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GENERALIZED WEISNER DESIGNS AND QUASIGROUPS

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Abstract

An n-ary quasigroup (Q, A) such that for some $i \in \{1, \ldots, n\}$ the identity

$$A(A(x_1^n), A(x_2^{n-1}, x_1), \ldots, A(x_n, x_1^{n-1})) = x_i$$

holds is called an i-Weisner n-quasigroup (i-W-n-quasigroup). i-W-n-quasigroups represent a generalization of quasigroups satisfying Schröder law ($xy \cdot yx = x$) and quasigroups satisfying Stein's third law ($xy \cdot yx = y$).

Properties of i-W-n-quasigroups which are satisfied for all i are determined. Necessary and sufficient conditions for an n-quasigroup to be an i-W-n-quasigroup are obtained. It is proved that every i-W-n-quasigroup of order v defines an orthogonal set of n $(n-1)$-quasigroups of order v. It is shown that some i-W-n-quasigroups are equivalent to orthogonal arrays. Conjugates of i-W-n-quasigroups are investigated, connections among these conjugates for different values of n, i are established. The existence of several classes of i-W-n-quasigroups is proved.

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1. Introduction and Preliminaries

A Weisner design is a pair of superimposed orthogonal latin squares with the property that (x, y) is in position i, j if and only if (i, j) is in position x, y. A Weisner design is selforthogonal if the latin squares are transposes of each other. Every self-orthogonal Weisner design is equivalent to a quasigroup $(Q, \cdot)$ in which $xy \cdot yx = x$ (Schröder second law) holds [11], [13] . A quasigroup satisfying Schröder second law is called a Schröder quasigroup. Such quasigroups are self-orthogonal and they are associated to other combinatorial structures. Schröder quasigroups are equivalent to a class of $n^2 \times 4$ orthogonal arrays [7], [8], and idempotent Schröder quasigroups correspond to triple tournaments of Baker [1] and to a class of edge-coloured block designs with block size four [4].

The identity $xy \cdot yx = y$ is called Stein's third law [5]. Quasigroups satisfying this identity are also self-orthogonal. They are equivalent to a class of $n^2 \times 4$ orthogonal arrays and also to a class of perfect Mendelsohn designs [2].

Here we shall generalize Schröder and quasigroups satisfying Stein's third law, but first we give some necessary definitions and notations.

The sequence $x_m, x_{m+1}, \ldots, x_n$ we denote by x_m^n or $\{x_i\}_{i=m}^n$. If $m > n$ then x_m^n will be considered empty.

An n-ary groupoid (Q, A) is called an n-quasigroup if the equation $A(a_1^{i-1}, x, a_{i+1}^n) = a_{n+1}$ has a unique solution x for every $a_1^{n+1} \in Q$ and every $i \in \mathbb{N}_n = \{1, \ldots, n\}$. An n-quasigroup (Q, A) is called idempotent if for every $x \in Q \quad A(x, x, \ldots, x) = x$. An element $x \in Q$ is called an idempotent if $A(x, x, \ldots, x) = x$.

A set $\{A_1, \ldots, A_k\}$ of n-ary operations defined on the same nonempty set Q, $k \geq n$, is orthogonal if for each $(a_1^n) \in Q^n$ and each injection $\varphi : \mathbb{N}_n \to \mathbb{N}_k$ there exist a unique $(c_1^n) \in Q^n$ such that

$$(\forall i \in \mathbb{N}_n) A_{\varphi(i)}\binom{n}{1} = c_i.$$

If (Q, A) is an n-quasigroup and $\sigma \in S_{n+1}$, where S_{n+1} is the symmetric group of degree $n+1$, then the n-quasigroup (Q, A^σ) defined by

$$A^\sigma(x_{\sigma 1}^{\sigma n}) = x_{\sigma(n+1)} \Leftrightarrow A(x_1^n) = x_{n+1}$$

is called a σ-conjugate (or simply conjugate) of A. A conjugate A^σ such that $\sigma(n+1) = n+1$ is called principal. A permutation $\sigma \in S_{n+1}$ such that $\sigma(n+1) = n+1$ is called a principal permutation. If (Q, A) is an n-quasigroup, $\sigma, \tau \in S_{n+1}$, then $(A^\sigma)^\tau = A^{\sigma\tau}$. If (Q, A) is an n-quasigroup such that the set $\{A, A_1, \ldots, A_{n-1}\}$ is orthogonal, where A_i are conjugates of A defined by $A_i(x_1^n) = A(x_{i+1}^n, x_1^i)$, $i \in \mathbb{N}_{n-1}$, then (Q, A) is called a self-orthogonal n-quasigroup.

