NOVI SAD J. MATH.
VOL. 27, NO. 2, 1997, 101-115

HIGHER ORDER GAUGE-INVARIANT LAGRANGIANS

Gheorghe Munteanu
Department of Geometry,"Transilvania"University of Brasov
Faculty of Science, 2200, Brasov, Romania

Abstract

Generally, the gauge theories have as a geometrical support the associated bundles of a principal G-bundle.

An associate bundle of a k-principal frame bundle $P^k(M)$ is the k-osculator bundle, $Osc^{(k)}M$ over some manifold M ([8]).

In the paper [9], we defined and studied the general concepts concerning the higher order gauge transformations. The theory becomes consistent when investigating the gauge invariance of k-order Lagrangians.

In the present paper we shall study the global and local invariance of k-order Lagrangians under the gauge infinitesimal transformations from classical gauge theory, generalizing the result from [3].

AMS Mathematics Subject Classification (1991): 53B40
Key words and phrases: Gauge-theory, higher order Lagrange space

1. K-gauge osculator bundle

In many physical theories, in addition to the local transformations on an M manifold appears another transformation determined by one group G (sometimes group of internal symmetry) which acts globally or locally on a manifold F, and is called gauge transformation.

The physical framework of the study is based on the Lagrange formalism, and the adequate geometrical support is the associated bundles of a principal G-bundle.

The particular case when the associate bundle has vector structure presents the advantage of having useful local expressions for physicists([1]). Moreover, if a nonlinear connection on the associate bundle is given, then the calculus is simplified ([2],[3],[7]).

Let (P,π_P,M,G) be a principal bundle, (U,Ψ_U^P) a local map $\Psi_U^P : \pi_P^{-1}(U) \to U \times G$, $p \to (x=\pi_P(p), g=\varphi_U(p))$, and $L:G\times F \to F$, $(g,f)\to gf$, the action of the group G on F. An associate bundle of P has F as a fibre and is defined by the quotient $E=(P\times F)/G$, identifying the elements $(p,y) \sim (p.g, g^{-1}y)$. A local fibre map on E is (U,Ψ_U^E), where $\Psi_U^E:\pi_E^{-1}(U)\to U\times F, [p,y]\to(\pi_E(p),\varphi_U(p)y)$ and as a manifold the change of maps is expressed by $(x,f) \overset{\Psi_V^E\circ(\Psi_U^E)^{-1}}{\longrightarrow} (x, g_{UV}(x)y)$, with $g_{UV}(x) \in G$.

Let us consider $f^P:P\to P$ an automorphism of principal bundle, i. e. diffeomorphism and the equinvariant condition:

$$f^P(p.g) = f^P(p).g \quad . \tag{1.1}$$

A gauge transformation on P is a pair (f^P,f^0), where f^0 is a diffeomorphism on the base manifold M and $\pi_P \circ f^P = f^0 \circ \pi_P$.

Usually, f^0 is taken to be id_M, because G is considered as a group of internal symmetry ([4],[5]).

The automorphism f^P induces an automorphism f^E on E, $f^E[p,y]=[f^P(p),y]$.

In the particular case of a product bundle $P=M\times G$ the right translation $L_g:P\to P$ is a gauge transformation, $L_g(x,g')=(x,g.g')$ called global transformation. If $f^P(x,g')=(x,h(x)g')$ is a gauge transformation, then it is called local transformation.

Let $\dim M=n$, $F=R^m$ and G be a group of matrices. Then G acts linearly on R^m. An associate bundle E of (P,π_p,M,G) is said to be a gauge vectorial bundle. The local change of maps on E is of the form: $\tilde{x}^i=x^i(x)$, $\tilde{y}^a=M_b^a(x).y^b$, where $M_b^a(x)\in G$ and a gauge transformation (f^E,f^0) is locally expressed by: $\bar{x}^i=X^i(x)$, $\bar{y}^a=Y^a(x,y)$, with $\det\left(\frac{\partial X^i}{\partial x^j}\right).\det\left(\frac{\partial Y^a}{\partial y^b}\right) \neq 0$. This kind of gauge transformation is studied on a vector bundle in [1],[2],[7].

Generalizing such ideas a gauge transformation of higher order is studied in [9].

Let G be a compact subgroup in GL(m,R) and $G^{(k)}$ the k-prolongation of G (The elements of $G^{(k)}$ are k-jets of diffeomorfisms in R^m to R^m, preserving the origin and having a base of the R^m matrice from G.). Let $P_G^{(k)}(M)$ be a principal bundle over M and structural group $G^{(k)}$.

If $F=R^{mk}$, then we can consider the action of $G^{(k)}$ on F:

$$
\begin{aligned}
gy = (&g^a_{b_1} y^{(1)b_1}; g^a_{b_1 b_2} y^{(1)b_1} y^{(1)b_2} + g^a_{b_1} y^{(2)b_1};\\
&g^a_{b_1 b_2 b_3} y^{(1)b_1} y^{(1)b_2} y^{(1)b_3} + g^a_{b_1 b_2} y^{(1)b_1} y^{(2)b_2} +\\
&g^a_{b_1 b_3} y^{(1)b_1} y^{(2)b_3} + g^a_{b_1} y^{(3)b_1}; \ldots),
\end{aligned} \tag{1.2}
$$

where $g = (g^a_{b_1}, \ldots, g^a_{b_1 b_2 \ldots b_k}) \in G^{(k)}$ and $y = (y^{(1)a}, \ldots, y^{(k)a}) \in F$.

The associated bundle $(E_k, \pi^k, M, F, G^{(k)})$ of $P_G^{(k)}(M)$ is obtained, and the local coordinates on manifold E_k are changed according to the rules:

$$
\begin{cases}
\tilde{x}^i = \tilde{x}^i(x)\\
\tilde{y}^{(1)a} = g^a_{b_1}(x) y^{(1)b_1}\\
\tilde{y}^{(2)a} = g^a_{b_1 b_2}(x) y^{(1)b_1} y^{(1)b_2} + g^a_{b_1}(x) y^{(2)b_1}\\
\ldots
\end{cases} \tag{1.3}
$$

