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A BINARY SEARCH PROBLEM ON GRAPHS

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Abstract

The determination of defective elements in a population by a series of group tests goes back to questions arising in connection with medical examinations during the second world war. Some natural generalizations to graphs have been studied by Aigner, Andreae, Chang, Hwang, Lin and others. In this paper we investigate the following binary variant of search problem: Given a graph G with vertex-set V and edge-set E, and an unknown edge $e^* \in E$. In order to find e^* we choose a sequence of test-sets $A \subseteq V$ where after every test we are told whether e^* has at least one end-vertex in A or none. We are asked to find the minimum number $\bar{c}(G)$ of tests required. Chang, Hwang and Lin have studied $\bar{c}$ for $G = K_{m,n}$ and $G = K_n$ in [5,6]. We extend some of their results to some other classes of graphs, which include planar graphs.

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1. Introduction

Suppose that in a set S with n elements there are exactly d "defective" elements which are unknown. We wish to determine the set of defectives by

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a series of group tests. A test consists of selecting a subset A of S and, as the ontcome of a test, we receive the answer whether at least one of defectives belong to A or none.

Let $f_d(n)$ be the minimal number of tests sufficient to determine the defectives. It is well known that $f_1(n) = \lceil \log_2 n \rceil$. For $d \geq 2$, the question of determining the exact values of $f_d(n)$ is open.

Let $d = 2$ and assume that we have performed some tests on S. Then, certain pairs are already excluded while others are still candidates for being the defectives. Thus we have a graph-structure on S.

This observation leads to the following generalization of the above search problem:

Problem. Given a finite simple graph G with vertex-set $V(G)$ and edge-set $E(G)$, and an unknown edge $e^* \in E(G)$. In order to find e^* we choose a sequence of test-sets $A \subseteq V(G)$ where after each test we are told whether e^* has at least one end-vertex in A, or none. Find the minimum number $\bar{c}(G)$ of tests required.

This problem was studied in [5] for $G = K_{m,n}$, and in [6] and [12] for $G = K_n$. Some ternary variants of one search problem have been studied in [1, 2, 13, 14]. For some early papers see e.g. Dorfman [7], Stevett [11] and Cairns [4]. Bellman and Gluss [3] also studied the problem of identifying two irregular coins in a set of n coins with a balance scale. They wrote: "A small amount of analysis discloses the enormons difference in complexity between the one-coin and the two-coin problem". Katona [10] gives an excellent overview of the subject. Hwang [9] gives a classification of search models under the test-type device.

2. Some definitions and notation

All graphs considered in this paper are simple graphs. For a given graph G we denote by $V(G)$ and $E(G)$ its vertex-set and edge-set respectively. By $\delta_G(v)$ we denote the degree of the vertex $v \in V(G)$. We adopt also the following notation:

$$\delta(G) = \min\{\delta_G(v) \mid v \in V(G)\},$$

$$\Delta(G) = \max\{\delta_G(v) \mid v \in V(G)\},$$

$< W >_G$ is the subgraph of G induced by the set $W \subset V(G)$,

$q(G) =| E(G) |$ is the number of edges of G,

$< F >_G$ is the subgraph of G induced by the set $F \subset E(G)$.

We say that G is a non-trivial graph if $q(G) > 0$.

$\lceil x \rceil$ is the least integer $\geq x$.

Whenever $\log n$ is written without specifying the base we mean the binary logarithm $\log_2 n$.

For all graph terms not defined in the paper the reader is referred to Harary [8].

We introduce now some new definitions.

