\Gamma^i_{gj,f}+{}^\theta\Gamma^p_{fj}\,{}^\theta\Gamma^i_{gp}-{}^\theta\Gamma^p_{gj}\,{}^\theta\Gamma^i_{fp},\quad \theta\in\{',''\},
\tag{26}
$$

is a curvature tensor of the 2^{nd} *kind in* O_N, *obtained by means of* ${}^\theta\Gamma$.

2.3. In the case $(\lambda,\mu;\nu,\omega)=(1,2;1,2)$ for the term in the first brackets on the right side at (15) we have

$$
\begin{aligned}
&P^f_mV^{p_1\cdots p_u}_{q_1\cdots q_v\underset{1}{|}f}P^{a_\alpha}_{p_\alpha\underset{2}{|}n}-P^f_nV^{p_1\cdots p_u}_{q_1\cdots q_v\underset{1}{|}f}P^{a_\alpha}_{p_\alpha\underset{2}{|}m}=\\
&\qquad=V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{1}{|}f}P^{a_\alpha}_{p_\alpha\underset{2}{|}g}(P^f_m\delta^g_n-P^f_n\delta^g_m),
\end{aligned}
$$

for the term in the second brackets:

$$P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}f} P^{q_\beta}_{b_\beta}{}_{\underset{2}{|}n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}f} P^{q_\beta}_{b_\beta}{}_{\underset{2}{|}m} = \\ = V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}f} P^{q_\beta}_{b_\beta}{}_{\underset{2}{|}g} (P^f_m \delta^g_n - P^f_n \delta^g_m),$$

and in the third brackets:

$$P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}f\underset{2}{|}n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}f\underset{2}{|}m} = V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}f\underset{2}{|}g} (P^f_m \delta^g_n - P^f_n \delta^g_m).$$

Presenting $V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}f\underset{2}{|}g}$ analogously to (19) and putting the previous expressions into (15), using the value for $V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}[f\underset{2}{|}g]}$ acording to (20) in [3] (the 3^{rd} Ricci type identity for basic differentiation), we obtain the 3^{rd} Ricci type identity for non-basic differentiation in O_N:

$$\begin{aligned} V^{i_1\cdots i_u}_{j_1\cdots j_v}{}_{\underset{1}{||}m\underset{2}{||}n} - V^{i_1\cdots i_u}_{j_1\cdots j_v}{}_{\underset{1}{||}n\underset{2}{||}m} = \\ &= P^f_{[m}\delta^g_{n]} \Big\{ \sum_{\alpha=1}^{u} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}} P^{i_\alpha}_{a_\alpha} \mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}} \cdots \mathcal{M}^{i_u}_{p_u} \cdot \\ &\cdot \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_v}_{j_v} V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}f} P^{a_\alpha}_{p_\alpha}{}_{\underset{2}{|}g} + \\ &+ \sum_{\beta=1}^{v} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}} P^{b_\beta}_{j_\beta} \cdot \\ &\cdot \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}} \cdots \mathcal{M}^{q_v}_{j_v} V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}f} P^{q_\beta}_{b_\beta}{}_{\underset{2}{|}g} + \\ &+ \frac{1}{2} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_v}_{j_v} \Big[V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}(f\ \underset{2}{|}g)} + \\ &+ \sum_{\alpha=1}^{u} {}' \underset{1}{A}{}^{p_\alpha}_{sfg} \binom{s}{p_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} {}'' \underset{2}{A}{}^{s}_{q_\beta fg} \binom{q_\beta}{s} V^{\cdots}_{\cdots} + \\ &+ V^{p_1\cdots p_u}_{q_1\cdots q_v <[fg]>} + V^{p_1\cdots p_u}_{q_1\cdots q_v \leq[fg]\geq} + {}''\Gamma^s_{[fg]} V^{p_1\cdots p_u}_{q_1\cdots q_v}{}_{\underset{1}{|}s} \Big] \Big\}. \end{aligned} \tag{27}$$

So, we have

Theorem 3. The 3^{rd} Ricci type identity for non-basic differentiation in O_N is given by the equation (27), where [3]:

$$' \underset{1}{A}{}^i_{jfg} = {}'\Gamma^i_{jf,g} - {}'\Gamma^i_{jg,f} + {}'\Gamma^p_{jf}{}'\Gamma^i_{gp} - {}'\Gamma^p_{jg}{}'\Gamma^i_{fp}, \tag{28}$$

$$'' \underset{2}{A}{}^i_{jfg} = {}''\Gamma^i_{jf,g} - {}''\Gamma^i_{jg,f} + {}''\Gamma^p_{fj}{}''\Gamma^i_{pg} - {}''\Gamma^p_{gj}{}''\Gamma^i_{pf}, \tag{29}$$