In the particular situation when n=m and $G \subset GL(n,R)$, $P_G^{(k)}(M)$ is a G -structure of k-order. Moreover, if G=GL(n,R), taking y of the form $y = (y^{(1)i}, 2! y^{(2)i}, \ldots, k! y^{(k)i}) \in R^{nk}$ and $\tilde{x}^i = \tilde{x}^i(x)$ the local change of coordinates for the maps (U, φ) and $(\tilde{U}, \tilde{\varphi})$ in $x \in M$, the k-jet of $\tilde{\varphi} \circ \varphi^{-1}$from $G^{(k)}$ is:

$$
g^i_{j_1} = \frac{\partial \tilde{x}^i}{\partial x^{j_1}};\ g^i_{j_1 j_2} = \frac{\partial^2 \tilde{x}^i}{\partial x^{j_1} \partial x^{j_2}}; \ldots; g^i_{j_1 j_2 \ldots j_k} = \frac{\partial^k \tilde{x}^i}{\partial x^{j_1} \partial x^{j_2} \ldots \partial x^{j_k}}
$$

and the (1.3) local changes of coordinates $u = (x^i, y^{(1)i}, \ldots y^{(k)i})$

$$
\begin{cases}
\tilde{x}^i = \tilde{x}^i(x),\ \det\left(\frac{\partial \tilde{x}^i}{\partial x^j}\right) \neq 0\\
\tilde{y}^{(1)i} = \frac{\partial \tilde{x}^i}{\partial x^j} y^{(1)j}\\
2\, \tilde{y}^{(2)i} = \frac{\partial \tilde{y}^{(1)i}}{\partial x^j} y^{(1)j} + 2 \frac{\tilde{y}^{(1)i}}{y^{(1)j}} y^{(2)j}\\
k\, \tilde{y}^{(k)i} = \frac{\partial \tilde{y}^{(k-1)i}}{\partial x^j} y^{(1)j} + 2 \frac{\partial \tilde{y}^{(k-1)i}}{\partial y^{(1)j}} y^{(2)j} + \ldots + k \frac{\partial \tilde{y}^{(k-1)i}}{\partial y^{(k-1)j}} y^{(k)j}.
\end{cases} \tag{1.4}
$$

The formulas (1.4) represent exactly the change of local coordinates on the k-osculator bundle, $\mathrm{Osc}^{(k)}M$, systematically studied by R. Miron ([8]).

We emphasize here the structure of the $\mathrm{Osc}^{(k)}M$ bundle as an associated bundle of principal k-frame bundle.

Definition 1.1 *A k-order G-structure of $P_G^{(k)}(M)$ is said to be a gauge k-osculator bundle and is denoted by $GOsc^{(k)}M$.*

It follows that in $u = (x^i, y^{(1)i}, ..., y^{(k)i}) \in \mathrm{GOsc}^{(k)}M$ the change of coordinates (1.4) is restricted to $\tilde{x}^i = \tilde{x}^i(x)$ for $\left(\frac{\partial \tilde{x}^i}{\partial x^j}\right)(x) \in G$. The whole construction made for the k-osculator bundle remains valid by restriction to $G^{(k)}$.

The bundle ($\mathrm{GOsc}^{(k)}M$,π^k,M) is also a fibre bundle over $\mathrm{GOsc}^{(h)}M$, with $0 \leq h \leq k$ ($\mathrm{GOsc}^{(0)}M \equiv M$). Thus, we have the bundle ($\mathrm{GOsc}^{(k)}M$,π_h^k,$\mathrm{GOsc}^{(h)}M$), where $\pi_h^k(x, y^{(1)}, ...y^{(k)}) = (x, y^{(1)}, ..., y^{(h)})$.

Definition 1.2. *A gauge transformation in $GOsc^{(k)}M$ is a set of diffemorphisms $T=\left\{f^{(0)}, f^{(1)}, ...f^{(k)}\right\}$, $f^{(h)} \in Diff\, GOsc^{(h)}M$, $h=\overline{0,k}$, that satisfy the conditions:*

$$\pi_h^k \circ f^{(k)} = f^{(k)} \circ \pi_h^k (h = \overline{0,k}) \tag{1.5}$$

The set of all gauge transfomations in $\mathrm{GOsc}^{(k)}M$ bundle is a group and a study of various geometric objects (nonlinear gauge connections, gauge covariant derivatives, metric gauge structures) related to the gauge transformations is developed in [9]. Here we shall only summarize some ideas.

In local coordinates on the $\mathrm{GOsc}^{(k)}M$ bundle a gauge transformation T $= \left\{f^{(0)}, f^{(1)}, ..., f^{(k)}\right\}$ is represented by the equations:

$$\begin{cases} \bar{x}^i = X^i(x) \\ \bar{y}^{(1)i} = Y^{(1)i}(x, y^{(1)}) \\ \dots \\ \bar{y}^{(k)i} = Y^{(k)i}(x, y^{(1)}, ..., y^{(k)}) \end{cases} \tag{1.6}$$

with $\det\left(\frac{\partial X^i}{\partial x^j}\right)$, $\det\left(\frac{\partial Y^{(1)i}}{\partial y^{(1)j}}\right)$,...,$\det\left(\frac{\partial Y^{(k)i}}{\partial y^{(k)j}}\right) \neq 0$, and globally defined in an open set $U \subset \mathrm{GOsc}^{(k)}M$.

A gauge d-tensor is a system of functions $W^{i_1...i_r}_{j_1...j_s}(u)$ which has the following properties:

$$\bar{W}^{i_1...i_r}_{j_1...j_s}(\bar{u}) = X^{i_1}_{h_1}...X^{i_r}_{h_r}\cdot \bar{X}^{l_1}_{j_1} ... \bar{X}^{l_s}_{j_s} W^{h_1...h_r}_{l_1...l_s}(u)$$

$$\tilde{W}^{i_1...i_r}_{j_1...j_r}(\tilde{u}) = \frac{\partial \tilde{x}^{i_1}}{\partial x^{h_1}}...\frac{\partial \tilde{x}^{i_r}}{\partial x^{h_s}}\cdot\frac{\partial x^{l_1}}{\partial \tilde{x}^{j_1}}...\frac{\partial x^{l_s}}{\partial \tilde{x}^{j_s}} W^{h_1...h_r}_{l_1...l_s}(u)$$

where $X^i_j = \frac{\partial X^i}{\partial x^j}$ and $\bar{X}^i_j \,.X^j_m = \delta^i_m$.

A gauge nonlinear collection is a nonlinear collection N ([8]) with the property that the tangent map T_* preserves the distributions defined by N.