Definition 1. *Let G be a non-trivial graph. We say that the sequence*

$$s = (G_0, A_1, r_1, G_1, A_2, r_2, G_2, ..., G_{N-1}, A_N, r_n, G_N) \quad (*)$$

is a search procedure on G if

1^0 $G_0 = G$;

2^0 $A_i \subset V(G_{i-1})$, *for* $i = 1, 2, ..., N$;

3^0 $r_i \in \{0, 1\}$, *for* $i = 1, 2, ..., N$;

4^0 $G_i = \begin{cases} < V(G_{i-1}) \backslash A_i >_{G_{i-1}}, & \text{if } r_i = 0 \\ < \{e \in E(G_{i-1}) \mid e \cap A_i \neq \emptyset\} >_{G_{i-1}}, & \text{if } r_i = 1, \end{cases}$
for $i = 1, 2, ..., N$;

5^0 $q(G_i) > 1$, *for* $i = 1, 2, ..., N-1$; $q(G_N) = 1$.

The set A_i we call the i−th test, r_i is the answer to the i−th test, the unique edge e^* of the graph G_N represents the pair of defective items, N is the length of the search procedure s. We usually write: $len(s) = N$.

Definition 2. *The sequence*

$$(q(G_0), q(G_1), ..., q(G_n))$$

is said to be the profile of the search procedure $(*)$.

Obviously, $q(G_{i-1}) \geq q(G_i)$, for $i = 1, 2, ..., n$.

We adopt also the following notation:

$$\text{sub } G = \{< W >_G \mid W \subset V(G), q(< W >_G) > 0\};$$

$$\delta^+(G) = \begin{cases} 0, & \text{if } q(G) = 0 \\ \min\{\delta_G(v) \mid v \in V(G), \delta_G(v) > 0\}, & \text{if } q(G) > 0, \end{cases}$$

$\inf G = k$, if

$1^0 q(G) > 0$,

$2^0 (\forall H \in \text{sub } G)\delta^+(H) \leq k$,

$3^0 (\exists H \in \text{sub } G)\delta^+(H) = k$.

We state the following observations for non-trivial graphs G and H :

(a) $\text{sub } G \neq \emptyset$ iff $q(G) > 0$;

(b) $\inf G = \max\{\delta^+(H) \mid H \in \text{sub } G\}$;

(c) If $q(G) \geq 2$, then $\inf G \leq q(G) - 1$;

(d) If H is a subgraph of G then $\inf H \leq \inf G$.

3. The results

For the proof of Theorem 1 we need two lemmas which also will be used throughout the paper.

Lemma 1. *For each non-trivial graph G there is a set $B \subset V(G)$ such that*

$$\left| \frac{1}{2}q(G) - q(< V(G)\backslash B >_G) \right| \leq \frac{1}{2} \inf G.$$

Proof. Consider the sequence of graphs

$$H_0, H_1, H_2, ..., H_N, H_{N+1}$$

constructed in the following way:

$1^0 \;\; H_0 = G$,

$2^0 \;\; H_{i+1} = H_i - v_i$, where $v_i \in V(H_i)$ such that $\delta_{H_i}(v_i) = \delta^+(H_i)$, for $i = 0, 1, ..., n$,

3^0 $q(H_n) > 0$, $q(H_{n+1}) = 0$.

Since $\delta_{H_i}(v_i) = \delta^+(H_i)$, for $i = 0, 1, ..., n$, and, by construction, $H_i \in$ sub G, taking into account observation (b), it follows that

$$\delta_{H_i}(v_i) \leq \inf G, \tag{1}$$

for $i = 0, 1, ..., n$.

If for some $k \in \{0, 1, ..., n\}$, $q(H_k) = \frac{1}{2}q(G)$, we take $B = V(G) \backslash V(H_k)$.

Suppose now that $q(H_k) \neq \frac{1}{2}q(G)$, for each $k \in \{0, 1, ..., n\}$. Then there is a $k \in \{0, 1, ..., n\}$, such that

$$q(H_k) > \frac{1}{2}q(G) > q(H_{k+1}). \tag{2}$$

If $q(H_k) - \frac{1}{2}q(G) \leq \frac{1}{2}\inf G$, we can take $B = V(G) \backslash V(H_k)$, and the theorem is proved. Otherwise, suppose that