$$V^{p_1\cdots p_u}_{q_1\cdots q_v <fg>} = \sum_{\alpha=1}^{u} {}'\Gamma^{p_\alpha}_{[sf]} \binom{s}{p_\alpha} V^{\cdots}_{\cdots,g} - \sum_{\beta=1}^{v} {}''\Gamma^s_{[q_\beta f]} \binom{q_\beta}{s} V^{\cdots}_{\cdots,g}, \tag{30}$$

$$
\begin{aligned}
V^{p_1\cdots p_u}_{q_1\cdots q_v\le fg\ge} &= \sum_{\alpha=1}^{u-1}\sum_{\substack{\beta=2\\(\alpha<\beta)}}^{u}({}'\Gamma^{p_\alpha}_{pf}{}'\Gamma^{p_\beta}_{gs}-{}'\Gamma^{p_\alpha}_{fp}{}'\Gamma^{p_\beta}_{sg})\binom{p}{p_\alpha}\binom{s}{p_\beta}V^{\cdots}_{\cdots}-\\
&-\sum_{\alpha=1}^{u}\sum_{\beta=1}^{v}({}'\Gamma^{p_\alpha}_{pf}{}''\Gamma^{s}_{gq_\beta}-{}'\Gamma^{p_\alpha}_{fp}{}''\Gamma^{s}_{q_\beta g})\binom{p}{p_\alpha}\binom{q_\beta}{s}V^{\cdots}_{\cdots}+\\
&+\sum_{\alpha=1}^{v-1}\sum_{\substack{\beta=2\\(\alpha<\beta)}}^{v}({}''\Gamma^{p}_{q_\alpha f}{}''\Gamma^{s}_{gq_\beta}-{}''\Gamma^{p}_{fq_\alpha}{}''\Gamma^{s}_{q_\beta g})\binom{q_\alpha}{p}\binom{q_\beta}{s}V^{\cdots}_{\cdots}.
\end{aligned}\tag{31}
$$

2.4. The next theorem is proved analogously to the previous one, using the corresponding identity for basic differentiation [3].

Theorem 4. *In the space O_N is in force the 4^{th} Ricci type identity for non-basic differentiation:*

$$
\begin{aligned}
&V^{i_1\cdots i_u}_{j_1\cdots j_v\underset{2}{||}m\underset{1}{||}n}-V^{i_1\cdots i_u}_{j_1\cdots j_v\underset{2}{||}n\underset{1}{||}m}=\\
&=P^f_{[m}\delta^g_{n]}\Big\{\sum_{\alpha=1}^{u}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}}P^{i_\alpha}_{a_\alpha}\mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}}\cdots\mathcal{M}^{i_u}_{p_u}\cdot\\
&\cdot\ \mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{2}{|}f}P^{a_\alpha}_{p_\alpha\underset{1}{|}g}+\\
&+\sum_{\beta=1}^{v}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}}P^{b_\beta}_{j_\beta}\cdot\\
&\cdot\ \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{2}{|}f}P^{q_\beta}_{b_\beta\underset{1}{|}g}+\\
&+\frac{1}{2}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{2}{|}(f\underset{1}{|}g)}+\\
&+\sum_{\alpha=1}^{u}{}'\underset{3}{A}{}^{p_\alpha}_{sfg}\binom{s}{p_\alpha}V^{\cdots}_{\cdots}-\sum_{\beta=1}^{v}{}''\underset{4}{A}{}^{s}_{q_\beta fg}\binom{q_\beta}{s}V^{\cdots}_{\cdots}-\\
&-V^{p_1\cdots p_u}_{q_1\cdots q_v<[fg]>}-V^{p_1\cdots p_u}_{q_1\cdots q_v\le[fg]\ge}-{}''\Gamma^{s}_{[fg]}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{2}{|}s}\Big]\Big\},
\end{aligned}\tag{32}
$$

where

$$
{}'\underset{3}{A}{}^{i}_{jfg}={}'\Gamma^{i}_{jf,g}-{}'\Gamma^{i}_{gj,f}+{}'\Gamma^{p}_{fj}{}'\Gamma^{i}_{pg}-{}'\Gamma^{p}_{gj}{}'\Gamma^{i}_{pf},\tag{33}
$$

$$
{}''\underset{4}{A}{}^{i}_{jfg}={}''\Gamma^{i}_{fj,g}-{}''\Gamma^{i}_{gj,f}+{}''\Gamma^{p}_{jf}{}''\Gamma^{i}_{gp}-{}''\Gamma^{p}_{jg}{}''\Gamma^{i}_{fp},\tag{34}
$$

and the rest designations are explained previously.