A natural generalization of Schröder and quasigroups satisfying Stein's third law to n-ary case is given in the following definition.

Definition 1. *An n-quasigroup (Q, A) such that for some $i \in \mathbb{N}_n$ the identity*

$$A(A(x_1^n), A(x_2^n, x_1), \ldots, A(x_n, x_1^{n-1}) = x_i$$

holds is called an i-Weisner n-quasigroup (i-W-n-quasigroup).

From the preceding definition for $n = 2$, $i = 1$ Schröder quasigroups are obtained and for $n = 2$, $i = 2$ we get quasigroups satisfying Stein's third law. These two classes of quasigroups, being special cases of a more general class, have many common properties, but on the other hand, some of their properties are different. For example, there are no Schröder quasigroups of order 5, but quasigroups satisfying Stein's third law of order 5 do exist.

2. Properties of i-W-n-quasigroups

An important characterization of i-W-n-quasigroups is given in the following theorem which can be proved easily.

Theorem 1. *An n-quasigroup (Q, A) is an i-W-n-quasigroup if and only if the following equivalence is valid*

$$\left.\begin{array}{l} A(x_1^n) = a_1, \\ A(x_2^n, x_1) = a_2, \\ \ldots\ldots\ldots\ldots\ldots \\ A(x_n, x_1^{n-1}) = a_n, \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} A(a_1^n) = x_i, \\ A(a_2^n, a_1) = x_{i+1}, \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ A(a_{n-i+1}^n, a_1^{n-i}) = x_n, \\ A(a_{n-i+2}^n, a_1^{n-i+1}) = x_1, \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ A(a_n, a_1^{n-1}) = x_{i-1}. \end{array}\right.$$

Corollary 1. *Every i-W-n-quasigroup is selforthogonal.*

Corollary 2. *If (Q, A) is an i-W-n-quasigroup , then for all $x, y \in Q$*

$$A(x, \ldots, x) = y \Leftrightarrow A(y, \ldots, y) = x.$$

Corollary 3. *If (Q, A) is an i-W-n-quasigroup and for some $x, y \in Q$*

$$A(x, \ldots, x) = A(y, \ldots, y),$$

then $x = y$.

Corollary 4. *If (Q, A) is a finite i-W-n-quasigroup, then*

$$\{A(x, \ldots, x) \mid x \in Q\} = Q.$$

Corollary 5. *Every i-W-n-quasigroup of odd order has at least one idempotent.*

The condition for an n-quasigroup to be an i-W-n quasigroup given in Theorem 1 for $i = 1$ is called the Weisner property. In [11] it was proved that for any $n \geq 3$ and every odd v none of whose prime divisors divides n there exists a self-orthogonal n-quasigroup of order v having the Weisner property. The condition given in Theorem 1 where i is arbitrary we call the generalized Weisner property.

3. Orthogonal sets of n-quasigroups

We have seen that every i-W-n-quasigroup is self-orthogonal. Now we shall prove that every i-W-n quasigroup of order v also defines an orthogonal set of n $(n-1)$-quasigroups of order v.

Theorem 2. *Let (Q, A) be an i-W-n-quasigroup of order v and $a \in Q$. If n $(n-1)$-ary operations are defined on Q by*

$$\begin{aligned} B_1(x_1^{n-1}) &= A(x_1^{n-1}, a), \\ B_2(x_1^{n-1}) &= A(x_2^{n-1}, a, x_1), \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ B_n(x_1^{n-1}) &= A(a, x_1^{n-1}), \end{aligned}$$

then $\{B_1, \ldots, B_n\}$ is an orthogonal set of n $(n-1)$-quasigroups of order v.

Proof. Let b_1^{n-1} be arbitrary elements from Q. Since A is an n-quasigroup the equation $A(b_{n-i+1}^{n-1}, x, b_1^{n-i}) = a$ has a unique solution $x = c$.

