Novi Sad J. Math.
Vol. 28, No. 2, 1998, 163-171

# PLANNING OF THE TRAJECTORY FOR THE TIP OF REDUNDANT ROBOTIC MECHANISMS IN THE PRESENCE OF OBSTACLES

**Nataša Divljak, Miloš Racković**[1]
Institute of Mathematics, Faculty of Science,
21000 Novi Sad, Yugoslavia

**Abstract**

We consider the problem of determining the trajectory for the tip of redundant mechanisms, by optimisation of the given optimality criterion. The shortest distance between the initial and terminal position of the mechanism tip has been adopted as the optimality criterion.

The multi-linked robotic mechanism with spherical joints is considered. The obstacles are given in the form of simple polyhedra.

An algorithm for generation a set of admissible trajectories for the tip of robotic mechanism is given. On the basis of this set, using an optimisation method based on the $\psi$-transformation, a near-optimal admissible trajectory for the mechanism tip is determined. Since the mechanism is redundant, we give a local deterministic criterion for motion of mechanism while tip is tracking the polygonal line.

*AMS Mathematics Subject Classification (1991):* 68U05
*Key words and phrases:* computer geometry, planning of motion

[1]The work is supported by Science Fund of Serbia, through Institute of Mathematics, Novi Sad

# 1. Introduction

One of the main tasks in robotics is the planning of the mechanism's trajectories. The problem of synthesis of a nominal trajectory for the redundant mechanisms in the presence and in the absence of obstacles, so that the mechanism passes from a given initial state to the terminal state has been considered in [1-3]. The task is solved in the following way. First, a set of trajectories for all degrees of freedom of the mechanism is generated in the form of spline functions with the given number of knots and for the given boundary conditions. Then, for the generated set of trajectories, the corresponding value of the optimality criterion is calculated. Finally, on the basis of these values, using the optimisation procedure based on $\psi$-transformation, the knots of spline functions are determined for all those degrees of freedom for which the criterion value is minimal.

In [4] the authors considered the problem of synthesis of a nominal trajectory for which a planar mechanism passes from a given initial state to the given terminal position of the mechanism tip. In this case the optimality criterion was the length of the trajectory of the mechanism tip. This trajectory is a polygonal line whose origin is the initial point of the tip, and the end is terminal point of the mechanism tip. This polygonal line is also determined by the optimisation method employed in [1-3]. In this paper, we consider the solving of this problem in three-dimensional space.

# 2. Robotic mechanisms

The $n$-segment mechanism with spherical joints is considered (Fig. 1). We designate spherical joints $M_i$, $i = 0, ..., n-1$, tip of the mechanism $M_n$ and length of $i$-th segment $l_i$, $i = 1, ..., n$. In view of redundancy of the mechanism, the position of the mechanism for the initial state of the mechanism tip has to be known in advance. The position of the mechanism is given by coordinates of joints $M_i(x_i, y_i, z_i)$, $i = 0, ..., n-1$.

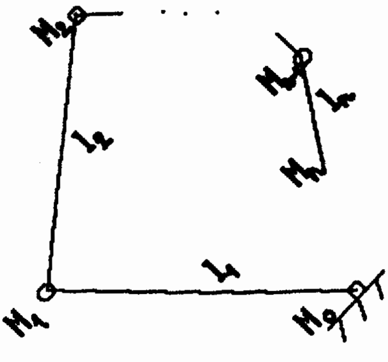

Fig. 1. $n$-segment robotic mechanism

## 3. The algorithm for determining the near-optimal trajectory of mechanism tip

The initial position of the robotic mechanism, obstacles in the form of the simple polyhedra and terminal point are given. Our goal is to determine the near-optimal admissible trajectory of mechanism tip. The algorithm is divided in three parts:

**I)** determination of $s$ admissible trajectories,

**II)** determination of the near-optimal trajectory,

**III)** admissibility test of the obtained near-optimal trajectory.

**I)** The admissible trajectory is a polygonal line $X = Z_0 Z_1 Z_2 ... Z_t Z_{t+1} = Y$, where $X$ is initial and $Y$ is terminal position of the mechanism tip, such that the intersection of the mechanism links and the interior regions of the given obstacles, while the mechanism tip is tracking this polygonal line, is an empty set. The points $Z_i$, $i = 1, ..., t$ are produced by the generator of uniform distribution random numbers. When the point $Z_i$ is determined, the admissibility of the obtained segment $Z_{i-1}Z_i$ is tested. In the case of admissibility, the next point $Z_{i+1}$ is generated (except, for $i = t$), otherwise a new point $Z_i$ is determined ([5]). The admissibility of segment $Z_{i-1}Z_i$ is tested in this way:

- the segment $Z_{i-1}Z_i$ is divided into $k+1$ subsegments $Z_{i-1}Z_{i-1}^1 ... Z_{i-1}^k Z_i$,
- for each partition point $Z_{i-1}^j$, $j = 1, ..., k$ and point $Z_i$ the position of the mechanism is determined;

  Since the mechanism is redundant, we give a local deterministic criterion for motion of mechanism while tip is tracking the polygonal line. The next position of the mechanism is determined in the following way:

  Let $M_0M_1...M_n$ denote the position of the mechanism, and $M_0'M_1'...M_n'$ the position which has to be determined for the point $Z_{i-1}^j$. $M_n' = Z_{i-1}^j$, $M_0' = M_0$, while the points $M_i'$, $i = 1, ..., n-1$ are determined as follows:

  When the point $M_{i+1}'$ is determined, for the point $M_i'$ we choose the point on the sphere $S(M_{i+1}', l_{i+1})$ which is the closest to the point $M_i$. So, the point $M_i'$ is determined from the intersection of the sphere $S(M_{i+1}', l_{i+1})$ and the line through points $M_i$ and $M_{i+1}'$, where from two intersection points, we take the one closer to the point $M_i$. The point $M_1'$ is determined from the intersection of spheres $S(M_2', l_2)$ and $S(M_0, l_1)$. For $M_1'$ we take the point from the intersection circumference, which is the closest to the projection of the point $M_1$ in the circumference plane. If these two spheres do not intersect, we conclude that the mechanism cannot reach the point $Z_{i-1}^j$ by this algorithm.

- collision between segments of the mechanism and given obstacles is tested;

  The algorithm for detection the intersection of segment and simple polyhedra is given in [6,7].

The structure of procedure for generating the trajectories is same as in [2]. The basic procedures for determination of the admissible trajectories are:

```
FUNCTION Next_position(T:point):BOOLEAN;
  BEGIN
    Next_position:=FALSE;
    M[n]:=T;
    FOR i:=n-1 DOWNTO 2 DO
      BEGIN
```

```
      Intersection_line_sphere(M[i+1],M[i],l[i+1]);
      (* determination of the new point M[i] *)
      IF Collision(M[i],M[i+1]) THEN EXIT
      (* tests the collision between segment M[i]M[i+1]
         and obstacles *)
    END;
  Intersection_sphere_sphere(M[2],l[2],l[1],M[1],achieve);
  (* determination of the point M[1] *)
  IF NOT achieve THEN EXIT;
  (* spheres don't intersect *)
  IF Collision(M[0],M[1]) THEN EXIT;
  Next_position:=TRUE
END;

FUNCTION Admissible(A,B:point):BOOLEAN;
BEGIN
  Admissible:=FALSE;
  k:=[p*d(A,B)]; (* k is the number of partition points *)
  FOR i:=1 TO 3 DO
    d[i]:=(B[i] - A[i])/k;
  FOR i:=1 TO k DO
    BEGIN
      FOR j:=1 TO 3 DO
        C[j]:=A[j] + i*d[j];
      IF NOT Next_position(C) THEN EXIT
    END;
  Admissible:=TRUE
END;
```