2.5. The following case, according to (12,13), one obtains for $(\lambda,\mu;\nu,\omega)=(1,1;2,2)$. Then for the term in the third brackets on the right side in (15) one gets

$$
\begin{aligned}
&P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{1}{\mid}f\underset{1}{\mid}n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{2}{\mid}f\underset{2}{\mid}m} = \\
&\qquad = \frac{1}{2}P^f_m\big(V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{1}{\mid}(f\underset{1}{\mid}n)} + V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{1}{\mid}[f\underset{1}{\mid}n]}\big) - \\
&\qquad - \frac{1}{2}P^f_n\big(V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{2}{\mid}(f\underset{2}{\mid}m)} + V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{2}{\mid}[f\underset{2}{\mid}m]}\big).
\end{aligned}
\tag{35}
$$

Expressing $V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{1}{\mid}[f\underset{1}{\mid}n]}$ by virtue of (22) and $V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{2}{\mid}[f\underset{2}{\mid}m]}$ as in the proof of Theorem 2, based on (15) and (35), we get

$$
\begin{aligned}
&V^{i_1\cdots i_u}_{j_1\cdots j_v\,\underset{1}{\|}m\underset{1}{\|}n} - V^{i_1\cdots i_u}_{j_1\cdots j_v\,\underset{2}{\|}n\underset{2}{\|}m} = \\
&= \sum_{\alpha=1}^{u}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}}P^{i_\alpha}_{a_\alpha}\mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}\cdot \\
&\cdot\ \big(P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{1}{\mid}f}P^{a_\alpha}_{p_\alpha\underset{1}{\mid}n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{2}{\mid}f}P^{a_\alpha}_{p_\alpha\underset{2}{\mid}m}\big) + \\
&+ \sum_{\beta=1}^{v}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}}P^{b_\beta}_{j_\beta}\mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}}\cdots\mathcal{M}^{q_v}_{j_v}\cdot \\
&\cdot\ \big(P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{1}{\mid}f}P^{q_\beta}_{b_\beta\underset{1}{\mid}n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{2}{\mid}f}P^{q_\beta}_{b_\beta\underset{2}{\mid}m}\big) + \\
&+ \frac{1}{2}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}\Big\{P^f_m\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{1}{\mid}(f\underset{1}{\mid}n)} + \\
&+ \sum_{\alpha=1}^{u}{}'\underset{1}{R}{}^{p_\alpha}_{sfn}\binom{s}{p_\alpha}V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}''\underset{1}{R}{}^{s}_{q_\beta fn}\binom{q_\beta}{s}V^{\cdots}_{\cdots} - {}''\Gamma^s_{[fn]}V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{1}{\mid}s}\Big] - \\
&- P^f_n\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{2}{\mid}(f\ \underset{2}{\mid}m)} + \sum_{\alpha=1}^{u}{}'\underset{2}{R}{}^{p_\alpha}_{sfm}\binom{s}{p_\alpha}V^{\cdots}_{\cdots} - \\
&- \sum_{\beta=1}^{v}{}''\underset{2}{R}{}^{s}_{q_\beta fm}\binom{q_\beta}{s}V^{\cdots}_{\cdots} + {}''\Gamma^s_{[fm]}V^{p_1\cdots p_u}_{q_1\cdots q_v\,\underset{2}{\mid}s}\Big]\Big\},
\end{aligned}
\tag{36}
$$

where the curvature tensors ${}^\theta\underset{1}{R}$, ${}^\theta\underset{2}{R}$ $(\theta = {}', {}'')$ are given by virtue of (23), respectively (25). Thus we have proved

Theorem 5. *The* 5^{th} *Ricci type identity for non-basic differentiation in the space* O_N *is given by the equation* (36).

2.6. Let us consider the case $(\lambda,\mu;\nu,\omega) = (1,1;1,2)$. For the term in the 3^{rd} brackets of (15) one obtains

$$
\begin{aligned}
&P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f \underset{1}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f \underset{2}{|} m} = \\
&\qquad = \frac{1}{2} P^f_m \big(V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} (f \underset{1}{|} n)} + V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} [f \underset{1}{|} n]}\big) - \\
&\qquad - \frac{1}{2} P^f_n \big(V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} (f \underset{2}{|} m)} + V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} [f \underset{2}{|} m]}\big).
\end{aligned}
$$

Presenting $V^{\cdots}_{\cdots \underset{1}{|} [f \underset{1}{|} n]}$ according to (22), and $V^{\cdots}_{\cdots \underset{1}{|} [f \underset{2}{|} m]}$ as in the proof of Theorem 3, we obtain from (15) *the* 6^{th} *Ricci type identity for non-basic differentiation* in O_N:

$$
\begin{aligned}
&V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{1}{||} m \underset{1}{||} n} - V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{1}{||} n \underset{2}{||} m} = \\
&= \sum_{\alpha=1}^{u} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}} P^{i_\alpha}_{a_\alpha} \mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_v}_{j_v} V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} \cdot \\
&\cdot\ (P^f_m P^{a_\alpha}_{p_\alpha \underset{1}{|} n} - P^f_n P^{a_\alpha}_{p_\alpha \underset{2}{|} m}) + \\
&+ \sum_{\beta=1}^{v} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}} P^{b_\beta}_{j_\beta} \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}} \cdots \mathcal{M}^{q_v}_{j_v} \cdot \\
&\cdot\ V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} (P^f_m P^{q_\beta}_{b_\beta \underset{1}{|} n} - P^f_n P^{q_\beta}_{b_\beta \underset{2}{|} m}) + \\
&+ \frac{1}{2} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_v}_{j_v} \Big\{ P^f_m \Big[V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} (f \ \underset{1}{|} n)} + \\
&+ \sum_{\alpha=1}^{u}{}' \underset{1}{R}{}^{p_\alpha}_{sfn} \binom{s}{p_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}'' \underset{1}{R}{}^{s}_{q_\beta fn} \binom{q_\beta}{s} V^{\cdots}_{\cdots} - {}''\Gamma^s_{[fn]} V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} s} \Big] - \\
&- P^f_n \Big[V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} (f \ \underset{2}{|} m)} + \sum_{\alpha=1}^{u}{}' \mathring{A}_1{}^{p_\alpha}_{sfm} \binom{s}{p_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}'' \underset{2}{A}{}^{s}_{q_\beta fm} \binom{q_\beta}{s} V^{\cdots}_{\cdots} + \\
&+ V^{p_1\cdots p_u}_{q_1\cdots q_v <[fm]>} + V^{p_1\cdots p_u}_{q_1\cdots q_v \leq[fm]\geq} + {}''\Gamma^s_{[fm]} V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} s} \Big] \Big\}.
\end{aligned}
\tag{37}
$$

In this manner we have proved

Theorem 6. *The* 6^{th} *Ricci type identity for non-basic differentiation in* O_N *is given by the equation* (37)*, where the introduced designations are explained in the previous displaying.*

2.7. Let $(\lambda, \mu; \nu, \omega) = (1, 1; 2, 1)$. Then for the term in the 3^{rd} brackets of (15) one gets

$$P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f \underset{1}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} f \underset{1}{|} m} =$$
$$= \frac{1}{2}P^f_m(V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|}(f \underset{1}{|} n)} + V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|}[f \underset{1}{|} n]}) -$$
$$- \frac{1}{2}P^f_n(V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|}(f \underset{1}{|} m)} + V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|}[f \underset{1}{|} m]}),$$

and using the identity for $V^{\cdots}_{\cdots \underset{2}{|}[f \underset{1}{|} m]}$ as in Theorem 4, we obtain

Theorem 7. *The* 7^{th} *Ricci type identity for non-basic differentiation in* O_N *is*

$$V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{1}{||} m \underset{1}{||} n} - V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{2}{||} n \underset{1}{||} m} =$$
$$= \sum_{\alpha=1}^{u} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}} P^{i_\alpha}_{a_\alpha} \mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_v}_{j_v} \cdot$$
$$\cdot\ (P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} P^{a_\alpha}_{p_\alpha \underset{1}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} f} P^{a_\alpha}_{p_\alpha \underset{1}{|} m}) +$$
$$+ \sum_{\beta=1}^{v} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}} P^{b_\beta}_{j_\beta} \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}} \cdots \mathcal{M}^{q_v}_{j_v} \cdot$$
$$\cdot\ (P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} P^{q_\beta}_{b_\beta \underset{1}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} f} P^{q_\beta}_{b_\beta \underset{1}{|} m}) + \tag{38}$$
$$+ \frac{1}{2}\mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_v}_{j_v} \Big\{ P^f_m \Big[V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|}(f \underset{1}{|} n)} +$$
$$+ \sum_{\alpha=1}^{u} {}' \underset{1}{R}{}^{p_\alpha}_{sfn} \binom{s}{p_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} {}'' \underset{1}{R}{}^{s}_{q_\beta fn} \binom{q_\beta}{s} V^{\cdots}_{\cdots} - {}''\Gamma^s_{[fn]} V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} s} \Big] -$$
$$- P^f_n \Big[V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|}(f \ \underset{1}{|} m)} + \sum_{\alpha=1}^{u} {}' \underset{3}{A}{}^{p_\alpha}_{sfm} \binom{s}{p_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} {}'' \underset{4}{A}{}^{s}_{q_\beta fm} \binom{q_\beta}{s} V^{\cdots}_{\cdots} -$$
$$- V^{p_1\cdots p_u}_{q_1\cdots q_v <[fm]>} - V^{p_1\cdots p_u}_{q_1\cdots q_v \leq[fm]\geq} - {}''\Gamma^s_{[fm]} V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} s} \Big] \Big\}$$