If $\left(\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}}, ..., \frac{\delta}{\delta y^{(k)i}}\right)$ is the adapted base determined by N:

$$(1.7)\quad \begin{cases} \frac{\delta}{\delta x^i} = \frac{\partial}{\partial x^i} - N^j_{(1)i}\frac{\partial}{\partial y^{(1)j}} - ... - N^j_{(k)i}\frac{\partial}{\partial y^{(k)j}} \\ \frac{\delta}{\delta y^{(\alpha)i}} = \frac{\partial}{\partial y^{(\alpha)i}} - N^J_{(1)i}\frac{\partial}{\partial y^{(\alpha)j}} - ... - N^j_{(k-\alpha)i}\frac{\partial}{\partial y^{(k)j}}, \alpha = \overline{1, k-1} \\ \frac{\delta}{\delta y^{(k)i}} = \frac{\partial}{\partial y^{(k)i}}, \end{cases}$$

where $N^j_{(h)i}$ are the coefficients of nonlinear connection N, then N is a gauge nonlinear connection iff the vectors $\left(\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}}, ..., \frac{\delta}{\delta y^{(k)i}}\right)$ are all gauge d-tensors.

Let $g_{ij}(u)$ be a gauge d-tensor, symmetric and positively defined, rank(g_{ij}) =n. If $\{dx^i, \delta y^{(1)i}, ..., \delta y^{(k)i}\}$ is the dual base of $\left(\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}}, ..., \frac{\delta}{\delta y^{(k)i}}\right)$, then a metric gauge structure on $GOsc^{(k)}M$ is defined by the pair (G,$GOsc^{(k)}M$), where G is the structure:

$$(1.8)\quad G = g_{ij}dx^i \otimes dx^j + g_{ij}\delta y^{(1)i} \otimes \delta y^{(1)j} + ... + g_{ij}\delta y^{(k)i} \otimes \delta y^{(k)j}.$$

2. K-order Euler-Lagrange equations

Let us consider $L_0(u)$, u=(x^i,$y^{(1)i}$,...$y^{(k)i}$), a Lagrangian defined on the domain $\Omega \subset R^{n(k+1)}$ and $N = (N^j_{(1)i}, ..., N^j_{(k)i})$ a nonlinear gauge connection, G=(g_{ij}) a metric structure on $GOsc^{(k)}M$. Certainly, the Lagrangian L_0 depends on the local maps in u, so that the action $\int_\Omega L_0(u)d\omega$ is not independent of the choice of coordinates. To eliminate this inconvenience we shall consider the Lagrangian density:

$$(2.1)\qquad \mathcal{L}(x, y^{(1)}, ..., y^{(k)}) = L_0(x, y^{(1)}, ..., y^{(k)}) \cdot (\sqrt{g})^{k+1},$$

with $g = \det(g_{ij})$, that is independent of the system of local coordinates.

In gauge theories the Lagrangian L_0 depends on the coordinates of the point u by means of matter fields $\Phi^A, A = \overline{1,p}$ and their derivatives $\frac{\delta\Phi^A}{\delta x^i}$, $\frac{\delta\Phi^A}{\delta y^{(\alpha)i}}$, so that we have:

$$(2.2)\qquad L_0(x, y^{(1)}, ...y^{(k)}) = L(\Phi^A, \frac{\delta\Phi^A}{\delta x^i}, \frac{\delta\Phi^A}{\delta y^{(1)i}}, ...\frac{\delta\Phi^A}{\delta y^{(k)}}) \ , \ A = \overline{1,p}.$$

The matter fields Φ^A are supposed to be gauge scalars and because the nonlinear connection is a gauge, their derivatives are gauge vectors.

The extremal value of the action $I(\Phi^A) = \int_\Omega \mathcal{L}(\Phi^A)d\omega$ results from the variational principle $\delta I(\Phi^A) = 0$ and involves the following k-order Euler-Lagrange equation ([9]):

$$(2.3)\frac{\partial\mathcal{L}}{\partial\Phi^A} - \frac{\partial}{\partial x^i}\frac{\partial\mathcal{L}}{\partial\left(\frac{\partial\Phi^A}{\partial x^i}\right)} - \frac{\partial}{\partial y^{(1)i}}\frac{\partial\mathcal{L}}{\partial\left(\frac{\partial\Phi^A}{\partial y^{(1)i}}\right)} - ... - \frac{\partial}{\partial y^{(k)i}}\frac{\partial\mathcal{L}}{\partial\left(\frac{\partial\Phi^A}{\partial y^{(k)i}}\right)} = 0$$

Taking into account the fact that L depends on $\frac{\partial\Phi^A}{\partial x^i}$ by means of $\frac{\delta\Phi^A}{\delta x^i}$, and $\frac{\partial\Phi^A}{\partial y^{(1)i}}$ by means of $\frac{\delta\Phi^A}{\delta x^i}$ and $\frac{\delta\Phi^A}{\delta y^{(1)i}}$, e.t.c., after a direct calculus we obtain the k-order (E-L) equation in adapted base (1.7):

$$(2.4)\qquad \begin{aligned} &\sqrt{g^{k+1}}\{\tfrac{\partial L}{\partial\Phi^A} - \tfrac{\delta}{\delta x^i}(\overset{h}{\Phi}{}^i_A) - \tfrac{\delta}{\delta y^{(1)i}}(\overset{v_1}{\Phi}{}^i_A) - ... - \tfrac{\delta}{\delta y^{(k)i}}(\overset{v_k}{\Phi}{}^i_A) + \\ &\quad + \tfrac{\delta N^i_{(1)j}}{\delta y^{(1)i}}\overset{h}{\Phi}{}^j_A + \tfrac{\delta N^i_{(1)j}}{\delta y^{(2)i}}\overset{v_1}{\Phi}{}^i_A + ... + \tfrac{\delta N^i_{(k)j}}{\delta y^{(k)i}}\overset{v_{k-1}}{\Phi}{}^j_A + \\ &\quad + \tfrac{\delta N^i_{(2)j}}{\delta y^{(k)i}}\overset{v_{k-2}}{\Phi}{}^j_A + ... + \tfrac{\delta N^i_{(k)j}}{\delta y^{(k)i}}\overset{h}{\Phi}{}^j_A\} = \\ &\quad = \tfrac{\delta\sqrt{g^{k+1}}}{\delta x^i}\overset{h}{\Phi}{}^i_A + \tfrac{\delta\sqrt{g^{k+1}}}{\delta y^{(1)i}}\overset{v_1}{\Phi}{}^i_A + ... + \tfrac{\delta\sqrt{g^{k+1}}}{\delta y^{(k)i}}\overset{v_k}{\Phi}{}^i_A\ . \end{aligned}$$

where we use the gauge vectors:

$$\overset{h}{\Phi}{}^i_A = \frac{\partial L}{\partial\left(\frac{\delta\Phi^A}{\delta x^i}\right)}\ , \ \overset{v_\alpha}{\Phi}{}^i_A = \frac{\partial L}{\partial\left(\frac{\delta\Phi^A}{\delta y^{(\alpha)i}}\right)|c_1 c_2 ... c_{\alpha-1}},$$

with $c_1 = \frac{\delta\Phi}{\delta x^j} = const, c_2 = \frac{\delta\Phi}{\delta y^{(1)j}} = const$, etc.

