Novi Sad J. Math.
Vol. 28, No. 3, 1998, 29-39

A COINCIDENCE POINT THEOREM FOR MULTIVALUED MAPPINGS IN 2-MENGER SPACES

Olga Hadžić
Institute of Mathematics, Faculty of Science, University of Novi Sad
Trg Dositeja Obradovića 4, 21000 Novi Sad, Yugoslavia

Zoran Ovcin
Faculty of Engineering, University of Novi Sad
Trg Dositeja Obradovića 6, 21000 Novi Sad, Yugoslavia

Abstract

The notion of the 2-Menger space is a probabilistic generalization of the 2-metric space introduced by Gähler [1] in 1964.

A coincidence point theorem is proved in such kind of spaces using a generalization of Hicks' probabilistic contraction.

AMS Mathematics Subject Classification (1991): 47H10
Key words and phrases: fixed point, 2-metric spaces, 2-Menger spaces

1. Introduction

The theory of probabilistic metric spaces is an important part of Stochastic Analysis, and so it is of interest to develop the fixed point theory in such spaces. There are many results in fixed point theory in probabilistic metric spaces ([10], [11]).

2-metric spaces were introduced by Gähler [1] in 1964, and since then there have been many fixed point theorems proved in 2-metric spaces, [8].

Since the introduction of 2-Menger spaces, as a generalization of 2-metric spaces [12], there have been only a few results in fixed point theory (see [12], [13], [14]).

In this paper we shall prove a coincidence point theorem for multivalued mappings satisfying generalized Hicks' contraction principle in 2-Menger spaces.

2. Preliminaries

Let X be a nonempty set, and let the mapping $d : X \times X \times X \to [0, +\infty)$ satisfy the following conditions:

1. For each pair of points $(x, y) \in X \times X$ with $x \neq y$ there is $z \in X$ such that $d(x, y, z) \neq 0$.
2. $d(x, y, z) = 0$ when at least two of three points are equal.
3. For all $x, y, z \in X$:
$$d(x, y, z) = d(x, z, y) = d(y, z, x)\,.$$
4. For all $x, y, z, u \in X$:
$$d(x, y, z) \leq d(x, y, u) + d(x, u, z) + d(u, y, z)\,.$$

Then (X, d) is said to be a *2-metric space*. It is easy to see that d is a non-negative functional.

The convergence in a 2-metric space is introduced in the following way. Let $\{x_n\}_{n\in\mathbf{N}}$ be a sequence of X and $x \in X$. We say that the sequence $\{x_n\}_{n\in\mathbf{N}}$ *converges to* x if for every $a \in X$, $\lim_{n\to\infty} d(x_n, x, a) = 0$. The sequence $\{x_n\}_{n\in\mathbf{N}}$ is a *Cauchy sequence* if for every $a \in X$, $\lim_{m,n\to\infty} d(x_m, x_n, a) = 0$. If every Cauchy sequence in X is convergent to a point in X we say that (X, d) is a *complete* 2-metric space.

Two argument function $t : [0,1] \times [0,1] \to [0,1]$ is called *t-norm* if it is symmetric, non-decreasing in both arguments, associative and $\forall a \in [0,1]$, $t(a, 1) = t(1, a) = a$.

We denote by Δ the set of *distribution functions* $F : \mathbf{R} \to [0,1]$ (F is non-decreasing, lower semicontinuous, $\inf_{s\in\mathbf{R}} F(s) = 0$ and $\sup_{s\in\mathbf{R}} F(s) = 1$).

We denote by Δ^+ the set of distribution functions which satisfy the condition $F(0) = 0$.