$$q(H_k) - \frac{1}{2}q(G) > \frac{1}{2}\inf G. \tag{3}$$

According to (1) and (2),

$$\delta_{H_k}(v_k) + q(H_{k+1}) < \frac{1}{2}q(G) + \inf G,$$

i.e.,

$$q(H_{k+1}) - \frac{1}{2}q(G) < \inf G - \delta_{H_k}(v_k) \tag{4}$$

By construction $H_{k+1} = H_k - v_k$, hence $q(H_{k+1}) = q(H_k) - \delta_{H_k}(v_k)$, i.e.,

$$q(H_k) = q(H_{k+1}) + \delta_{H_k}(v_k). \tag{5}$$

Combining (3) and (5) we obtain

$$q(H_{k+1}) - \frac{1}{2}q(G) > \frac{1}{2}\inf G - \delta_{H_k}(v_k), \tag{6}$$

and, according to (1) ,

$$q(H_{k+1}) - \frac{1}{2}q(G) > -\frac{1}{2}\inf G. \tag{7}$$

From (2) follows the inequality

$$-q(H_{k+1}) > -\frac{1}{2}q(G),$$

which combined with (6) gives

$$\inf G - \delta_{H_k}(v_k) < \frac{1}{2}\inf G. \tag{8}$$

From (8) and (4) we obtain

$$q(H_{k+1}) - \frac{1}{2}q(G) < \frac{1}{2}\inf G, \tag{9}$$

and from (9) and (7) follows

$$\left| \frac{1}{2}q(G) - q(H_{k+1}) \right| < \frac{1}{2}\inf G.$$

Now follows the statement because we can take $B = V(G)\backslash V(H_{k+1})$. □

The validity of the following statement can be checked by the method of exhaustion.

Lemma 2. *Let G be a non-trivial graph with at most eight edges. Then there is a set $C \subset V(G)$ such that*

$$\left| \frac{1}{2}q(G) - q(< V(G)\backslash C >_G) \right| \leq \frac{1}{2}.$$

Definition 3. *Let G be a nontrivial graph. A search procedure $(*)$ is said to be good (GSP) if for each $i \in \{1, 2, ..., N\}$,*

(a) $q(G_{i-1}) \leq 8 \Rightarrow \left| \frac{1}{2}q(G_{i-1}) - q(G_i) \right| \leq \frac{1}{2}$

and

(b) $q(G_{i-1}) > 8 \Rightarrow \left| \frac{1}{2}q(G_{i-1}) - q(G_i) \right| \leq \frac{1}{2}\inf G_0$.

We denote by $S^*(G)$ the set of all good search procedures on G.

Theorem 1. *For each non-trivial graph G and each edge $e^* \in E(G)$ there is a good search procedure s such that*

$$E(G_{len(s)}) = \{e^*\}.$$

Proof. Let G be a non-trivial graph and $e^* \in E(G)$.

If $q(G) = 1$ then $E(G) = \{e^*\}$, and the corresponding GSP is (G).

If $q(G) > 1$, we construct A_i, r_i and G_i inductively as follows:

Step 1. $G_0 = G$.

Step 2. Let $i > 0$ and $q(G_{i-1}) > 1$. We take:

$$A_i = \begin{cases} B, & \text{if } q(G_{i-1}) > 8 \\ C, & \text{if } q(G_{i-1}) \leq 8, \end{cases}$$

where B and C are the sets constructed according to Lemmas 1 and 2, respectively.

$$r_i = \begin{cases} 0, & \text{if } e^* \cap A_i = \emptyset \\ 1, & \text{if } e^* \cap A_i \neq \emptyset; \end{cases}$$

$$G_i = \begin{cases} < V(G_{i-1}) \backslash A_i >_{G_{i-1}}, & \text{if } r_i = 0 \\ < e \in E(G_{i-1}) \mid e \cap A_i \neq 0\} >_{G_{i-1}}, & \text{if } r_i = 1. \end{cases}$$

We repeat Step 2 while $q(G_i) > 1$.