2.8. For the case $(\lambda, \mu; \nu, \omega) = (2, 2; 1, 2)$ one obtains

$$P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} f \underset{2}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f \underset{2}{|} m} =$$
$$= \frac{1}{2}P^f_m(V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|}(f \underset{2}{|} n)} + V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|}[f \underset{2}{|} n]}) -$$
$$- \frac{1}{2}P^f_n(V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|}(f \underset{2}{|} m)} + V^{p_1\cdots p_u}_{q_1\cdots q_v |[f \underset{2}{|} m]}),$$

and expressing $V^{\cdots}_{\cdots \underset{2}{|}[f \underset{2}{|} n]}$ like in Theorem 2, and $V^{\cdots}_{\cdots \underset{1}{|}[f \underset{2}{|} n]}$ like in Theorem 3, we get

Theorem 8. *The* 8^{th} *Ricci type identity for non-basic differentiation in* O_N *is*

$$
\begin{aligned}
&V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{2}{\|} m \underset{2}{\|} n} - V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{1}{\|} n \underset{2}{\|} m} = \\
&= \sum_{\alpha=1}^{u} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}} P^{i_\alpha}_{a_\alpha} \mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_v}_{j_v} \cdot \\
&\cdot\ (P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} f} P^{a_\alpha}_{p_\alpha \underset{2}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} P^{a_\alpha}_{p_\alpha \underset{2}{|} m}) + \\
&+ \sum_{\beta=1}^{v} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}} P^{b_\beta}_{j_\beta} \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}} \cdots \mathcal{M}^{q_v}_{j_v} \cdot \\
(39)\quad &\cdot\ (P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} f} P^{q_\beta}_{b_\beta \underset{2}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} f} P^{q_\beta}_{b_\beta \underset{2}{|} m}) + \\
&+ \frac{1}{2} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_v}_{j_v} \Big\{ P^f_m \Big[V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} (f \underset{2}{|} n)} + \\
&+ \sum_{\alpha=1}^{u}{}' \underset{2}{R}{}^{p_\alpha}_{sfn} \binom{s}{p_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}'' \underset{2}{R}{}^{s}_{q_\beta fn} \binom{q_\beta}{s} V^{\cdots}_{\cdots} + {}''\Gamma^s_{[fn]} V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} s} \Big] - \\
&- P^f_n \Big[V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} (f \ \underset{2}{|} m)} + \sum_{\alpha=1}^{u}{}' \underset{1}{A}{}^{p_\alpha}_{sfm} \binom{s}{p_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}'' \underset{2}{A}{}^{s}_{q_\beta fm} \binom{q_\beta}{s} V^{\cdots}_{\cdots} + \\
&+ V^{p_1\cdots p_u}_{q_1\cdots q_v <[fm]>} + V^{p_1\cdots p_u}_{q_1\cdots q_v \le[fm]\ge} + {}''\Gamma^s_{[fm]} V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{1}{|} s} \Big] \Big\}
\end{aligned}
$$

2.9. Analogously to the previous cases one proves

Theorem 9. *In* O_N *is in force the* 9^{th} *Ricci type identity for non-basic differentiation*

$$
\begin{aligned}
&V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{2}{\|} m \underset{2}{\|} n} - V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{2}{\|} n \underset{1}{\|} m} = \\
&= \sum_{\alpha=1}^{u} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}} P^{i_\alpha}_{a_\alpha} \mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_v}_{j_v} \cdot \\
&\cdot\ V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} f} (P^f_m P^{a_\alpha}_{p_\alpha \underset{2}{|} n} - P^f_n P^{a_\alpha}_{p_\alpha \underset{1}{|} m}) + \\
(40)\quad &+ \sum_{\beta=1}^{v} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}} \mathcal{M}^{b_\beta}_{j_\beta} \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}} \cdots \mathcal{M}^{q_v}_{j_v} \cdot \\
&\cdot\ V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} f} (P^f_m P^{q_\beta}_{b_\beta \underset{2}{|} n} - P^f_n P^{q_\beta}_{b_\beta \underset{1}{|} m}) + \\
&+ \frac{1}{2} \mathcal{M}^{i_1}_{p_1} \cdots \mathcal{M}^{i_u}_{p_u} \mathcal{M}^{q_1}_{j_1} \cdots \mathcal{M}^{q_v}_{j_v} \Big\{ P^f_m \Big[V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} (f \underset{2}{|} n)} + \\
&+ \sum_{\alpha=1}^{u}{}' \underset{2}{R}{}^{p_\alpha}_{sfn} \binom{s}{p_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}'' \underset{2}{R}{}^{s}_{q_\beta fn} \binom{q_\beta}{s} V^{\cdots}_{\cdots} - {}''\Gamma^s_{[fn]} V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{2}{|} s} \Big] -
\end{aligned}
$$

$$- P^f_n\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v\ \underset{2}{|}(f\ \underset{1}{|}m)} + \sum_{\alpha=1}^{u}{}'\underset{3}{A}{}^{p_\alpha}_{sfm}\binom{s}{p_\alpha}V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}''\underset{4}{A}{}^{s}_{q_\beta fm}\binom{q_\beta}{s}V^{\cdots}_{\cdots} -$$
$$- V^{p_1\cdots p_u}_{q_1\cdots q_v<[fm]>} - V^{p_1\cdots p_u}_{q_1\cdots q_v\le[fm]\ge} - {}''\Gamma^s_{[fm]}V^{p_1\cdots p_u}_{q_1\cdots q_v\ \underset{2}{|}s}\Big]\Big\}.$$

