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PLANNING OF THE TRAJECTORY FOR THE TIP OF NON-REDUNDANT ROBOTIC MECHANISMS IN THE PRESENCE OF OBSTACLES

Dušan Surla, Miloš Racković[1]
Institute of Mathematics, University of Novi Sad
Trg Dositeja Obradovića 4, 21000 Novi Sad, Yugoslavia

Abstract

An algorithm is formed for the generation of a set of admissible trajectories for the tip of robotic mechanisms. On the basis of this set, using an optimisation method based on the ψ-transform, a near-optimal admissible trajectory for the mechanism tip is determined.

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Key words and phrases: Robotic mechanism, admissible trajectory.

1. Introduction

The problem of determining the admissible region and admissible trajectory for the tip of the two-link mechanism in the presence of obstacles has been considered in several papers [1-4]. Determination of the admissible region is based on the following statement: If the position of the two-link mechanism is admissible when the mechanism describes a closed Jordan curve, then, the mechanism position is also admissible when mechanism tip belongs to the interior region of this curve. This statement served as the basis for the algorithmic structures for determination of the admissible region [1, 2, 4].

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In this way, two-link mechanism has been reduced to the determination of the shortest path between the points in the given admissible region [3, 5].

In the present work we consider the problem of determining the trajectory for the tip of non-redundant mechanisms, by optimisation of the given optimality criterion. The shortest distance between the initial and terminal position of the mechanism tip has been adopted as the optimality criterion. The method used in [6] for the determination of the shortest polygonal line between the two given points in the presence of the polygonal obstacles has also been used in this work, whereas the problem in the absence of the obstacles was dealt with in [7].

2. Robotic mechanisms

Let $T \in R^d$ be the tip and $q \in R^d$ the vector generalized coordinates of the mechanism (R^d is a d-dimensional Euclidean space). Then, in a general case, the relation between T and q is of the form:

$$T = f(q) \tag{1}$$

where $f : R^d \to R^d$. Let a small change in the position of the mechanism tip ΔT causes a small change in the vector Δq. Then, for a given ΔT, the increment Δq can be calculated from the relation:

$$\Delta q = (\partial f(q)/\partial q)^{-1} \Delta T \tag{2}$$

where $\partial f(q)/\partial q$ is the Jacobian matrix.

Let us consider two examples of robotic mechanisms, one in R^2, and the other in R^3.

Let O be a fixed point in a plane. Consider the linkage OAT, such that $|OA| = l_1$, $|AT| = l_2$. By the rotation about the center O, the point A of the segment OA can reach any point of the circle (O, l_1). Similarly, by the rotation about A, the end point T of the segment AT can reach any point of the circle (A, l_2).

The coordinates of point T (Fig. 1) are:

$$T_x = l_1 \cos q_1 + l_2 \cos(q_1 + q_2) \tag{3}$$

$$T_y = l_1 \sin q_1 + l_2 \sin(q_1 + q_2) \tag{4}$$

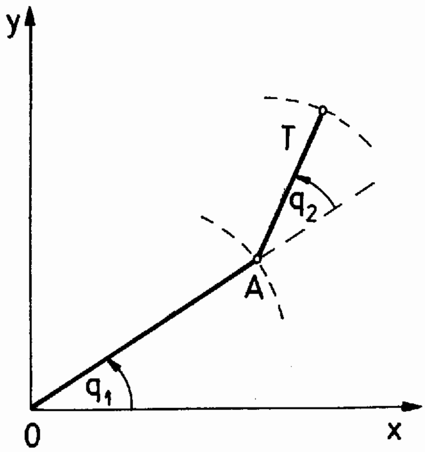

Fig. 1

In Fig. 2 is illustrated a mechanism having three degrees of freedom, for which relation (1) assumes the following form:

(5) $$T_x = (l_1 \cos q_2 + l_2 \cos(q_2 + q_3)) \cos q_1$$

(6) $$T_y = (l_1 \cos q_2 + l_2 \cos(q_2 + q_3)) \sin q_1$$

(7) $$T_z = -l_1 \sin q_2 - l_2 \sin(q_2 + q_3)$$

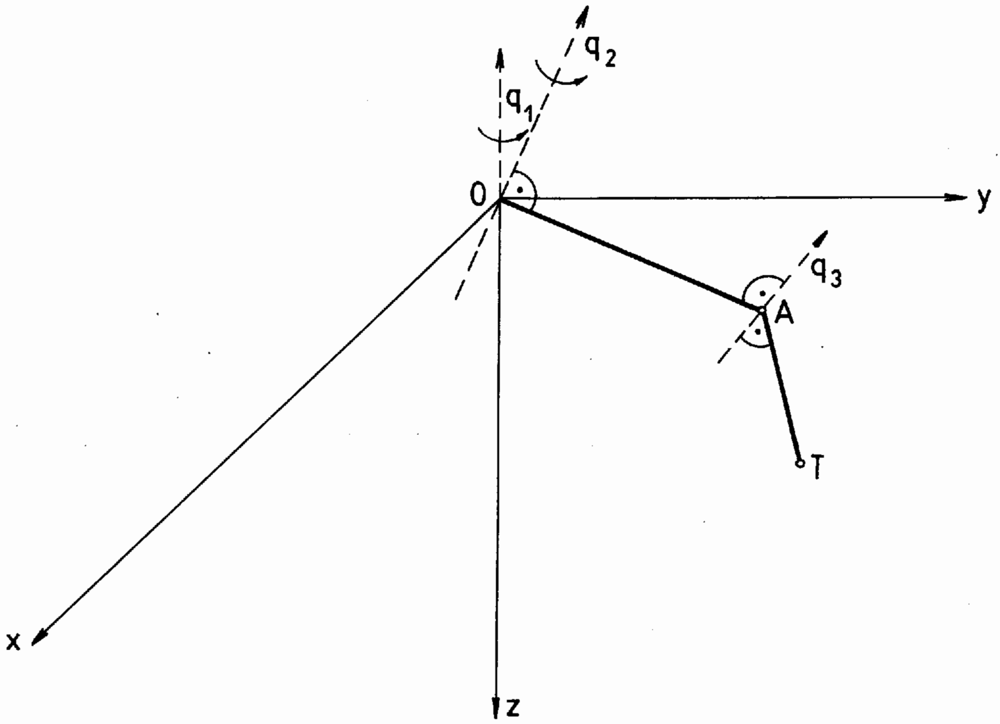

Fig. 2

3. Generating of admissible trajectories for the mechanism tip

Let us denote by $O_i, i = 1, 2, ..., m$ the obstacles, and by X and Y the initial and terminal position of the mechanism tip, respectively. Let $\Omega \subset R^d$ be the space in which X and Y are located. The task consists of the following: generate in Ω a polygonal line $XZ_1Z_2...Z_nY$, such that the intersection of the mechanisms links and the interior regions of the given obstacles, while the mechanism tip is tracking this polygonal line, is an empty set. The points $Z_i \in \Omega, i = 1, 2, ..., n$ will be produced by the generator of uniform distribution random numbers. First, the point Z_1 is generated, and the segment XZ_1 is divided into $k+1$ parts $XZ_1^1Z_1^2...Z_1kZ_1$. Then, for each partition point, the admissibility of the mechanism tip is checked. The mechanism position is admissible if the partition point is attainable for the mechanism tip, and if

$$OA\bigcap O_i = \emptyset \wedge AX\bigcap O_i = \emptyset, i = 1, 2, ..., m \tag{8}$$

Now, the procedure is repeated for each partition point of the segment XZ_1. If the condition (8) is satisfied for each of the partition points, then the point Z_2 is generated and the procedure repeated for the segment Z_1Z_2. If for any partition point of the segment XZ_1 the condition (8) is not fulfilled, a new point Z_1 is generated, and the procedure id repeated from the beginning. Thus, the admissible polygonal line $XZ_1...Z_nY$ can be generated by a back-track algorithm. The algorithm structure is as follows:

```
procedure admissiblepath(X,Y,O,m,n,maxbr,path,ind,dd);
  Input: points X and Y; obstacles O[i], i=1,2,...,m;
  n - the number of vertices of the  admissible path;
  maxbr - the number of attempts to generate each of
  vertices of the admissible path; dd - the parameter
  of the partition of the polygonal line.
  Output: path - the admissible polygonal line; ind -
  indicator, if ind=false, an admissible path has not
  been found.
begin
  path[0]:=X; path[n+1]:=Y;
  ind:=true; ist:=1; nuatt[ist]:=1;
  while ind and (ist<=n+1) do
    begin
      i:=1; ok:=true;
      if ist < n+1 then
        randu(path[ist]);
          (* generates an arbitrary point  under  the
             given constraints *)
      while ok and (i<=m) do
        begin
          k:=trunc(dd*distance(path[ist-1], path[ist]);
          j:=0;
```

```
          while ok and (j<=k) do
            begin
              generate(A[j],j,ok);
                (* A[j], the joint coordinates at the
                   j-th partition point *)
              if ok and (inters(A[j],O[i]) <> 0) then
                ok:=false;
              j:=j+1
            end;
          i:=i+1
        end;
      if ok then
        begin
          ist:=ist+1;
          nuatt[ist]:=1
        end
      else
        if nuatt[ist] >= maxbr then
          if (ist=1) then
            ind:=false
          else
            ist:=ist-1
        else
          nuatt[ist]:=nuatt[ist]+1
    end
end; (* admissiblepath *)

procedure generate(A,j,ok);
begin
  move(path[ist-1],path[ist],j,dd,v);
    (* Determine the point v by translating it  for
       (1/dd)*j along seg(path[ist-1],path[ist]) *)
  if (attainable(v)) then
    (* Checks if the point v is attainable for  the
       mechanism tip *)
    begin
      increment(path[ist-1],j,v,DV);
       (* Determines the increment in the mechanism
          tip DV *)
```

```
        change(DV,Dq);
        (* Via the inverse Jacobian matrix calculates
           the vector change Dq *)
        q:=q+Dq;
        position(q,A)
      (* Determines A via the formula for calculation
         of the mechanism position via the vector  q.
         The tip position thus  calculated  should be
         sufficiently close to the point v calculated
         by the procedure of  its  translation  along
         the segment *)
      end
    else
      ok:=false
  end; (* generate *)
```