The system

$$\begin{aligned}
&B_1(x_1^{n-1}) = b_1,\\
&B_2(x_1^{n-1}) = b_2,\\
&\dots\dots\dots\dots\dots\\
&B_{n-1}(x_1^{n-1}) = b_{n-1},
\end{aligned}$$

has a solution

$$\begin{aligned}
&x_1 = A(b_{n-i+2}^{n-1}, c, b_1^{n-i+1}),\\
&x_2 = A(b_{n-i+3}^{n-1}, c, b_1^{n-i+2}),\\
&\dots\dots\dots\dots\dots\dots\\
&x_{i-1} = A(c, b_1^{n-1})\\
&x_i = A(b_1^{n-1}, c)\\
&x_{i+1} = A(b_2^{n-1}, c, b_1)\\
&\dots\dots\dots\dots\dots\dots\\
&x_{n-1} = A(b_{n-i}^{n-1}, c, b_1^{n-i-1}),
\end{aligned}$$

since

$$\begin{aligned}
&B_1(A(b_{n-i+2}^{n-1}, c, b_1^{n-i+1}), A(b_{n-i+3}^{n-1}, c, b_1^{n-i+2}), \dots,\\
&A(c, b_1^{n-1}), A(b_1^{n-1}, c), A(b_2^{n-1}, c, b_1), \dots, A(b_{n-i}^{n-1}, c, b_1^{n-i-1}))\\
&= A(A(b_{n-i+2}^{n-1}, c, b_1^{n-i+1}), A(b_{n-i+3}^{n-1}, c, b_1^{n-i+2}), \dots,\\
&A(c, b_1^{n-1}), A(b_1^{n-1}, c), A(b_2^{n-1}, c, b_1), \dots, A(b_{n-i}^{n-1}, c, b_1^{n-i-1}), a) = b_1,
\end{aligned}$$

$$\begin{aligned}
&B_2(A(b_{n-i+2}^{n-1}, c, b_1^{n-i+1}), A(b_{n-i+3}^{n-1}, c, b_1^{n-i+2}), \dots,\\
&A(c, b_1^{n-1}), A(b_1^{n-1}, c), A(b_2^{n-1}, c, b_1), \dots, A(b_{n-i}^{n-1}, c, b_1^{n-i-1}))\\
&= A(A(b_{n-i+3}^{n-1}, c, b_1^{n-i+2}), A(b_{n-i+4}^{n-1}, c, b_1^{n-i+3}), \dots,\\
&A(c, b_1^{n-1}), A(b_1^{n-1}, c), A(b_2^{n-1}, c, b_1), \dots,\\
&A(b_{n-i}^{n-1}, c, b_1^{n-i-1}), a, A(b_{n-i+2}^{n-1}, c, b_1^{n-i+1})) = b_2,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
&B_{n-1}(A(b_{n-i+2}^{n-1}, c, b_1^{n-i+1}), A(b_{n-i+3}^{n-1}, c, b_1^{n-i+2}), \dots,\\
&A(c, b_1^{n-1}), A(b_1^{n-1}, c), A(b_2^{n-1}, c, b_1), \dots, A(b_{n-i}^{n-1}, c, b_1^{n-i-1}))\\
&= A(A(b_{n-i}^{n-1}, c, b_1^{n-i-1}), a, A(b_{n-i+2}^{n-1}, c, b_1^{n-i+1}), \dots,
\end{aligned}$$

$$A(c, b_1^{n-1}), A(b_1^{n-1}, c), A(b_2^{n-1}, c, b_1), \ldots, A(b_{n-i-1}^{n-1}, c, b_1^{n-i-2})) = b_{n-1}.$$

This solution is unique. Indeed, if we take any other solution x_1^n of the given system, we get

$$\begin{aligned} x_1 &= A(A(x_{n-i+2}^{n-1}, a, x_1^{n-i+1}), A(x_{n-i+3}^{n-1}, a, x_1^{n-i+2}), \ldots, \\ &\quad A(a, x_1^{n-1}), A(x_1^{n-1}, a), \ldots, A(x_{n-i+1}^{n-1}, a, x_1^{n-i})) \\ &= A(B_{n-i+2}(x_1^{n-1}), B_{n-i+3}(x_1^{n-1}), \ldots, \\ &\quad B_n(x_1^{n-1}), B_1(x_1^{n-1}), \ldots, B_{n-i+1}(x_1^{n-1})) \\ &= A(b_{n-i+2}^{n-1}, B_n(x_1^{n-1}), b_1^{n-i+1}). \end{aligned}$$

But since

$$\begin{aligned} a &= A(A(x_{n-i+1}^{n-1}, a, x_1^{n-i}), A(x_{n-i+2}^{n-1}, a, x_1^{n-i+1}), \ldots, \\ &\quad A(a, x_1^{n-1}), A(x_1^{n-1}, a), \ldots, A(x_{n-i}^{n-1}, a, x_1^{n-i-1})) \\ &= A(b_{n-i+1}^{n-1}, B_n(x_1^{n-1}), b_1^{n-i}) \end{aligned}$$

and $A(b_{n-i+1}^{n-1}, c, b_1^{n-i}) = a$ we obtain $B_n(x_1^{n-1}) = c$ which implies $x_1 = A(b_{n-i+2}^{n-1}, c, b_1^{n-i+1})$ and analogously for x_2^{n-1}.