We shall use function $H(s) := \begin{cases} 0, & s \le 0 \\ 1, & s > 0 . \end{cases}$

The triple $(X, \mathcal{F}, t)$ is a *2-Menger space* if X is a non-empty set, for all $x, y, z \in X$, $\mathcal{F} : (x, y, z) \mapsto \{F_{x,y,z}(s)\} \in \Delta^+$, t a t-norm and the following conditions are satisfied:

1. for each pair of points $(x, y) \in X \times X$ with $x \ne y$ there is $z \in X$ such that $F_{x,y,z} \ne H$,
2. $F_{x,y,z} = H$ when at least two of three points are equal.
3. for all $x, y, z \in X$:
$$F_{x,y,z} = F_{x,z,y} = F_{y,z,x} \,,$$
4. for all $x, y, z, u \in X$ and for all $t_1, t_2, t_3 \ge 0$
$$F_{x,y,z}(t_1 + t_2 + t_3) \ge T^2(F_{x,y,u}(t_1), F_{x,u,z}(t_2), F_{u,y,z}(t_3)) \,.$$

Remark 1. In this paper we shall use the following notation: $T^1(x, y) = T(x, y)$, $T^2(x, y, z) = T(x, T(y, z))$ and for every $n \ge 3$

$$T^n(x_1, x_2, \ldots, x_{n+1}) = T(x_1, T^{n-1}(x_2, x_3, \ldots, x_{n+1})) \,.$$

A sequence $\{x_n\}_{n \in \mathbf{N}}$ of X *converges to* $x \in X$ if for every $a \in X$, every $\epsilon > 0$ and every $\lambda \in (0, 1)$ there exists $n(\epsilon, \lambda, a) \in \mathbf{N}$ such that

$$F_{x,x_n,a}(\epsilon) > 1 - \lambda \text{ for all } n \ge n(\epsilon, \lambda, a) \,.$$

A sequence $\{x_n\}_{n \in \mathbf{N}}$ from X is a *Cauchy sequence* if for every $a \in X$, $\epsilon > 0$ and $\lambda \in (0, 1)$ there exists $n(\epsilon, \lambda, a) \in \mathbf{N}$ such that

$$F_{x_n,x_{n+p},a}(\epsilon) > 1 - \lambda \text{ for all } n \ge n(\epsilon, \lambda, a) \text{ , and all } p \in \mathbf{N} \,.$$

A 2-Menger space is *complete* if every Cauchy sequence $\{x_n\}_{n \in \mathbf{N}}$ of elements of X converges to an element of X.

Example 1. Let (M, d) be a separable 2-metric space with the area function d continuous in all variables such that there exists a continuous function

$z : M\times M\backslash\Delta_M \to M$, so that for all $(x,y)\in M\times M\backslash\Delta_M, d(x,y,z(x,y))\neq 0$. Let $(\Omega,\mathcal{A},P)$ be a probability measure space. We shall denote by S the set of all the equivalence classes of measurable mappings $X:\Omega\to M$. If $X,Y,Z\in S$ and $s\in\mathbf{R}$ then $F_{X,Y,Z}(s)$ is defined in the following way:

$$F_{X,Y,Z}(s) = P(\{\omega : \omega\in\Omega, d(X(\omega),Y(\omega),Z(\omega)) < s\}) .$$

Knowing that d is continuous we know that $F_{X,Y,Z}(s)$ is a probability distribution function.

Let us prove that $(S,\mathcal{F},T)$ is a 2-Menger space, where

$$T(a,b) = t_m(a,b) := \max(a+b-1,0) .$$

1. If $X,Y\in S$ and $X\neq Y$, let $A:=\{\omega : X(\omega)\neq Y(\omega)\}$. Obviously, there exists $A'\subseteq A$ such that $P(A')>0$ and $A'\in\mathcal{A}$. Let us define a mapping $Z:\Omega\to M$ in the following way:

$$Z(\omega) := \begin{cases} z(\omega), & \omega\in A' \\ X(\omega), & \omega\notin A' , \end{cases}$$

where $z(\omega) := z(X(\omega),Y(\omega))$. X and Y are measurable mappings and z is a continuous mapping so $z(\omega)$ is a measurable mapping. By definition $d(X(\omega),Y(\omega),z(X(\omega),Y(\omega)))\neq 0$. We have only to prove that Z is a measurable mapping since