It is clear that the sequence

$$s = (G_0, A_1, r_1, G_1, A_2, r_2, G_2, ..., G_{N-1}, A_n, r_n, G_N)$$

constructed in this way satisfies the conditions $1^0 - 4^0$ and also the first part of the condition 5^0 of Definition 1. We are going to pove that the second part of the condition 5^0 is also satisfied, i.e., $q(G_N) = 1$.

By construction, $q(G_N) \leq 1$ and it suffices to prove that $q(G_N) \geq 1$. By construction, $q(G_{N-1}) \geq 2$.

If $q(G_{N-1}) > 8$, then according to Lemma 1,

$$(10) \qquad \left| \frac{1}{2} q(G_{N-1}) - q(< V(G_{N-1}) \backslash A_N >_{G_{N-1}}) \right| \leq \frac{1}{2} \inf G_{N-1}.$$

Now, if $r_N = 0$, then $G_N =< V(G_{N-1}) \backslash A_N >_{G_{N-1}}$, and (10) becomes

$$\left| \frac{1}{2} q(G_{N-1}) - q(G_N) \right| \leq \frac{1}{2} \inf G_{N-1},$$

i.e.,

$$(11) \qquad q(G_N) \geq \frac{1}{2}(q(G_{N-1}) - \inf G_{N-1}).$$

Since $q(G_{N-1}) \geq 2$, according to observation (c), $q(G_{N-1}) - \inf G_{N-1} \geq 1$, i.e., $q(G_N) \geq \frac{1}{2}$. Since $q(G_N)$ must be integer, it follows $q(G_N) \geq 1$.

If , on the other hand, $r_N = 1$, then

$$G_N =< \{e \in E(G_{N-1}) \mid e \cap A_n \neq \emptyset\} >_{G_{N-1}},$$

and

$$q(G_N) = q(G_{N-1}) - q(< V(G_{N-1}) \backslash A_n >_{G_{N-1}}).$$

Now, (10) becomes

$$\mid q(G_N) - \frac{1}{2} q(G_{N-1}) \mid \leq \frac{1}{2} \inf G_{N-1},$$

and quite similar as in the case $r_N = 0$, we prove that $q(G_N) \geq 1$.

The proof that $q(G_N) \geq 1$ in the case $q(G_{N-1}) \leq 8$, is quite similar.

So, in any case $q(G_N) = 1$. It means that s is a search procedure on G.

Now, we are going to prove that s is a GSP. For $i \in \{1, 2, ..., N\}$ there are two possible cases: $q(G_i) > 8$ and $q(G_i) \leq 8$.

If $q(G_i) > 8$, then

$$\mid \frac{1}{2} q(G_{i-1}) - q(< V(G_{i-1}) \backslash A_i >_{G_{i-1}}) \mid \leq \frac{1}{2} \inf G_{i-1}. \tag{12}$$

Now, if $r_i = 0$, then $G_i =< V(G_{i-1}) \backslash A_i >_{G_{i-1}}$, and (12) becomes

$$\mid \frac{1}{2} q(G_{i-1}) - q(G_i) \mid \leq \frac{1}{2} \inf G_{i-1}. \tag{13}$$

Obviously, G_{i-1} is a non-trivial subgraph of G_0, and according to observation (d), $\inf G_{i-1} \leq \inf G_0$, so, from (13) we obtain

$$\mid \frac{1}{2} q(G_{i-1}) - q(G_i) \mid \leq \frac{1}{2} \inf G_0. \tag{14}$$

If, however, $r_i = 1$, then

$$G_i =< \{e \in E(G_{i-1}) \mid e \cap A_i \neq \emptyset\} >_{G_{i-1}}, \quad \text{and}$$

$$q(G_i) = q(G_{i-1}) - q(< V(G_{i-1}) \backslash A_i >_{G_{i-1}} .$$

Now, (12) becomes

$$| q(G_i) - \frac{1}{2}q(G_{i-1}) | \leq \frac{1}{2} \inf G_{i-1},$$

and, as in the case $r_i = 0$, we obtain (14).

If $q(G_i) \leq 8$, then in quite similar way as for $q(G_i) > 8$, we prove that

$$| q(G_i) - \frac{1}{2}q(G_{i-1}) | \leq \frac{1}{2}.$$

So, we proved that s is a GSP.