Since the adapted base changes with respect to (1.4) according to the rules: $\frac{\delta}{\delta x^i} = \frac{\partial\tilde{x}^j}{\partial x^i}\frac{\delta}{\delta\tilde{y}^j}$, $\frac{\delta}{\delta y^{(\alpha)i}} = \frac{\partial\tilde{x}^j}{\partial x^i}\frac{\delta}{\delta\tilde{y}^{(\alpha)j}}$, $\alpha = \overline{1,k}$, it is easy to check

A gauge d-tensor is a system of functions $W^{i_1...i_r}_{j_1...j_s}(u)$ which has the following properties:

$$\bar{W}^{i_1...i_r}_{j_1...j_s}(\bar{u}) = X^{i_1}_{h_1}...X^{i_r}_{h_r}.\ \bar{X}^{l_1}_{j_1}\ ...\ \bar{X}^{l_s}_{j_s}\ W^{h_1...h_r}_{l_1...l_s}(u)$$

$$\widetilde{W}^{i_1...i_r}_{j_1...j_r}(\widetilde{u}) = \frac{\partial \widetilde{x}^{i_1}}{\partial x^{h_1}}...\frac{\partial \widetilde{x}^{i_r}}{\partial x^{h_s}}.\frac{\partial x^{l_1}}{\partial \widetilde{x}^{j_1}}...\frac{\partial x^{l_s}}{\partial \widetilde{x}^{j_s}} W^{h_1...h_r}_{l_1...l_s}(u)$$

where $X^i_j = \frac{\partial X^i}{\partial x^j}$ and $\bar{X}^i_j\ .X^j_m = \delta^i_m$.

A gauge nonlinear collection is a nonlinear collection N ([8]) with the property that the tangent map T_* preserves the distributions defined by N.

If $\left(\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}}, ..., \frac{\delta}{\delta y^{(k)i}}\right)$ is the adapted base determined by N:

$$(1.7)\quad \begin{cases} \frac{\delta}{\delta x^i} = \frac{\partial}{\partial x^i} - N^j_{(1)i}\frac{\partial}{\partial y^{(1)j}} - ... - N^j_{(k)i}\frac{\partial}{\partial y^{(k)j}} \\ \frac{\delta}{\delta y^{(\alpha)i}} = \frac{\partial}{\partial y^{(\alpha)i}} - N^J_{(1)i}\frac{\partial}{\partial y^{(\alpha)j}} - ... - N^j_{(k-\alpha)i}\frac{\partial}{\partial y^{(k)j}}, \alpha = \overline{1, k-1} \\ \frac{\delta}{\delta y^{(k)i}} = \frac{\partial}{\partial y^{(k)i}}, \end{cases}$$

where $N^j_{(h)i}$ are the coefficients of nonlinear connection N, then N is a gauge nonlinear connection iff the vectors $\left(\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}}, ..., \frac{\delta}{\delta y^{(k)i}}\right)$ are all gauge d-tensors.

Let $g_{ij}(u)$ be a gauge d-tensor, symmetric and positively defined, rank(g_{ij}) =n. If $\{dx^i, \delta y^{(1)i}, ..., \delta y^{(k)i}\}$ is the dual base of $\left(\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}}, ..., \frac{\delta}{\delta y^{(k)i}}\right)$, then a metric gauge structure on $GOsc^{(k)}M$ is defined by the pair $(G, GOsc^{(k)}M)$, where G is the structure:

$$(1.8)\quad G = g_{ij}dx^i \otimes dx^j + g_{ij}\delta y^{(1)i} \otimes \delta y^{(1)j} + ... + g_{ij}\delta y^{(k)i} \otimes \delta y^{(k)j}.$$

2. K-order Euler-Lagrange equations

Let us consider $L_0(u)$, $u=(x^i, y^{(1)i}, ... y^{(k)i})$, a Lagrangian defined on the domain $\Omega \subset R^{n(k+1)}$ and $N = (N^j_{(1)i}, ..., N^j_{(k)i})$ a nonlinear gauge connection, $G=(g_{ij})$ a metric structure on $GOsc^{(k)}M$. Certainly, the Lagrangian L_0 depends on the local maps in u, so that the action $\int_\Omega L_0(u)d\omega$ is not independent of the choice of coordinates. To eliminate this inconvenience we shall consider the Lagrangian density:

$$(2.1)\qquad \mathcal{L}(x, y^{(1)}, ..., y^{(k)}) = L_0(x, y^{(1)}, ..., y^{(k)}) \cdot (\sqrt{g})^{k+1},$$

up the invariance of the equations (2.4) with respect to the local change of coordinates (1.4) on $GOsc^kM$. The gauge invariance of the equations (2.4) will be studied in the following sections in the special case of gauge infinitesimal transformations.

Now let us give a simplified form of the equations (2.4). Let us consider $(L^i_{kj}, C^i_{(\alpha)kj})$ the coefficients of a given d-connection on the $GOsc^kM$ bundle ([8]) and

$$X^i_{|k} = \frac{\delta X^i}{\delta x^k} + L^i_{kj}X^j \ ; \ X^i \overset{\alpha}{|}_k = \frac{\delta X^i}{\delta y^{(\alpha)k}} + C^i_{(\alpha)jk}\ , \ \alpha = \overline{1,k}$$

the rules of horizontal and v_α-vertical derivatives of one d-vector X. Then equations (2.4) are written in the equivalent form:

$$\frac{\partial L}{\partial \Phi^A} - \overset{h}{\Phi}{}^i_{A|i} - \overset{v_1}{\Phi}{}^i_{A} \overset{v_1}{|}_i - \ldots - \overset{v_k}{\Phi}{}^i_{A} \overset{v_k}{|}_i = E_A \tag{2.5}$$

where:

$$\begin{aligned} E_A = {} & \frac{1}{\sqrt{g^{k+1}}}\left\{\frac{\delta\sqrt{g^{k+1}}}{\delta x^i}\overset{h}{\Phi}{}^i_A + \ldots + \frac{\delta\sqrt{g^{k+1}}}{\delta y^{(k)i}}\overset{v_k}{\Phi}{}^i_A\right\} - \\ & -\left\{L^i_{ji} + \frac{\delta N^i_{(1)j}}{\delta y^{(1)i}} + \ldots + \frac{\delta N^i_{(k)j}}{\delta y^{(k)i}}\right\}\overset{h}{\Phi}{}^j_A - \\ & -\left\{C^i_{(1)ji} + \frac{\delta N^i_{(1)j}}{\delta y^{(2)i}} + \ldots + \frac{\delta N^i_{(k-1)j}}{\delta y^{(k)i}}\right\}\overset{v_1}{\Phi}{}^j_A - \ldots - \\ & -\left\{C^i_{(k-1)ji} + \frac{\delta N^i_{(1)j}}{\delta y^{(k)i}}\right\}\overset{v_{k-1}}{\Phi}{}^j_A - C^i_{(k)ji}\overset{v_k}{\Phi}{}^j_A\,. \end{aligned} \tag{2.6}$$

From the invariance of the left-hand side of equations (2.5) relative to the local changes of coordinates, it results that E_A are p scalar fields. Moreover, if the d-linear connection is gauge, then E_A becomes gauge scalar fields. For a particular choice of nonlinear connection and d-linear connection (Berwald type, for instance), the form (2.6) of E_A is simplified.

3. Global gauge invariance of k-order Lagrangians

Let us note that a gauge transformation $T = \{f^{(0)}, f^{(1)}, \ldots f^{(k)}\}$ locally represented by equations (1.6) in the form $\bar{u} = \mathcal{T}(u), u = (x, y^{(1)}, \ldots y^{(k)})$, is globally defined in a set $U \subset GOsc^kM$ and therefore depends implicitly on the elements of the Lie group G.

We shall suppose that dim G=m and $a = (a^1, a^2, \ldots, a^m)$ are the parameters of the group G and that the gauge transformation (1.6) preserves the identity. Hence, a gauge transformation is represented by the equations:

$$(3.1)\qquad \begin{cases} \bar{u}= \mathcal{T}(u,a) \\ u = \mathcal{T}(u,0) \end{cases}$$

Now let us consider an infinitesimal variation of parameters.

Taking the first terms in the Taylor series results in the infinitesimal transformation:

$$(3.2)\qquad \bar{u}= u + \xi_\lambda \epsilon^\lambda,$$

where $\xi_\lambda(u) = \frac{\partial \mathcal{T}}{\partial a^\lambda} |_{a^\lambda=0}$ and $\epsilon^\lambda = \delta a^\lambda$

The variation $\Delta u = \bar{u} - u = \xi_\lambda \epsilon^\lambda$ has the components :

$\Delta y^{(\alpha)i} = \xi_\lambda^{(\alpha)i}\epsilon^\lambda$, where $\xi_\lambda^{(\alpha)i} = \frac{\partial f^{(\alpha)}}{\partial a^\lambda} |_{a^\lambda=0}, \alpha = \overline{0,k}\ , y^{(0)i} = x^i$.

An infinitesimal transformation of the matter field $\Phi^A(u)$ will be

$$(3.3)\quad \bar{\Phi}^A - \Phi^A = \delta\Phi^A = \frac{\partial \phi^A}{\partial y^{(\alpha)i}} \Delta y^{(\alpha)i} = \xi_\lambda^{(\alpha)i} \frac{\partial \phi^A}{\partial y^{(\alpha)i}} \epsilon^\lambda = \left(X_\lambda \Phi^A\right) \epsilon^\lambda.$$

The operators $X_\lambda = \xi_\lambda^{(\alpha)i} \frac{\partial}{\partial y^{(\alpha)i}}$ are the generators of the transformation group and obey the structure equations $[X_\lambda, X_\mu] = C^\nu_{\lambda\mu} X_\nu$, where $C^\nu_{\lambda\mu}$ are the Cartan coefficients.

For all the p fields Φ^A we consider a real representation ρ in a p - dimensional space and $[X_\lambda]^A_B$ be the matrix of X_λ with respect to the representation ρ.

Therefore, an infinitesimal transformation of Φ^A is written:

$$(3.4)\qquad \bar{\Phi}^A = \Phi^A + \delta\Phi^A,$$

where $\delta\Phi^A = \epsilon^\lambda \, [X_\lambda]^A_B \, \Phi^B$.

Definition 3.1. *The transformation (3.2) in which ϵ^λ are constants, $\lambda = \overline{1,m}$, is called global gauge transformation.*

In this paragraph we shall study the invariance of k-order Lagrangians to the infinitesimal transformation (3.4) determined by a global gauge transformation.

Analoguous results can be inferred for $\overset{v_\alpha i}{\Phi}_A$.

From (3.7) it follows:

Theorem 3.1. *The global conservative laws of a k-order Lagrangian are the gauge scalar conditions:*

$$\overset{(h)^i}{J}_{\lambda\,|i} + \overset{(v_1)^i}{J}_{\lambda}\overset{v_1}{|}_i + \ldots + \overset{(v_k)^i}{J}_{\lambda}\overset{v_k}{|}_i = E_A\,[X_\lambda]^A_B\,\Phi^B, \lambda = \overline{1,m}. \tag{3.9}$$

4. Local gauge invariance of k-order Lagrangians.

Let us note that in infinitesimal transformations (3.2) the parameters ϵ^λ of the group can depend on the local point u in the GOsc^{k}M bundle. In this case the transformation (3.2) is called *local* gauge transformation.

The infinitesimal transformation (3.4) will be:

$$\overline{\Phi}^A = \Phi^A + \delta\Phi^A \quad \text{where } \delta\Phi^A = \epsilon^\lambda(u)\cdot[X_\lambda]^A_B\,\Phi^B \tag{4.1}$$

and the variations of the gauge fields $\frac{\delta\Phi^A}{\delta y^{(\alpha)i}}$ are:

$$\delta\left(\frac{\delta\Phi^A}{\delta y^{(\alpha)i}}\right) = \frac{\delta}{\delta y^{(\alpha)i}}(\delta\Phi^A) = \epsilon^\lambda\,[X_\lambda]^A_B\,\Phi^B + [X_\lambda]^A_B\,\Phi^B\,\frac{\delta\epsilon^\lambda}{\delta y^{(\alpha)i}}, \tag{4.2}$$

for $\alpha = \overline{0,k}, y^{(0)i} = x^i$.