$$P(\{\omega : d(X(\omega),Y(\omega),Z(\omega))\neq 0\}) = P(A') > 0 .$$

Take an arbitrary Borel set B. Then

$$Z^{-1}(B) = (X^{-1}(B)\cap \bar{A}')\cup(z^{-1}(B)\cap A')\in\mathcal{A} .$$

2. $\Leftarrow$ At least two of X, Y and Z are equal $\Rightarrow$ (for example $X = Y$, the same applies for other combinations) $P(\{\omega : X(\omega) = Y(\omega)\}) = 1 \leq P(\{\omega : d(X(\omega),Y(\omega),Z(\omega)) = 0\})$ since $X(\omega) = Y(\omega)$ implies $d(X(\omega),Y(\omega),Z(\omega)) = 0$. It follows that $F_{X,Y,Z} = H$.

3. Follows immediately from the definition of 2-metric space (3.).

4. Suppose x,y,z from X and t_1,t_2,t_3 are given. Let us denote by $D := \{\omega : d(X(\omega),Y(\omega),Z(\omega)) < t_1+t_2+t_3\}$,

$A := \{\omega : d(X(\omega), Y(\omega), U(\omega)) < t_1\}$,
$B := \{\omega : d(X(\omega), U(\omega), Z(\omega)) < t_2\}$ and
$C := \{\omega : d(U(\omega), Y(\omega), Z(\omega)) < t_3\}$.

Since for all x, y, z, u from M

$$d(x,y,z) \leq d(x,y,u) + d(x,u,z) + d(u,y,z),$$

it follows that $A \cap B \cap C \subseteq D$. Using

$$P(A \cap B \cap C) = P(A \cap B) + P(C) - P((A \cap B) \cup C) =$$

$$= P(A) + P(B) - P(A \cup B) + P(C) - P((A \cap B) \cup C)$$

there follows

$$F_{X,Y,Z}(t_1 + t_2 + t_3) = P(D) \geq P(A \cap B \cap C) \geq$$

$$\geq P(A) + P(B) + P(C) - 2 \geq P(A) + t_m(P(B), P(C)) - 1 \geq$$

$$\geq t_m(P(A), t_m(P(B), P(C))) = T^2(F_{x,y,u}(t_1), F_{x,u,z}(t_2), F_{u,y,z}(t_3)),$$

provided that $P(B)+P(C)-1 > 0$ and $P(A)+(P(B)+P(C)-1)-1 > 0$. If that is not the case, then

$$T^2(F_{x,y,u}(t_1), F_{x,u,z}(t_2), F_{u,y,z}(t_3)) = 0,$$

and the proof follows immediately. □

We shall denote by $\mathcal{M}$ the set of all functions $\psi : [0, +\infty) \to [0, +\infty)$ which are nondecreasing and such that for every $s > 0$ series $\sum_{n=1}^{\infty} \psi^n(s)$ converges. By 2^M we shall denote the family of all nonempty subsets of $M \subseteq S$ and by 2^M_C the family of all nonempty closed subsets of $M \subseteq X$.

A t-norm T is of the *h-type* if the family of functions $\{T_n(x)\}_{n\in\mathbf{N}}$ is equicontinuous at the point $x = 1$, where $T_1(x) = T(x,x)$ and $T_n(x) = T(x, T_{n-1}(x))$, for every $n \geq 2$.

A mapping $A : M \to 2^M$ is *weakly commuting* with $f : M \to M$ if for every $x \in M$, $f(Ax) \subseteq A(fx)$.