Hence $\{B_1, \ldots, B_{n-1}\}$ is an orthogonal set of $n-1$ $(n-1)$-quasigroups.

The proof is analogous for any other choice of $n-1$ $(n-1)$-quasigroups from the set $\{B_1, \ldots, B_n\}$. □

4. Orthogonal arrays and i-W-n-quasigroups

A $v^n \times m$, $n < m$, orthogonal array (OA) is a pair (P, B), where P is an v element set and B is a set of v^n ordered m-tuples of elements from P which called rows, such that if $i_1^n \in \mathbb{N}_>$, $i_1 < \ldots < i_k$, then for all $a_1^k \in P$ there exists exactly one row from B in which i_p-th coordinate is a_p, $p \in \mathbb{N}_\kappa$.

Theorem 3. *Every i-W-n-quasigroup, $n \in \{2, 3\}$, of order v defines an $v^n \times (2n)$ orthogonal array.*

Proof. It is well known that every self-orthogonal binary quasigroup (Q, A) of order v defines an $v^2 \times 4$ OA of order v, the rows of which are given by

$(x, y, A(x, y), A^{(12)}(x, y))$. Since 1-W-2-quasigroups and 2-W-2-quasigroups are self-orthogonal they define orthogonal arrays.

In [12] it was proved that every *1-W-3*-quasigroup (Q, A) of order v defines an $v^3 \times 6$ OA of order v where the rows of the OA are defined by $(x, y, z, A(x, y, z), A^p(x, y, z), A^{p^2}(x, y, z))$, $p = (123)$. It can be proved analogously that also *2-W-3*-quasigroups and *3-W-3*-quasigroups define $v^3 \times 6$ OAs. □

REMARK. It is interesting that Theorem 3 is not valid for $n \geq 4$. For example, if for $n = 4$ on $GF(5)$ we define an 1-W-4-quasigroup by

$$A(x_1^4) = 3(-x_1 + x_2 + x_3 + x_4)$$

then it does not define an $5^4 \times 8$ OA.

5. Conjugates of *i-W-n*-quasigroups

Theorem 4. *If n is odd, then for every i-W-n-quasigroup A and every $j \in \mathbb{N}_k$ there exists a principal conjugate A^p such that A^p is a j-W-n-quasigroup.*

If n is even and $i \in \mathbb{N}$ an odd integer, then for every i-W-n-quasigroup A and every odd $j \in \mathbb{N}_k$ there exists a principal conjugate A^p such that A^p is a j-W-n-quasigroup.

If n and $i \in \mathbb{N}$ are even, then for every i-W-n-quasigroup A and every even $j \in \mathbb{N}_k$ there exists a principal conjugate A^p such that A^p is a j-W-n-quasigroup.

Proof. Let (Q, A) be a 1-*W-n*-quasigroup and $c = (n\ n-1 \ldots 2\ 1) \in S_{n+1}$. Then the identity

$$A(A(x_1^n), A(x_2^n, x_1), \ldots, A(x_n, x_1^{n-1})) = x_1$$

which is satisfied by A, can be written as

$$A(A(x_1^n), A^c(x_1^n), \ldots, A^{c^{n-1}}(x_1^n)) = x_1. \tag{1}$$

Let p be an arbitrary principal permutation from S_{n+1}. If we denote $A^p = B$, then (1) becomes

$$B^{p^{-1}}(B^{p^{-1}}(x_1^n), B^{p^{-1}c}(x_1^n), \ldots, B^{p^{-1}c^{n-1}}(x_1^n)) = x_1$$

and

$$B(B^{p^{-1}c^{p(1)-1}}(x_1^n), B^{p^{-1}c^{p(2)-1}}(x_1^n), \ldots, B^{p^{-1}c^{p(n)-1}}(x_1^n)) = x_1$$

which implies

$$B(B(\{x_{c^{1-p(1)}p(k)}\}_{k=1}^n), B(\{x_{c^{1-p(2)}p(k)}\}_{k=1}^n), \ldots, B(\{x_{c^{1-p(n)}p(k)}\}_{k=1}^n)) = x_1. \quad (2)$$

We shall investigate under what conditions the preceding identity defines a j-W-n-quasigroup (Q, B) for some $j \in \mathbb{N}_k$.