2.10. The last case according to (13) is $(\lambda,\mu;\nu,\omega)=(1,2;2,1)$. Presenting $V^{\cdots}_{\cdots\underset{1}{|}[f\underset{2}{|}n]}$ as in the proof of Theorem 3, and $V^{\cdots}_{\cdots\underset{2}{|}[f\underset{1}{|}m]}$ as in the proof of Theorem 4, based on (15), we obtain

Theorem 10. *In O_N is valid the 10^{th} Ricci type identity for non-basic differentiation*

$$
\begin{aligned}
&V^{i_1\cdots i_u}_{j_1\cdots j_v\ \underset{1}{||}m\underset{2}{||}n} - V^{i_1\cdots i_u}_{j_1\cdots j_v\ \underset{2}{||}n\underset{1}{||}m} = \\
&= \sum_{\alpha=1}^{u}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}}P^{i_\alpha}_{a_\alpha}\mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}\cdot \\
&\cdot\ (P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v\ \underset{1}{|}f}P^{a_\alpha}_{p_\alpha\underset{2}{|}n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v\ \underset{2}{|}f}P^{a_\alpha}_{p_\alpha\underset{1}{|}m}) + \\
&+ \sum_{\beta=1}^{v}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}}P^{b_\beta}_{j_\beta}\mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}}\cdots\mathcal{M}^{q_v}_{j_v}\cdot \\
&\cdot\ (P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v\ \underset{1}{|}f}P^{q_\beta}_{b_\beta\underset{2}{|}n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v\ \underset{2}{|}f}P^{q_\beta}_{b_\beta\underset{1}{|}m}) + \\
&+ \frac{1}{2}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}\Big\{P^f_m\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v\ \underset{1}{|}(f\underset{2}{|}n)} + \\
&+ \sum_{\alpha=1}^{u}{}'\underset{1}{A}{}^{p_\alpha}_{sfn}\binom{s}{p_\alpha}V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}''\underset{2}{A}{}^{s}_{q_\beta fn}\binom{q_\beta}{s}V^{\cdots}_{\cdots} + \\
&+ V^{p_1\cdots p_u}_{q_1\cdots q_v<[fn]>} + V^{p_1\cdots p_u}_{q_1\cdots q_v\le[fn]\ge} + {}''\Gamma^s_{[fn]}V^{p_1\cdots p_u}_{q_1\cdots q_v\ \underset{1}{|}s}\Big] - \\
&- P^f_n\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v\ \underset{2}{|}(f\underset{1}{|}m)} + \sum_{\alpha=1}^{u}{}'\underset{3}{A}{}^{p_\alpha}_{sfm}\binom{s}{p_\alpha}V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}''\underset{4}{A}{}^{s}_{q_\beta fm}\binom{q_\beta}{s}V^{\cdots}_{\cdots} - \\
&- V^{p_1\cdots p_u}_{q_1\cdots q_v<[fm]>} - V^{p_1\cdots p_u}_{q_1\cdots q_v\le[fm]\ge} - {}''\Gamma^s_{[fm]}V^{p_1\cdots p_u}_{q_1\cdots q_v\ \underset{2}{|}s}\Big]\Big\}.
\end{aligned}
\tag{41}
$$

3. Ricci type identities for non-basic differentiation of the third and fourth kind

3.0. Analogously to (6,7), we can define in O_N *the* 3^{rd} *and* 4^{th} *kind of covariant derivative* for basic differentiation [3]:

$$(42)\, V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{3}{|} m} = V^{i_1\cdots i_u}_{j_1\cdots j_v ,m} + \sum_{\alpha=1}^{u} {}'\Gamma^{i_\alpha}_{pm} \binom{p}{i_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} {}''\Gamma^{p}_{mj_\beta} \binom{j_\beta}{p} V^{\cdots}_{\cdots},$$

$$(43)\, V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{4}{|} m} = V^{i_1\cdots i_u}_{j_1\cdots j_v ,m} + \sum_{\alpha=1}^{u} {}'\Gamma^{i_\alpha}_{mp} \binom{p}{i_\alpha} V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} {}''\Gamma^{p}_{j_\beta m} \binom{j_\beta}{p} V^{\cdots}_{\cdots},$$

and for the non-basic one:

$$(44) \qquad V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{\alpha}{||} m} = P^{i_1}_{a_1} \cdots P^{i_u}_{a_u} P^{b_1}_{j_1} \cdots P^{b_v}_{j_v} V^{i_1\cdots i_u}_{b_1\cdots b_v \underset{\alpha}{|} m}, \quad \alpha = 3, 4$$

Now for λ, μ, ν, $\omega \in \{3, 4\}$ we consider the difference (12) and conclude that (15) is valid, provided that, analogously to (13), we have 10 new cases, stating only certain of them.