Let $(X, \mathcal{F}, T)$ be a 2-Menger space, $\emptyset \neq M \subseteq S$, $f : M \to M$ and $A : M \to 2^M$. The mapping A is *f-strongly demicompact* if for every sequence $\{x_n\}_{n\in\mathbf{N}}$ from M, such that for all $a \in X$, $\lim_{n\to+\infty} F_{fx_n,y_n,a}(\epsilon) = 1$, for some sequence $\{y_n\}_{n\in\mathbf{N}}$, $y_n \in Ax_n$, $n \in \mathbf{N}$ and every $\epsilon > 0$, there exists a convergent subsequence $\{fx_{n_k}\}_{k\in\mathbf{N}}$ of the sequence $\{fx_n\}_{n\in\mathbf{N}}$.

3. A fixed point theorem

Theorem 1. *Let $(X, \mathcal{F}, T)$ be a complete 2-Menger space, $\sup_{x<1} T(x,x) = 1$, M a nonempty and closed subset of X, $f : M \to M$ a continuous mapping, $A, B : M \to 2_C^{f(M)}$, and $\psi \in \mathcal{M}$, so that the following implication holds for every $u, v \in M$ and every $\epsilon > 0$*

(*) $\forall a \in X, F_{fu,fv,a}(\epsilon) > 1 - \epsilon \Rightarrow$ *for every $p \in Au$ there exists $q \in Bv$ such that $\forall a \in X, F_{p,q,a}(\psi(s)) > 1 - \psi(\epsilon)$ and for every $p' \in Bv$ there exists $q' \in Au$ such that $\forall a \in X, F_{p',q',a}(\psi(s)) > 1 - \psi(\epsilon)$.*

If A and B are weakly commuting with f and a) or b) are satisfied, then there exists $x \in M$ such that $fx \in Ax \cap Bx$, where

a) A or B are f-strongly demicompact,

b) t-norm T is of the h-type.

Proof Like in [9] we shall choose x_0 and x_1 from M such that $fx_1 \in Ax_0$. Choose $s > 1$, then $\forall a \in X, F_{fx_0,fx_1,a}(s) > 1 - s$ and using (*) there exists $x_2 \in M$ such that $\forall a \in X, F_{fx_1,fx_2,a}(\psi(s)) > 1 - \psi(s)$ and $fx_2 \in Bx_1$. Continuing in this way we obtain a sequence $\{x_n\}_{n\in\mathbf{N}}$ in M such that for every $n \in \mathbf{N}$

i) $fx_{2n+1} \in Ax_{2n}$ and $fx_{2n+2} \in Bx_{2n+1}$,

ii) $\forall a \in X, F_{fx_n,fx_{n+1},a}(\psi^n(s)) > 1 - \psi^n(s)$.

Since $\lim_{n\to+\infty} \psi^n(s) = 0$, from ii) it is easy to prove that for every $\epsilon > 0$ and every $\lambda \in (0,1)$ there exists $n_1(\epsilon, \lambda) \in \mathbf{N}$ such that for every $n \geq n_1(\epsilon, \lambda)$, $\forall a \in X, F_{fx_n,fx_{n+1},a}(\epsilon) > 1 - \lambda$. This means that

$$\forall \epsilon > 0, \forall a \in X, \lim_{n\to\infty} F_{fx_n,fx_{n+1},a}(\epsilon) = 1\,. \tag{1}$$

a) If we suppose that A is f-strongly demicompact, using (1) and $fx_{2n+1} \in Ax_{2n}$ $(n \in \mathbf{N})$, we conclude that there exists a convergent subsequence $\{fx_{2n_k}\}_{k\in\mathbf{N}}$ of the sequence $\{fx_{2n}\}_{n\in\mathbf{N}}$.