(Q, B) will be a j-W-n-quasigroup if there exists a principal permutation $q \in S_{n+1}$ such that

$$\begin{aligned} c^{1-p(1)}p &= q, \\ c^{1-p(2)}p &= qc^{-1}, \\ c^{1-p(3)}p &= qc^{-2}, \\ &\ldots\ldots\ldots\ldots \\ c^{1-p(n)}p &= qc^{-(n-1)}. \end{aligned}$$

From the preceding system of equations it follows

$$c^{1-p(1)}p = c^{1-p(2)}pc = c^{1-p(3)}pc^2 = \ldots = c^{1-p(n)}pc^{n-1}$$

and

$$c^{p(k+1)-p(k)}p = pc, \quad k = 1, 2, \ldots, n-1.$$

Hence

$$c^{p(n)-p(n-1)} = c^{p(n-1)-p(n-2)} = \ldots = c^{p(2)-p(1)}$$

which implies

$$(3) \quad p(n) - p(n-1) \equiv p(n-1) - p(n-2) \equiv \ldots \equiv p(2) - p(1) \pmod{n}.$$

It is not difficult to see that (2) defines a j-W-n-quasigroup (Q, B) for some $j \in \mathbb{N}_k$ if and only if (3) holds.

The permutation

$$(4) \quad p = \begin{pmatrix} 1 & 2 & \ldots & t & t+1 & t+2 & \ldots & n & n+1 \\ n-t+1 & n-t+2 & \ldots & n & 1 & 2 & \ldots & n-t & n+1 \end{pmatrix}$$

where $t \in \mathbb{N}_{k-k'}$, satisfies (3) which means that (Q, B) is a j-W-n-quasigroup. The value of $j \in \mathbb{N}_k$ is determined by

$$j = p^{-1}c^{p(1)-1}(1) = p^{-1}c^{n-t}(1) = p^{-1}(t+1) \equiv 2t+1 \pmod{n}.$$

So, from $j \equiv 2t+1 \pmod{n}$, if n is odd varying t we can get every $j \in \mathbb{N}_k$.

If n is even, then for all values of t we get for j all odd numbers from $\mathbb{N}_k$.

Hence we have proved that when n is odd and (Q, A) is a 1-W-n-quasigroup, then for every $j \in \mathbb{N}_k$ there exists a principal conjugate A^p such that A^p is a j-W-n-quasigroup.

Since the set of all principal conjugates of an n-quasigroup with respect to multiplication of permutations is a group, we get that when n is odd for every i-W-n-quasigroup (Q, A) and every $j \in \mathbb{N}_k$ there exists a principal conjugate A^p such that A^p is a j-W-n-quasigroup.

Analogously, one can obtain that if n is even and $i \in \mathbb{N}$ is an odd integer, then for every i-W-n-quasigroup A and every odd $j \in \mathbb{N}_k$ there exists a principal conjugate A^p such that A^p is a j-W-n-quasigroup.

Also, if n and $i \in \mathbb{N}$ are even, then for every i-W-n-quasigroup A and every even $j \in \mathbb{N}_k$ there exists a principal conjugate A^p such that A^p is a j-W-n-quasigroup. □

Now we shall determine all values of j which can be obtained using all principal permutations $p \in S_{n+1}$ which satisfy (3).

When n is even the procedure described in the preceding theorem for permutation p defined by (4) gives as principal conjugates of a 1-W-n-quasigroup only j-W-n-quasigroups where j is odd. We shall prove that all other principal permutations from S_{n+1} satisfying (3) also give only j-W-n-quasigroups where j is odd.

If $p = \begin{pmatrix} 1 & 2 & \dots & n & n+1 \\ a_1 & a_2 & \dots & a_n & n+1 \end{pmatrix}$ is a principal permutation satisfying (3), then applying the procedure from Theorem 4 and using the same notation, we obtain

$$j = p^{-1}c^{p(1)-1}(1) = p^{-1}c^{a_1-1}(1) = p^{-1}(n - a_1 + 2).$$

Since n is even, we get that $n - a_1 + 2$ and a_1 are both even or both odd. But p satisfies (3), hence the differences $p(m) - p(m-1)$ must be relatively

prime to n for every $m = 2, \ldots . n$. Consequently, consecutive numbers a_i in p can not be both even or both odd, which implies that in p the number $n - a_1 + 2$ is at odd place (as a_1), so $j = p^{-1}(n - a_1 + 2)$ is always odd.