3.1. For $\lambda = \mu = \nu = \omega = 3$ for expressions in the brackets of (15) one obtains succesively:

$$P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f} P^{a_\alpha}_{\ p_\alpha \underset{3}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f} P^{a_\alpha}_{\ p_\alpha \underset{3}{|} m} = \\ = V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f} P^{a_\alpha}_{\ p_\alpha \underset{3}{|} g} (P^f_m \delta^g_n - P^f_n \delta^g_m),$$

$$P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f} P^{q_\beta}_{\ b_\beta \underset{3}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f} P^{q_\beta}_{\ b_\beta \underset{3}{|} m} = \\ = V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f} P^{q_\beta}_{\ b_\beta \underset{3}{|} g} (P^f_m \delta^g_n - P^f_n \delta^g_m),$$

$$P^f_m V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f \underset{3}{|} n} - P^f_n V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f \underset{3}{|} m} = V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f \underset{3}{|} g} (P^f_m \delta^g_n - P^f_n \delta^g_m).$$

In view of (62) from [3]

$$V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} [f \underset{3}{|} g]} = \sum_{\alpha=1}^{u} {}' \underset{1}{R}{}^{p_\alpha}_{sfg} \binom{s}{p_\alpha} V^{\cdots}_{\cdots} - \\ - \sum_{\beta=1}^{v} {}'' \underset{2}{R}{}^{s}_{q_\beta fg} \binom{q_\beta}{s} V^{\cdots}_{\cdots} + {}''\Gamma^s_{[fg]} V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} s},$$

and using an equation analogous to (19), based to (15), we get

$$\begin{aligned}
&V^{i_1\cdots i_u}_{j_1\cdots j_v\underset{3}{||}m\underset{3}{||}n} - V^{i_1\cdots i_u}_{j_1\cdots j_v\underset{3}{||}n\underset{3}{||}m} =\\
&\quad = P^f_{[m}\delta^g_{n]}\Big\{\sum_{\alpha=1}^{u}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}}P^{i_\alpha}_{a_\alpha}\mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}}\cdots\mathcal{M}^{i_u}_{p_u}\cdot\\
&\quad\cdot\ \mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{3}{|}f}P^{a_\alpha}_{p_\alpha\underset{3}{|}g}+\\
&\quad+\sum_{\beta=1}^{v}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}}P^{b_\beta}_{j_\beta}\cdot\\
&\quad\cdot\ \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{3}{|}f}P^{q_\beta}_{b_\beta\underset{3}{|}g}+\\
&\quad+\frac{1}{2}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{3}{|}(f\underset{3}{|}g)}+\\
&\quad+\sum_{\alpha=1}^{u}{}'\underset{1}{R}^{p_\alpha}_{sfg}\binom{s}{p_\alpha}V^{\cdots}_{\cdots}-\sum_{\beta=1}^{v}{}''\underset{2}{R}^{s}_{q_\beta fg}\binom{q_\beta}{s}V^{\cdots}_{\cdots}+\\
&\quad+{}''\Gamma^s_{[fg]}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{3}{|}s}\Big]\Big\}.
\end{aligned}\tag{45}$$

3.2. For $\lambda=\mu=\nu=\omega=4$, using (63) from [3], based on (15) it follows that

$$\begin{aligned}
&V^{i_1\cdots i_u}_{j_1\cdots j_v\underset{4}{||}m\underset{4}{||}n} - V^{i_1\cdots i_u}_{j_1\cdots j_v\underset{4}{||}n\underset{4}{||}m} =\\
&\quad = P^f_{[m}\delta^g_{n]}\Big\{\sum_{\alpha=1}^{u}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}}P^{i_\alpha}_{a_\alpha}\mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}}\cdots\mathcal{M}^{i_u}_{p_u}\cdot\\
&\quad\cdot\ \mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{4}{|}f}P^{a_\alpha}_{p_\alpha\underset{4}{|}g}+\\
&\quad+\sum_{\beta=1}^{v}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}}P^{b_\beta}_{j_\beta}\cdot\\
&\quad\cdot\ \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{4}{|}f}P^{q_\beta}_{b_\beta\underset{4}{|}g}+\\
&\quad+\frac{1}{2}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{4}{|}(f\underset{4}{|}g)}+\\
&\quad+\sum_{\alpha=1}^{u}{}'\underset{2}{R}^{p_\alpha}_{sfg}\binom{s}{p_\alpha}V^{\cdots}_{\cdots}-\sum_{\beta=1}^{v}{}''\underset{1}{R}^{s}_{q_\beta fg}\binom{q_\beta}{s}V^{\cdots}_{\cdots}-\\
&\quad-{}''\Gamma^s_{[fg]}V^{p_1\cdots p_u}_{q_1\cdots q_v\underset{4}{|}s}\Big]\Big\}.
\end{aligned}\tag{46}$$