b) We shall prove that if T is of the h-type, the sequence $\{fx_n\}_{n\in\mathbf{N}}$ is convergent. We shall use technique identical to the one used in [10]. Let $\epsilon > 0$, $\lambda \in (0,1)$ and $s > 0$ be given. $\sum_{n\in\mathbf{N}} \psi^n(s)$ is convergent so there exists $n'(\epsilon, s)$ such that $2 \cdot \sum\limits_{n\geq n'(\epsilon,s)} \psi^n(s) < \epsilon$. Then for all $n \geq n'(\epsilon, s)$ and for all $p \in \mathbf{N}$

$$\forall a \in X, F_{fx_{n+p},fx_n,a}(\epsilon) \geq F_{fx_{n+p},fx_n,a}\left(2 \cdot \sum_{m=n}^{n+p} \psi^m(s)\right) \geq$$

$$\geq T^2(F_{fx_n,fx_{n+1},fx_{n+p}}(\psi^n(s)), F_{fx_n,fx_{n+1},a}(\psi^n(s)),$$

$$F_{fx_{n+1},fx_{n+p},a}(2 \cdot \sum_{m=n+1}^{n+p} \psi^n(s))) \geq \cdots \geq$$

$$\geq T^{2p-3}(F_{fx_n,fx_{n+1},fx_{n+p}}(\psi^n(s)), F_{fx_n,fx_{n+1},a}(\psi^n(s)),$$

$$F_{fx_{n+1},fx_{n+2},fx_{n+p}}(\psi^{n+1}(s)), F_{fx_{n+1},fx_{n+2},a}(\psi^{n+1}(s)), \ldots$$

$$\ldots, F_{fx_{n+p-2},fx_{n+p-1},fx_{n+p}}(\psi^{n+p-2}(s)), F_{fx_{n+p-2},fx_{n+p-1},a}(\psi^{n+p-2}(s)))$$

Since $\psi^n(s) \to 0$ we can take $n(s)$ so that for all $n \geq n(s)$, $\psi^n(s) < 1$, then for all $n \geq \max(n(s), n'(\epsilon, s))$

$$F_{fx_n,fx_{n+p},a}(\epsilon) \geq T^{2p-3}(1 - \psi^n(s), 1 - \psi^n(s), 1 - \psi^{n+1}(s), 1 - \psi^{n+1}(s), \ldots,$$

$$1 - \psi^{n+p-2}(s) 1 - \psi^{n+p-2}(s)).$$

Since ψ is a nondecreasing function, it follows that for all $n \geq n(s)$

$$F_{fx_n,fx_{n+p},a}(\epsilon) \geq T_{2p-3}(1 - \psi^n(s)). \tag{2}$$

Let us assume that $\psi^n(s) \to 0$ and that T is of the h-type. From (2) we conclude that $\{fx_n\}_{n\in\mathbf{N}}$ is a Cauchy sequence.

In both cases there exists

$$x = \lim_{k\to\infty} fx_{2n_k}. \tag{3}$$

Now, we shall prove that

$$x = \lim_{k\to\infty} fx_{2n_k+1}, \tag{4}$$

although the case is less trivial than the one in PM-spaces.

Knowing that $\sup_{x<1} T(x,x) = 1$, for every $\lambda \in (0,1)$ we can find $\delta'(\lambda)$ such that

$$T^2(1-\delta'(\lambda), 1-\delta'(\lambda), 1-\delta'(\lambda)) > 1-\lambda\,.$$

Let $\epsilon > 0$, $\lambda \in (0,1)$ and $a \in X$ be given. Using (3) we can choose $k_1'(\epsilon,\lambda,a)$ so that

$$\forall k \geq k_1'(\epsilon,\lambda,a), F_{x,fx_{2n_k},a}(\frac{\epsilon}{3}) > 1-\delta'(\lambda)\,.$$

Using (1) we can choose $k_2'(\epsilon,\lambda)$ so that

$$\forall k \geq k_2'(\epsilon,\lambda), F_{x,fx_{2n_k+1},fx_{2n_k}}(\frac{\epsilon}{3}) > 1-\delta'(\lambda) \text{ and}$$

$$\forall k \geq k_2'(\epsilon,\lambda), F_{fx_{2n_k},fx_{2n_k+1},a}(\frac{\epsilon}{3}) > 1-\delta'(\lambda)\,.$$