Finally, we need to prove that $E(G_N) = \{e^*\}$. First, we are going to prove that $e^* \in E_i$, for every $i \in \{0, 1, ..., N\}$. The proof is by induction on i.

Obviously, $e^* \in E(G_0)$. Suppose that $e^* \in E(G_i)$, $i < N$. Then, taking into account how the set A_{i+1} and the graph G_{i+1} are constructed, we can easily check that in both cases ($r_{i+1} = 1$ and $r_{i+1} = 0$), $e^* \in E(G_{i+1})$.

So, $e^* \in E(G_N)$. Since $q(G_N) = 1$, it follows that $E(G_N) = \{e^*\}$. □

As a direct consequence of Theorem 1, we have:

Corollary 1. *Let G be a non-trivial graph. Then*

$$\bar{c}(G) \leq \max\{len(s) \mid s \in S^*(G)\}.$$

Lemma 3. *Let*

$$s = (G_0, A_1, r_1, G_1, A_2, r_2, G_2, ..., G_{N-1}, A_N, r_N, G_N)$$

be a GSP on a non-trivial graph G with $\inf G \leq 5$, *and let* $(m_0, m_1, ..., m_N)$ *be the profile of s. Then the following two statements are valid:*

(a) If $k \geq 2$ and $m_k \geq 3$ then $m_{k-2} \geq 9$;

(b) If $k \geq 1$ and $m_k \geq 9$, then $m_{k-1} \geq 2m_k - 5$.

Proof. (a) If $m_k \geq 9$, then obviously $m_{k-2} \geq 9$.

Let $m_k \leq 8$. Then, according to definition 3 (a), $| \frac{1}{2}m_{k-1} - m_k | \leq \frac{1}{2}$, or equivalently,

(15) $$2m_k - 1 \leq m_{k-1} \leq 2m_k + 1.$$

Now, if $5 \leq m_k \leq 8$, then (15) implies $m_{k-1} \geq 9$, and accordingly $m_{k-2} \geq 9$.

If $3 \leq m_k \leq 4$, then (15) implies $5 \leq m_{k-1} \leq 9$. Now, if $m_{k-1} = 9$, then obviously, $m_{k-2} \geq 9$. If $5 \leq m_{k-1} \leq 8$, then according to definition 3 (a), $| \frac{1}{2}m_{k-2} - m_{k-1} | \leq \frac{1}{2}$, or equivalently, $2m_{k-1} - 1 \leq m_{k-2} \leq 2m_{k-1} + 1$. Since $m_{k-1} \geq 5$, it follows $m_{k-2} \geq 9$.

(b) If $m_k \geq 9$, then according to definition 3 (b), $| \frac{1}{2}m_{k-1} - m_k | \leq \frac{1}{2} \inf G$, or, equivalently

(16) $$m_{k-1} \geq 2m_k - \inf G.$$

Taking into account that $\inf G_0 = \inf G \leq 5$, from (16) follows $m_{k-1} \geq 2m_k - 5$. □

Theorem 2. *Let G be a non-trivial graph such that* $\inf G \leq 5$. *Then, for each $s \in S^*(G)$,*

$$len(s) \leq \lceil \log_2 q(G) \rceil + 1.$$

Proof. Let

$$s = (G_0, A_1, r_1, G_1, A_2, r_2, G_2, ..., G_{N-1}, A_N, r_N, G_N)$$

be a GSP on G and let $(m_0, m_1, ..., m_N)$ be the profile of s.

Let $L = \lceil \log_2 q(G) \rceil$. If $L \leq 2$, then $q(G) \leq 4$, and the profile of s is one of the following five sequences: $(4,2,1); (3,2,1); (3,1); (2,1); (1)$. In each of these five cases, $N \leq L$.

Let $L \geq 3$ and $N \geq L$. (For $N < L$ the statement is obviously true). We are going to show that $m_L \leq 2$.