3.3. Using (15) and taking $(\lambda,\mu;\nu,\omega)=(3,4;3,4)$, using (64) from [3], one gets

$$
\begin{aligned}
&V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{3}{||} m \underset{4}{||} n} - V^{i_1\cdots i_u}_{j_1\cdots j_v \underset{3}{||} n \underset{4}{||} m} = \\
&= P^f_{[m}\delta^g_{n]}\Big\{\sum_{\alpha=1}^{u}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_{\alpha-1}}_{p_{\alpha-1}}P^{i_\alpha}_{a_\alpha}\mathcal{M}^{i_{\alpha+1}}_{p_{\alpha+1}}\cdots\mathcal{M}^{i_u}_{p_u}\cdot \\
&\cdot\ \mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f}P^{a_\alpha}_{p_\alpha \underset{4}{|} g} + \\
&+\sum_{\beta=1}^{v}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_{\beta-1}}_{j_{\beta-1}}P^{b_\beta}_{j_\beta}\cdot \\
&\cdot\ \mathcal{M}^{q_{\beta+1}}_{j_{\beta+1}}\cdots\mathcal{M}^{q_v}_{j_v}V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} f}P^{q_\beta}_{b_\beta \underset{4}{|} g} + \\
&+\frac{1}{2}\mathcal{M}^{i_1}_{p_1}\cdots\mathcal{M}^{i_u}_{p_u}\mathcal{M}^{q_1}_{j_1}\cdots\mathcal{M}^{q_v}_{j_v}\Big[V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|}(f \underset{4}{|} g)} + \\
&+\sum_{\alpha=1}^{u}{}'\underset{1}{A}{}^{p_\alpha}_{sfg}\binom{s}{p_\alpha}V^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}{}''\underset{4}{A}{}^{s}_{q_\beta fg}\binom{q_\beta}{s}V^{\cdots}_{\cdots} + \\
&+\overline{V}^{p_1\cdots p_u}_{q_1\cdots q_v<[fg]>} + \overline{V}^{p_1\cdots p_u}_{q_1\cdots q_v\leq[fg]\geq} - {}''\Gamma^s_{[fg]}V^{p_1\cdots p_u}_{q_1\cdots q_v \underset{3}{|} s}\Big]\Big\},
\end{aligned}
\tag{47}
$$

where

$$
\overline{V}^{p_1\cdots p_u}_{q_1\cdots q_v<fg>} = \sum_{\alpha=1}^{u}{}'\Gamma^{p_\alpha}_{[sf]}\binom{s}{p_\alpha}V^{\cdots}_{\cdots,g} - \sum_{\beta=1}^{v}{}''\Gamma^{s}_{[fq_\beta]}\binom{q_\beta}{s}V^{\cdots}_{\cdots,g},
\tag{48}
$$

$$
\begin{aligned}
\overline{V}^{p_1\cdots p_u}_{q_1\cdots q_v\leq fg\geq} &= \sum_{\alpha=1}^{u-1}\sum_{\substack{\beta=2\\(\alpha<\beta)}}^{u}({}'\Gamma^{p_\alpha}_{sf}{}'\Gamma^{p_\beta}_{gt} - {}'\Gamma^{p_\alpha}_{fs}{}'\Gamma^{p_\beta}_{tg})\binom{s}{p_\alpha}\binom{t}{p_\beta}V^{\cdots}_{\cdots} - \\
&-\sum_{\alpha=1}^{u}\sum_{\beta=1}^{v}({}'\Gamma^{p_\alpha}_{sf}{}''\Gamma^{t}_{q_\beta g} - {}'\Gamma^{p_\alpha}_{fs}{}''\Gamma^{t}_{gq_\beta})\binom{s}{p_\alpha}\binom{q_\beta}{t}V^{\cdots}_{\cdots} + \\
&+\sum_{\alpha=1}^{v-1}\sum_{\substack{\beta=2\\(\alpha<\beta)}}^{v}({}''\Gamma^{s}_{fq_\alpha}{}''\Gamma^{t}_{q_\beta g} - {}''\Gamma^{s}_{q_\alpha f}{}''\Gamma^{t}_{gq_\beta})\binom{q_\alpha}{s}\binom{q_\beta}{t}V^{\cdots}_{\cdots}.
\end{aligned}
\tag{49}
$$

If $P^i_j = \delta^i_j$, then from (1) it follows that ${}'\Gamma = {}''\Gamma = \Gamma$ and the obtained formulas reduce to the corresponding formulas for (nonsymmetric) affine connection (if Γ is nonsymmetric, see [1], [2]).

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