The local gauge invariance of a k-order Lagrangian L, δL=0, in addition to (3.6), implies the condition:

$$\left\{\overset{h\,i}{\Phi}_A\,\frac{\delta\epsilon^\lambda}{\delta x^i} + \overset{v_1 i}{\Phi}_A\,\frac{\delta\epsilon^\lambda}{\delta y^{(1)i}} + \ldots + \overset{v_k i}{\Phi}_A\,\frac{\delta\epsilon^\lambda}{\delta y(k)i}\right\}[X_\lambda]^A_B\,\Phi^B = 0 \tag{4.3}$$

Like in classical gauge theories, for obeying also the condition (4.3), a new Lagrangian is considered:

$$L_0(u) = L'(\Phi^A, \frac{\delta\Phi^A}{\delta x^i}, \ldots \frac{\delta\Phi^A}{\delta y^{(k)i}}, H_i^\lambda(u), \overset{(1)^\lambda}{V}_i(u), \ldots \overset{(k)^\lambda}{V}_i(u)) \tag{4.4}$$

in which H_i^λ and V_i^λ are the components of the (k+1) gauge d-covectors, called *local gauge fields,* satisfying the following nonhomogeneity conditions of variations:

$$\delta H_i^\lambda = \epsilon^\mu [P_\mu]_\nu^\lambda \cdot H_i^\nu + \frac{\delta \epsilon^\lambda}{\delta x^i} \tag{4.5}$$

$$\delta \overset{(\alpha)}{V}{}_i^\lambda = \epsilon^\mu \left[\overset{(\alpha)}{Q}_\mu\right]_\nu^\lambda \cdot \overset{(\alpha)}{V}{}_i^\nu + \frac{\delta \epsilon^\lambda}{\delta y^{(\alpha)i}}, \alpha = \overline{1,k}, i = \overline{1,n}, \lambda, \mu, \nu = \overline{1,m}.$$

where $[P_\mu]$ and $\left[\overset{(\alpha)}{Q}\right]$ are still unknown matrices.

The condition of local gauge invariance of L' , $\delta L' = 0$, is:

$$\begin{aligned} &\frac{\partial L'}{\partial \Phi^A} \delta \Phi^A + \frac{\partial L'}{\partial \left(\frac{\delta \Phi^A}{\delta x^i}\right)} \delta \left(\frac{\delta \Phi^A}{\delta x^i}\right) + \ldots + \frac{\partial L'}{\partial \left(\frac{\delta \Phi^A}{\delta y^{(k)i}}\right)} \delta \left(\frac{\delta \Phi^A}{\delta y^{(k)i}}\right) + \\ &+ \frac{\partial L'}{\partial H_i^\lambda} \delta H_i^\lambda + \frac{\partial L'}{\partial \overset{(1)}{V}{}_i^\lambda} \left(\overset{(1)}{V}{}_i^\lambda\right) + \ldots + \frac{\partial L'}{\partial \overset{(k)}{V}{}_i^\lambda} \delta \left(\overset{(k)}{V}{}_i^\lambda\right) = 0 \end{aligned} \tag{4.6}$$

Using the formulas (4.1), (4.2) and (4.5) in (4.6), after separating the independet variations ϵ^λ , $\frac{\delta \epsilon^\lambda}{\delta x^i}$ and $\frac{\delta \epsilon^\lambda}{\delta y^{(\alpha)i}}$, the following local conditions of invariance are obtained:

$$\begin{aligned} &\left\{ \frac{\partial L'}{\partial \Phi^A} \Phi^B + \frac{\partial L'}{\partial \left(\frac{\delta \Phi^A}{\delta x^i}\right)} \frac{\delta \Phi^B}{\delta x^i} + \ldots + \frac{\partial L'}{\partial \left(\frac{\delta \Phi^A}{\delta y^{(k)i}}\right)} \frac{\delta \Phi^B}{\delta y^{(k)i}} \right\} [X_\lambda]_B^A + \\ &+ \frac{\partial L'}{\partial H_i^\mu} [P_\lambda]_\nu^\mu \cdot H_i^\nu + \textstyle\sum_{\alpha=1}^k \frac{\partial L'}{\partial \overset{(\alpha)}{V}{}_i^\mu} \left[\overset{(\alpha)}{Q}_\lambda\right]_\nu^\mu \cdot \overset{(\alpha)}{V}{}_i^A = 0 \end{aligned} \tag{4.7}$$

$$\frac{\partial L'}{\partial \left(\frac{\partial \Phi^A}{\partial x^i}\right)} [X_\lambda]_B^A \, \Phi^B + \frac{\partial L'}{\partial H_i^\lambda} = 0 \tag{4.7'}$$

$$\frac{\partial L'}{\partial \left(\frac{\delta \Phi^A}{\delta y^{(\alpha)i}}\right)} [X_\lambda]_B^A \, \Phi^B + \frac{\partial L'}{\partial \overset{(\alpha)}{V}{}_i^\lambda} = 0, \alpha = \overline{1,k}. \tag{4.7''}$$

The conditions (4.7') and (4.7") suggest that H_i^λ and $\overset{(\alpha)}{V}{}_i^\lambda$ should enter L' as a combination of $\frac{\delta \Phi^A}{\delta x^i}$, respectively $\frac{\delta \Phi^A}{\delta y^{(\alpha)i}}$ and $[X_\lambda]_B^A$.

Let us define the following gauge derivative operators:

$$\overset{h}{\mathbf{D}}_i \, \Phi^A = \frac{\delta \Phi^A}{\delta x^i} - H_i^\lambda [X_\lambda]_B^A \, \Phi^B \tag{4.8}$$

$$\overset{v_\alpha}{\mathbf{D}}_i\, \Phi^A = \frac{\delta \Phi^A}{\delta y^{(\alpha)i}} - \overset{(\alpha)}{V}{}_i^\lambda\, [X_\lambda]^A_B\, \Phi^B\ ,\ \alpha = \overline{1,k}. \tag{4.9}$$

These are gauge tensors because $H_i^\lambda, \overset{(\alpha)}{V}{}_i^\lambda$ and $\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(\alpha)i}}$ are gauge tensors.