**II)** Using an optimisation method based on $\psi$-transformation, we determine the trajectory for which the value of $F(U)$, $U_{3i-2} = Z_{i1}$, $U_{3i-1} = Z_{i2}$, $U_{3i} = Z_{i3}$, $i = 1, ..., t$ is minimal. $F(U)$ is length of the trajectory $U$:

$$\begin{aligned} F(U) &= \sqrt{\sum_{j=1}^{3}(U_j - X_j)^2} + \\ &+ \sum_{i=1}^{t-1} \sqrt{\sum_{j=1}^{3}(U_{3i+3+j} - U_{3i+j})^2} + \\ &+ \sqrt{\sum_{j=1}^{3}(Y_j - U_{3t-3+j})^2} \end{aligned} \tag{1}$$

Algorithm for this optimisation method is given in [5].

**III)** Since the obstacles are not include in this optimisation method, an additional test is needed. We test the admissibility of the obtained near-optimal trajectory using the same algorithm.

## 4. Numerical examples

**Example 1.** (Fig. 2) In this example we consider the 3-segment mechanism. The obstacles are given in the form of simple polyhedra. Data structure of the polyhedron is given in Tables 1-3.

Table 1.

| Ord. no. of vert. | vert. of poly. |
|---|---|
| 1 | (0,1,0) |
| 2 | (2,1,0) |
| 3 | (2,3,0) |
| 4 | (0,3,0) |
| 5 | (0,1,2) |
| 6 | (2,1,2) |
| 7 | (2,3,2) |
| 8 | (0,3,2) |

Table 2.

| Ord. no. of edge | Edge det. by vert. |
|---|---|
| 1 | 1,2 |
| 2 | 2,3 |
| 3 | 3,4 |
| 4 | 1,4 |
| 5 | 1,5 |
| 6 | 2,6 |
| 7 | 3,7 |
| 8 | 4,8 |
| 9 | 5,6 |
| 10 | 6,7 |
| 11 | 7,8 |
| 12 | 5,8 |

Table 3.

| Ord. no. of facet | Fac. det. by vert. |
|---|---|
| 1 | 1,2,3,4 |
| 2 | 5,6,7,8 |
| 3 | 1,2,6,5 |
| 4 | 2,3,7,6 |
| 5 | 3,4,8,7 |
| 6 | 1,4,8,5 |

The initial point $X$ for the mechanism tip is $(1,0,2)$, and terminal point $Y$ is $(2,4,0)$. The initial positions of mechanism joints are $M_0 = (0,0,0)$, $M_1 = (0,0,3)$, $M_2 = (2,0,4)$. We consider the case with one point between $X$ and $Y$. The output data are presented in Table 4.

Table 4.

| Path | Point between $X$ and $Y$ | Path length |
|---|---|---|
| 1 | (2.1586,0.7044,1.2330) | 5.0801 |
| 2 | (2.5867,1.4329,1.3013) | 5.1865 |