Now, for all $k \geq \max(k_1'(\epsilon,\lambda,a), k_2'(\epsilon,\lambda)$

$$F_{x,fx_{2n_k+1},a}(\epsilon) \geq$$

$$\geq T^2(F_{x,fx_{2n_k},a}(\frac{\epsilon}{3}), F_{x,fx_{2n_k+1},fx_{2n_k}}(\frac{\epsilon}{3}), F_{fx_{2n_k},fx_{2n_k+1},a}(\frac{\epsilon}{3})) \geq$$
$$\geq T^2(1-\delta'(\lambda), 1-\delta'(\lambda), 1-\delta'(\lambda)) > 1-\lambda\,,$$

and the proof follows from the definition of convergence.

Now, we have to show that $fx \in Ax \cap Bx$. Since Ax and Bx are closed it remains to prove that $fx \in \overline{Ax} \cap \overline{Bx}$.

From the continuity of f and (1) follows

(5) $$\forall \epsilon > 0, \forall a \in X, \lim_{n\to\infty} F_{ffx_{2n_k},ffx_{2n_k+1},a}(\epsilon) = 0\,.$$

From the continuity of f, (3) and (4) follows

(6) $$fx = \lim_{n\to\infty} ffx_{2n_k}\,,$$

(7) $$fx = \lim_{n\to\infty} ffx_{2n_k+1}\,.$$

Suppose $\epsilon > 0$, $\lambda \in (0,1)$ and $a \in X$ are given. Let us find $r(\epsilon,\lambda,a) \in Bx$ such that

$$F_{fx,r(\epsilon,\lambda,a),a}(\epsilon) > 1-\lambda\,.$$

Since $\sup_{x<1} T(x,x) = 1$ it is easy to see that there exists $\delta(\lambda) \in (0,1)$ such that

$$T^4(1-\delta(\lambda), 1-\delta(\lambda), 1-\delta(\lambda), 1-\delta(\lambda), 1-\delta(\lambda)) \geq 1-\lambda\,.$$

Let $t_0 \in [0,+\infty)$ be such that $\psi(t_0) < \min(\epsilon/5, \delta(\lambda))$. Using (6) we can choose $k_1(\epsilon,\lambda,a)$ such that

$$\forall k \geq k_1(\epsilon,\lambda,a), F_{fx,ffx_{2n_k},a}(t_0) > 1-t_0\,.$$

Since $fx_{2n_k+1} \in Ax_{2n_k}$, using (*) there exists $r(\epsilon,\lambda,a) \in Bx$ such that

$$\forall k \geq k_1(\epsilon,\lambda,a), F_{fx_{2n_k+1},r(\epsilon,\lambda,a),a}(\psi(t_0)) > 1-\psi(t_0) > 1-\delta(\lambda)\,.$$

Since distribution functions are nondecreasing and $\psi(t_0) \leq \epsilon/5$, there follows that

$$\forall k \geq k_1(\epsilon,\lambda,a), F_{fx_{2n_k+1},r(\epsilon,\lambda,a),a}(\frac{\epsilon}{5}) > 1-\delta(\lambda)\,.$$

(5) implies that there exists $k_2(\epsilon,\lambda)$ such that

$$\forall k \geq k_2(\epsilon,\lambda), F_{ffx_{2n_k},ffx_{2n_k+1},r(\epsilon,\lambda,a)}(\frac{\epsilon}{5}) > 1-\delta(\lambda) \text{ and}$$

$$\forall k \geq k_2(\epsilon,\lambda), F_{ffx_{2n_k},ffx_{2n_k+1},a}(\frac{\epsilon}{5}) > 1-\delta(\lambda)\,.$$

(3) implies that there exists $k_3(\epsilon,\lambda,a)$ such that

$$\forall k \geq k_3(\epsilon,\lambda,a), F_{fx,ffx_{2n_k},r(\epsilon,\lambda,a)}(\frac{\epsilon}{5}) > 1-\delta(\lambda)\,.$$