Now we can look at the Lagrangian L_0 as being

$$L_0(u) = L"(\Phi^A, \overset{h}{\mathbf{D}}\, \Phi^A, \overset{v_1}{\mathbf{D}}\, \Phi^A, ..., \overset{v_k}{\mathbf{D}}\, \Phi^A) \tag{4.10}$$

The relationships between L' and $L"$ will be:

$$\frac{\partial L'}{\partial \Phi^A} = \frac{\partial L"}{\partial(\overset{h}{\mathbf{D}}_i \Phi^B)} H_i^\lambda\, [X_\lambda]^B_A - \textstyle\sum_{\alpha=1}^k \frac{\partial L"}{\partial(\overset{v_\alpha}{\mathbf{D}}_i \Phi^B)} \cdot \overset{(\alpha)}{V}{}_i^\lambda\, [X_\lambda]^B_A$$

$$\frac{\partial L'}{\partial\left(\frac{\delta \Phi^A}{\delta x^i}\right)} = \frac{\partial L"}{\partial(\overset{h}{\mathbf{D}}_i \Phi^A)} \text{ and } \frac{\partial L'}{\partial\left(\frac{\delta \Phi^A}{\delta y^{(\alpha)i}}\right)} = \frac{\partial L"}{\partial(\overset{v_\alpha}{\mathbf{D}}_i \Phi^A)}$$

$$\frac{\partial L'}{\partial H_i^\lambda} = -\frac{\partial L"}{\partial(\overset{h}{\mathbf{D}}_i \Phi^A)} [X_\lambda]^A_B \text{ and } \frac{\partial L'}{\partial \overset{(\alpha)}{V}{}_i^\lambda} = -\frac{\partial L"}{\partial(\overset{v_\alpha}{\mathbf{D}}_i \Phi^A)} [X_\lambda]^A_B\, \Phi^B.$$

Looking at this relation it is easy to see that the conditions (4.7') and (4.7") will be obeyed by $L"$, and taking into account the expression of the bracket $[X_\lambda, X_\mu]$, the condition (4.7) becomes:

$$\frac{\partial L"}{\partial \Phi^A} [X_\lambda]^A_B\, \Phi^B + \frac{\partial L"}{\partial(\overset{h}{\mathbf{D}}_i \Phi^A)} \left\{[X_\lambda, X_\mu]^A_B - [X_\nu]^A_B \cdot [P_\lambda]^\nu_\mu\right\} \cdot \Phi^B \cdot H_i^\mu +$$

$$+ \frac{\partial L"}{\partial(\overset{h}{\mathbf{D}}_i \Phi^A)} [X_\lambda]^A_B \cdot \overset{h}{\mathbf{D}}_i\, \Phi^B + \textstyle\sum_{\alpha=1}^k \frac{\partial L"}{\partial(\overset{v_\alpha}{\mathbf{D}}_i \Phi^A)} \left\{[X_\lambda, X_\mu]^A_B - [X_\nu]^A_B \cdot \left[\overset{(\alpha)}{Q}{}_\lambda\right]^\nu_\mu\right\} \cdot$$

$$\cdot \Phi^B \cdot \overset{(\alpha)}{V}{}_i^\mu + \textstyle\sum_{\alpha=1}^k \frac{\partial L"}{\partial(\overset{v_\alpha}{\mathbf{D}}_i \Phi^A)} [X_\lambda]^A_B \cdot \overset{v_\alpha}{\mathbf{D}}_i\, \Phi^B = 0$$

This condition is simplifed if the matrices of gauge fields have as elements the structure constants of group G , $[P_\lambda]^\nu_\mu = \left[\overset{(\alpha)}{Q}{}_\lambda\right]^\nu_\mu = C^\nu_{\lambda\mu}$, where $[X_\lambda, X_\mu] = C^\nu_{\lambda\mu} X_\nu$.

It results in the equinvariant condition:

$$\left\{\frac{\partial L"}{\partial \Phi^A}\Phi^B + \frac{\partial L"}{\partial(\overset{h}{\mathbf{D}}_i\, \Phi^A)}\, \overset{h}{\mathbf{D}}_i\, \Phi^B + \sum_{\alpha=1}^{k} \frac{\partial L"}{\partial(\overset{v_\alpha}{\mathbf{D}}_i\, \Phi^A)}\, \overset{v_\alpha}{\mathbf{D}}_i\, \Phi^B\right\} [X_\lambda]^A_B = 0 \tag{4.11}$$

For the chosen matrices $[P_\lambda]$ and $[Q_\lambda]$, using the formulas (4.5), (4.8), and (4.9), the following variation of gauge derivatives is obtained,

$$\delta(\overset{h}{\mathbf{D}}_i\,\Phi^A) = \epsilon^\lambda(u)\,[X_\lambda]^A_B\,\overset{h}{\mathbf{D}}_i\,\Phi^B \text{ and } \delta(\overset{v_\alpha}{\mathbf{D}}_i\,\Phi^A) = \epsilon^\lambda(u)\,[X_\lambda]^A_B\,\overset{v_\alpha}{\mathbf{D}}_i\,\Phi^B. \tag{4.12}$$

Therefore, we have:

Proposition 4.1. *The local gauge invariance of the k-order Lagrangian L'' is deduced from the global gauge invariance of the Lagrangian L.*

The conditions (4.11) and (4.12) are obtained from (3.6) and (3.5) respectively, by replacing $\frac{\delta}{\delta x^i}$, $\frac{\delta}{\delta y^{(\alpha)i}}$ with $\overset{h}{\mathbf{D}}_i$ and $\overset{v_\alpha}{\mathbf{D}}_i$.

However, we want to give a local gauge invariant Lagrangian L$'$ that depends on both $\Phi^A, \frac{\delta\Phi^A}{\delta x^i}, \frac{\delta\Phi^A}{\delta y^{(\alpha)i}}$ and H_i^λ, V_i^λ.

Having this in mind, we act as in classical electromagnetism ([6]). A local gauge invariant Lagrangian depending on H_i^λ and $\overset{(\alpha)}{V_i^\lambda}$ is built up, and according to general theory ([5]) the local gauge invariant Lagrangian L$'$ is the sum of this with L$''$.