Suppose, on the contrary, that $m_L \geq 3$. Then, according to Lemma 3 (a),

(17) $$m_{L-2} \geq 9.$$

Since $L-2 \geq 1$ and $m_{L-2} \geq 9$, from Lemma 3 (b) if follows that

(18) $$m_{L-3} \geq 2m_{L-2} - 5.$$

Taking into account that $m_{i-1} \geq m_i$, (17) and (18) give us the following system of inequalityes:

$$m_0 \geq 2m_1 - 5$$

$$m_1 \geq 2m_2 - 5$$
$$\vdots$$
$$m_{L-3} \geq 2m_{L-2} - 5$$
$$m_{L-2} \geq 9.$$

Multiplying the inequality $m_{i-1} \geq 2m_i - 5$ by 2^i (for $i = 1, 2, ..., L-1$), we obtain:

$$m_0 \geq 9 \cdot 2^{L-2} - 5(1 + 2 + ... + 2^{L-3}),$$

or equivalently,

$$m_0 \geq 9 \cdot 2^{L-2} - 5 \cdot 2^{L-2} + 5,$$

and finally, $m_0 \geq 2^L + 5$.

On the other hand,

$$L = \lceil \log_2 q(G) \rceil = \lceil \log_2 m_0 \rceil,$$

i.e., $m_0 \leq 2^L$. Contradiction. Thus we proved that $m_L \leq 2$.

If $m_L = 1$, then $N = L$. If $m_L = 2$ then $m_{L+1} = 1$, and $N = L + 1$.

In any case, $N = len(s) \leq L + 1$. □

Corollary 2. *Let G be a non-trivial graph such that* $\inf G \leq 5$. *Then*

$$\bar{c}(G) \leq \lceil \log_2 q(G) \rceil + 1.$$

Proof. Follows from Theorem 2, taking into account that

$$\bar{c}(G) \leq \max\{len(s) \mid s \in S^*(G)\}. \quad \square$$

Corollary 3. *Let G be a non-trivial graph such that* $\Delta(G) \leq 5$. *Then*

$$\bar{c}(G) \leq \lceil \log_2 q(G) \rceil + 1.$$

Proof. Follows from Corollary 2, taking into account that $\inf G \leq \Delta(G)$. □

Corollary 4. *Let G be a non-trivial planar graph. Then*

$$\bar{c}(G) \leq \lceil \log_2 q(G) \rceil + 1.$$

Proof. If G is a planar graph, then each $H \in \text{sub}\, G$ is also a planar graph. It is known that each planar graph has at least one vertex of degree ≤ 5. According to observation (b), $\inf G \leq 5$. Now, the statement follows from Corollary 2. □

The most interesting open questions in connection with this search variant concern optimal graphs. Let us call a graph $G = (V, E)$ with at least two edges 2-optimal (Aigner [1]) if

$$\bar{c}(G) = \lceil \log_2 q(G) \rceil,$$

i.e., if $\bar{c}(G)$ achives the information theoretical lower bound. For complete bipartite graphs Chang and Hwang have proved in [5] that these graphs are optimal. Aigner in [1] posed the corresponding conjecture for all bipartite graphs. We close with the conjecture for all planar graphs.

Conjecture. Every planar graph is 2-optimal.

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REZIME

JEDAN PROBLEM BINARNOG TRAŽENJA U GRAFOVIMA

U radu se ispituje sledeća binarna varijanta traženja u grafu: Dat je graf G sa skupom čvorova V i skupom grana E, i nepoznata grana $e^* \in E$. Da bi odredili e^* biramo niz test-skupova $A \subseteq V$, gde posle svakog testa dobijamo odgovor da li je bar jedan čvor incidentan sa e^* u skupu A ili ne. Treba odrediti minimalan broj $\bar{c}(G)$ testova. Chang, Hwang i Lin su izučavali broj $\bar{c}$ za $G = K_{m,n}$ i $G = K_n$. Mi proširujemo njihove rezultate na neke druge klase grafova, uključujući planarne.

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