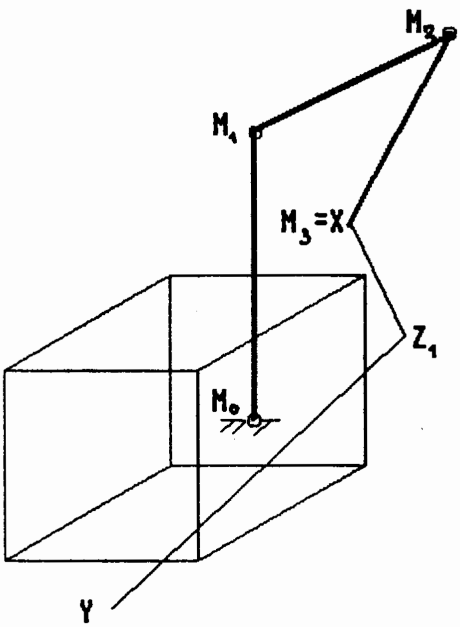


Fig. 2. Near-optimal trajectory $XZ_1Y$ for 3-segment mechanism

**Example 2.** (Fig. 3) Now we consider the 4-segment mechanism. Polyhedron, initial and terminal point are same as in Example 1, while the initial position of the mechanism joints are $M_1 = (0,0,3)$, $M_2 = (2,0,4)$, $M_3 = (1,0,3)$, $M_4 = (1,0,2)$. Again, we consider the case with one point between $X$ and $Y$. The output data are also presented in Table 4.

## 5. Conclusion

The problem of planning the trajectories for the tip of redundant mechanism in the presence of the obstacles is solved using the optimisation method. The length of the trajectory of the mechanism tip is adopted as the optimality criterion. To move a redundant mechanism from one discrete position to another, a local criterion is introduced for each joint of the mechanism. Since, the new position of the tip is known (tracking the trajectory), the position of the adjacent joint is chosen from the condition that the movement of this joint is minimal. If the motion of the mechanism tip along the chosen path is not admissible in regard to the local criterion, a new path is to be chosen. This does not mean that mechanism tip, in general, cannot track this path. However, this problem is not considered in the present paper.

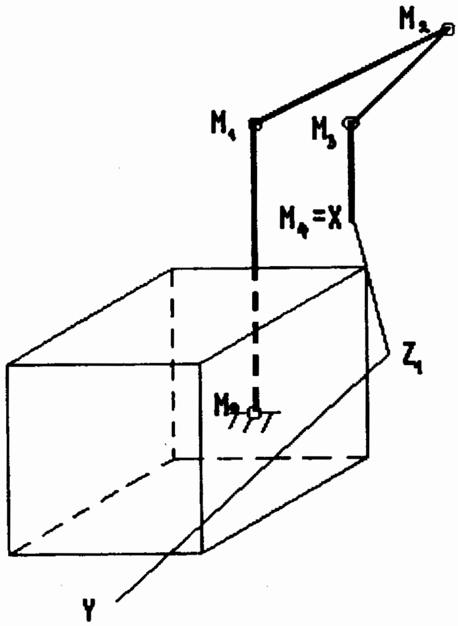


Fig. 3. Near-optimal trajectory $XZ_1Y$ for 4-segment mechanism

## References

[1] Surla D., Konjović Z., Path planning for $n$-DOF planar mechanism, Proc. of VI Yugoslav Symposium for Applied Robotics and Flexible Automatisation, Novi Sad 1989., 73-79 (in Serbian).

[2] Surla D., Obradović D., Konjović Z., Planning of trajectories for the motion of planar mechanism in the presence of obstacles, Automatika 31 (1990) 1-2, A.287-A.295.

[3] Surla D., Konjović Z., Synthesis of the optimal nominal control for manipulation robots by using $\psi$-transformation, Inter. Sym. MIM-S, '90, Brussels, (1990), IV.B.6.1-IV.B.6.6.

[4] Divljak N., Surla D., Planning of the trajectory for the tip of redundant planar robotic mechanisms in the presence of polygonal obstacles, Proc.

of XV Information Technology Conference, Sarajevo-Jahorina (1991) 243.1-243.7 (in Serbian).

[5] Surla D., Racković M., An algorithm for determining near-optimal distance between two points in a plane containing obstacles, Automatika 31 (1990), 1-2, 55-58.

[6] Surla D., Racković M., Planning of the trajectory for the tip of non-redundant robotic mechanisms in the presence of obstacles, Univ. Novom Sadu Zb. Rad. Prirod.-Mat. Fak. Ser. Mat. 23, 2 (1993), 331-344.

[7] Surla D., Budimac Z., Detection of the intersection of two simple polyhedra, Informatica 1(1990), pp. 49-54.

*Received by the editors Januar 18, 1997.*