Let us consider the brackets ([6]) of two gauge derivatives $\mathbf{D}_j$ and $\mathbf{D}_l$. If $[\mathbf{D}_j, \mathbf{D}_l] = -F^\lambda_{jl} X_\lambda$, where $\mathbf{D}_j$ or $\mathbf{D}_l$ are one of the gauge derivative (4.8) or (4.9), then the (0.2) gauge d-tensors F^λ_{jl} have the following form for any $\lambda = \overline{1, m}$:

$$\overset{(h)}{F^\lambda_{jl}} = \frac{\delta H_l^\lambda}{\delta x^j} - \frac{\delta H_j^\lambda}{\delta x^l} - \frac{1}{2}C^\lambda_{\mu\nu}(H_j^\mu H_l^\nu - H_l^\mu H_j^\nu) \tag{4.13}$$

$$\overset{(h,v_\alpha)}{F^\lambda_{jl}} = \frac{\delta \overset{(\alpha)}{V_l^\lambda}}{\delta x^j_{(\beta)}} - \frac{\delta H_j^\lambda}{\delta y^{(\alpha)l}} - \frac{1}{2}C^\lambda_{\mu\nu}(H_j^\mu \overset{(\alpha)}{V_l^\nu} - H_l^\mu \overset{(\alpha)}{V_j^\nu}) \tag{4.14}$$

$$\overset{(v_\alpha v_\beta)}{F^\lambda_{jl}} = \frac{\delta V_l^\lambda}{\delta y^{(\alpha)j}} - \frac{\delta \overset{(\alpha)}{V_j^\lambda}}{\delta y^{(\beta)l}} - \frac{1}{2}C^\lambda_{\mu\nu}(\overset{(\alpha)}{V_j^\mu}\overset{(\beta)}{V_l^\nu} - \overset{(\beta)}{V_l^\mu}\overset{(\alpha)}{V_j^\nu}) \tag{4.15}$$

Starting from (4.8), direct calculus yields the product $\overset{h}{\mathbf{D}}_j\overset{h}{\mathbf{D}}_l\ \Phi^A$ and the variation of $\overset{(h)}{F_{jl}^{\lambda}}$ of bracket:

$$(4.16)\delta\left(\overset{(h)}{F_{jl}^{\lambda}}\right)=\epsilon^{\nu}(\mu)C_{\mu\nu}^{\lambda}\ \overset{(h)}{F_{jl}^{\mu}},\quad\text{and similar for}\quad\overset{(hv_\alpha)}{F_{jl}^{\lambda}}\quad\text{and}\quad\overset{(v_\alpha,v_\beta)}{F_{jl}^{\lambda}}\ .$$

Following the ideas from electromagnetism, with this and d-gauge tensors (4.13), (4.14), and (4.15) a set of local gauge invariant Lagrangians with respect to H_i^{λ} and $\overset{\alpha}{V_i^{\lambda}}$ can be built up. Let us consider for this the Cartan-Killing metric of group G, $h_{\lambda\mu}=C_{\lambda\theta}^{\nu}.C_{\mu\nu}^{\theta}$ and $G=(g_{ij}(u))$ the metric structure on the $\mathrm{GOsc}^{(k)}$M bundle considering it in Lagrangian density.

By direct calculus, as in classical gauge theory, it is checked up that:

Proposition 4.2 *The following Lagrangians:*

$$(4.17)\qquad \overset{(h)}{L^{(F)}}=-\frac{1}{4}h_{\lambda\mu}g^{ij}g^{lm}\ \overset{(h)}{F_{il}^{\lambda}}\ .\ \overset{(h)}{F_{jm}^{\mu}}$$

$$\overset{(h,v_\alpha)}{L^{(F)}}=-\tfrac{1}{2}h_{\lambda\mu}g^{ij}g^{lm}\ \overset{(hv_\alpha)}{F_{il}^{\lambda}}\ .\ \overset{(hv_\alpha)}{F_{jm}^{\mu}}$$

$$\overset{(v_\alpha,v_\beta)}{L^{(F)}}=-\tfrac{1}{4}h_{\lambda\mu}g^{ij}g^{lm}\ \overset{(v_\alpha v_\beta)}{F_{il}^{\lambda}}\ .\ \overset{(v_\alpha v_\beta)}{F_{jm}^{\mu}}$$

are local gauge invariant Lagrangians with respect to H_i^{λ} , $\overset{(\alpha)}{V_i^{\lambda}}$, *i.e.,*

$$\delta L^{(F)}\left(H_i^{\lambda},\overset{(\alpha)}{V_i^{\lambda}},\frac{\delta H_i^{\lambda}}{\delta x^i},\frac{\delta H_i^{\lambda}}{\delta y^{(\alpha)i}},\frac{\delta \overset{(\alpha)}{V_i^{\lambda}}}{\delta x^i},\frac{\delta \overset{(\alpha)}{V_i^{\lambda}}}{\delta y^{(\beta)i}}\right)=0.$$

Theorem 4.1. *The full local gauge invariant Lagrangian L' from (4.4) is the sum of the Lagrangian L'' obtained as in proposition 4.1 and one of the Lagrangians in (4.17):*

$$(4.18)\qquad \overset{(h)}{L'}=L"+\overset{(h)}{L^{(F)}},\quad \overset{(h,v_\alpha)}{L'}=L"+\overset{(h,v_\alpha)}{L^{(F)}},\quad \overset{(v_\alpha,v_\beta)}{L'}=L"+\overset{(v_\alpha,v_\beta)}{L^{(F)}}$$

References

[1] Asanov G.S., Finsler geometry, relativity and gauge theories, D.Reidel, Dordrecht , 1985.

[2] Balan V., A class of solutions for the generalized gauge Asanov equations, Math. Reports. Academy Eds. Bucharest, 45, 4 (1993), 289-292.

[3] Bejancu A., Foundatios of direction-dependent gauge theories, Seminarul de Mecanica (13).Univ. Timisoara, 1988, 1-60.

[4] Bleecker D., Gauge theory and variational principles, Addison -Wesley Publishing Company, 1981.

[5] Chaichian M., Nelipa N.F., Introduction to gauge field theories, Springer-Verlag, 1984.

[6] Drechsler W., Mayer M.E., Fiber bundle techniques in gauge theories, Lect. Notes in Physics 67, Spriger-Verlag, 1977.

[7] Miron R., Anastasiei M., The Geometry of Lagrange Spaces.Theory and Applications, Kluwer Acad. Publ. 1993.

[8] Miron R., Atanasiu Gh., Compendium sur les espaces Lagrange d'ordre superieur, Seminarul de Mecanica, Univ. Timisoara, 1994, 1-27.

[9] Munteanu Gh., Techniques of higher order osculator bundle in generalized gauge theory, Proc. of Conf. on Diff. Geom. and Appl., Brno , 1995.

[10] Utiyama R., Invariant theoretical interpretation of interaction, Phys. Rev. 101 (1956), 1957-1607.

Received by the editors October 26, 1996.