(3) implies that there exists $k_4(\epsilon,\lambda,a)$ such that

$$\forall k \geq k_4(\epsilon,\lambda,a), F_{fx,ffx_{2n_k},a}(\frac{\epsilon}{5}) > 1-\delta(\lambda)\,.$$

Now, using 4. from the definition of 2-Menger spaces twice, we have

$$F_{fx,r(\epsilon,\lambda,a),a}(\epsilon) \geq$$

$$\geq T^2(F_{fx,fx_{2n_k},a}(\frac{\epsilon}{5}), F_{fx,fx_{2n_k},r(\epsilon,\lambda,a)}(\frac{\epsilon}{5}), F_{fx_{2n_k},r(\epsilon,\lambda,a),a}(\frac{3\epsilon}{5})) \geq$$

$$\geq T^4(F_{fx,fx_{2n_k},a}(\frac{\epsilon}{5}), F_{fx,fx_{2n_k},r(\epsilon,\lambda,a)}(\frac{\epsilon}{5}), F_{fx_{2n_k},fx_{2n_k+1},a}(\frac{\epsilon}{5}),$$

$$F_{fx_{2n_k}fx_{2n_k+1},r(\epsilon,\lambda,a)}(\frac{\epsilon}{5}), F_{fx_{2n_k+1},r(\epsilon,\lambda,a)}(\frac{\epsilon}{5})) \geq$$
$$\geq T^4(1-\delta(\lambda), 1-\delta(\lambda), 1-\delta(\lambda), 1-\delta(\lambda), 1-\delta(\lambda)) > 1-\lambda\,.$$

In the same manner, using (7) we can show that for all $\epsilon > 0$, $\lambda \in (0,1)$ and $a \in X$ we can find $q(\epsilon, \lambda, a) \in Ax$ such that

$$F_{fx,q(\epsilon,\lambda,a),a}(\epsilon) > 1-\lambda$$

which completes the proof.

References

[1] Gähler, S., 2-metrische Räume und ihre topologische Struktur, Math. Nachr., 26 1963/1964, 115-148

[2] Gähler, S., Linear 2-normierte Räume, Math. Nachr., 28 1965, 1-43.

[3] Gähler, S., Über 2-Banach Räume, Math. Nachr., 42 1969, 335-347.

[4] White, A., 2-Banach Spaces, Math. Nachr., 42 1969, 43-60

[5] Andalafte, E., Freese R., Existence of 2-segment in 2-metric spaces, FUNDAMENTA MATHEMATICAE, LX 1967, 201-208

[6] Freese R., A 2-metric characterization of the Euclidean Plane, Math. Ann. 206, 285-294 1973

[7] Rhoades B., Contraction Type Mappings on a 2-Metric Space Math. Nach. 91 1979, 151-155

[8] Hadžić, O., On Common Fixed Point Theorems in 2-metric Spaces, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak., Ser. Mat. 12, 1982, 7-18

[9] Hadžić, O., A Fixed Point Theorem for Multivalued Mappings in 2-Menger Spaces, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak., Ser. Mat. 24, 2 1994, 1-7

[10] Hadžić, O., On Coincidence Point Theorem for Multivalued Mappings in Probabilistic Metric Spaces, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak., Ser. Mat. 25, 1 1995, 1-7

[11] Hicks, T. L., Fixed Point Theory in Probabilistic Metric Spaces, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak., Ser. Mat. 13, 1 1983, 63-72

[12] Zeng,-Wen-Zhi, Probabilistic 2-metric spaces, J. Math. Research Expo. 2 1987, 241-245

[13] Chang,-Shi-sen, Huan,-Nan-jing, On the generalized 2-metric spaces and probabilistic 2-metric spaces with application to fixed point theory, Math. Japon. 34 1989, 885-900

[14] Singh,-Shyam-Lal, Talwar, -Rekha, Zeng,-Wen-Zhi, Common fixed point theorems in 2-Menger spaces and applications, Math.-Student 63 1994, no 1-4, 74-80