Analogously, one obtains that when n and i are even and (Q, A) is a i-W-n-quasigroup, then for every $p \in S_n$ satisfying (3) the procedure described in Theorem 4 gives only j-W-n-quasigroups (Q, A^p) where j is even.

6. The existence of i-W-n-quasigroups

We have already noted that spectra of i-W-n-quasigroups for $n = 2$, $i = 1$ and $i = 2$ are different. In fact, the spectrum of Schröder quasigroups consists of all positive integers $v \equiv 0, 1 \pmod 4$ except $n = 5$ ([4],[6], [9]) and the spectrum of quasigroups satisfying Stein's third law consists of all positive integers $v \equiv 0, 1 \pmod 4$ ([10],[6]).

On the other hand, some of the i-W-n-quasigroups are closely related and can be transformed one into another.

The existence of 1-W-3-quasigroups was considered in [12] where it was proved that idempotent 1-W-3-quasigroups of order v exist for every odd v which is not divisible by 3, and nonidempotent 1-W-3-quasigroups of order v exist for every $v = 4^\alpha k$, where α is a nonnegative integer and k is an odd integer not divisible by 3. From Theorem 4 it follows that the spectra of i-W-3-quasigroups are equal for all $i \in \mathbb{N}_{\not 4}$, hence we have the next theorem.

Theorem 5. *For all $i \in \mathbb{N}_{\not 4}$ there exists an i-W-n-quasigroup of order v, where $v \equiv 1, 5 \pmod 6$ or $v \equiv 4, 20 \pmod{24}$ or $v = 4^k$, $k \in \mathbb{N}$.*

From Theorem 4 and [11] we get the next theorem.

Theorem 6. *For every odd n, every $i \in \mathbb{N}$ and every odd v such that $(n, v) = 1$ there exists an i-W-n-quasigroup of order v.*

For every even n, every odd $i \in \mathbb{N}$ and every v such that $(n, v) = 1$ there exists an i-W-n-quasigroup of order v.

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Corrigenda

In the paper **"GENERALIZED WEISNER DESIGNS AND QUASI-GROUPS" by Zoran Stojaković and Wiesław A. Dudek,** published in the Novi Sad Journal of Mathematics, Vol. 28, No. 2, 1998, 143-153. a number of errors appeared as a consequence of using different version of LATEX. The following table gives a list of the errors and corrections.

page	line	error	correction
144	-10	$i \in \mathbb{N}_{\aleph} = \{\beth, \ldots, \aleph\}$	$i \in \mathbb{N}_n = \{1, \ldots, n\}$
144	-6	$\varphi : \mathbb{N}_{\aleph} \to \mathbb{N}_{\neg}$	$\varphi : \mathbb{N}_n \to \mathbb{N}_k$
144	-4	$(\forall i \in \mathbb{N}_{\aleph})\mathbb{A}_{\varphi(\beth)}\binom{\aleph}{\beth} = \eth\beth.$	$(\forall i \in \mathbb{N}_n)A_{\varphi(i)}(c_1^n) = a_i.$
145	6	$i \in \mathbb{N}_{\aleph-\beth}$	$i \in \mathbb{N}_{n-1}$
145	10	$i \in \mathbb{N}_{\aleph}$	$i \in \mathbb{N}_n$
148	-6	$i_1^n \in \mathbb{N}_{\geqslant}$	$i_1^n \in \mathbb{N}_m$
148	-6	i_k	i_n
148	-6	a_1^k	a_1^n
148	-5	$p \in \mathbb{N}_{\aleph}$	$p \in \mathbb{N}_n$
149	-15,-13,-10	$j \in \mathbb{N}_{\aleph}$	$j \in \mathbb{N}_n$
150	6,-3	$j \in \mathbb{N}_{\aleph}$	$j \in \mathbb{N}_n$
151	2,4,6,8,12,15,18	$j \in \mathbb{N}_{\aleph}$	$j \in \mathbb{N}_n$
152	-8, -7	$i \in \mathbb{N}_{\beth}$	$i \in \mathbb{N}_3$
152	-7	i-W-n	i-W-3

We apologize to the authors and readers for these mistakes.

Editors