4. The procedure for determining the global minimum using ψ-transform

Given $F(z), z \in R^d$ and $f : R^d \to R$, find z^* to satisfy

$$F(z^*) = \min_{z\in\Omega} F(z) \tag{9}$$

where $\Omega \subset R^d$. The determination of z^* by ψ-transform consists of the following [8]. Let

$$E_\xi^* = \{z : z \in \Omega, F(z) \le \xi\} \tag{10}$$

The function $\psi : R \to R_+$ is defined in the following way:

$$\psi(\xi) = m(E_\xi^*) \tag{11}$$

where $m(E_\xi^*)$ is the Lebeagne measure over set E_ξ^*. If $\xi^* = F(z^*)$, then

$$\psi(\xi^*) = 0, z^* = z(\xi^*) \tag{12}$$

On the basis of the above, z^* can be determined numerically in the following way. Let us generate the uniformly distributed random vectors $z^i \in \Omega, i = 1, 2, ..., s$ and calculates $F_i, i = 1, 2, ..., s$. On the basis of the values of the elements of matrix Z, whose rows are the vectors $z^i, i = 1, 2, ..., s$ and $F_i, i = 1, 2, ..., s$ the functions (12) are approximated, and from them z^* is calculated. This procedure is denoted by:
procedure minpsi(Z, F, s, z^*).

5. The algorithm for the determining the near-optimal trajectory of mechanism tip

Let $z_{d(i-1)+j}$ =path[i,j], for $i = 1,2,...,n, j = 1,2,...,d$. Then, according to the Euclidean norm, the length of the path, path[i] $i = 0,2,...,n+1$ obtained by the admissible path procedure can be calculated from the expression:

$$
\begin{array}{rcl}
F(z) & = & \sqrt{\sum_{j=1}^{d}(z_j - X_j)^2}+ \\
(13) \qquad & + & \sum_{i=1}^{n-1} \sqrt{\sum_{j=1}^{d}(z_{d(i+1)+j} - z_{d\cdot i+j})^2}+ \\
& + & \sqrt{\sum_{j=1}^{d}(Y_j - z_{(n-1)d+j})^2}
\end{array}
$$

Hence, the task is reduced to the determination of the vector x of dimension $d{\cdot}n$, so that function (13) has its minimum, and the corresponding polygonal line is admissible. Using the procedures described above, the algorithm for determining the shortest distance becomes of the form:

```
   Input: points X and Y; obstacles O[i], i=1,2,...,m
   n - the number of vertices of the admissible path.
   Output:  path[i],  i=0,1,...,n+1 -  the admissible
   polygonal  line; ind - indicator, if ind=false, an
   admissible path has not been found.
 begin
   for i:=1 to s do
     begin
       admissiblepath(X,Y,O,m,n,maxbr,path,ind,dd);
       if not ind then exit;
       F[i]:=distance(path[0],path[1]);
       for j:=1 to n do
         begin
           for l:=1 to d do
             z[i,d*(j-1)+l]:=path[j+1];
           F[i]:=F[i]+distance(path[j],path[j+1])
         end
     end;
   minpsi(z,F,s,k,1,2*n,minz,ind1);
   if not ind1 then exit;
   i:=1; path[0]:=X; path[n+1]:=Y; minF:=0;
   while ind and (i<=n+1) do
```

```
    begin
      for l:=1 to d do
        path[i,l]:=minz[d*(i-1)+l];
      minF:=minF + distance(path[i-1],path[i]);
      j:=1;
      while ind and (j<=m) do
        begin
          k:=trunc(dd*distance(path[i-1], path[i]);
          l:=0;
          while ok ind (l<=k) do
            begin
              generate(A[l],l,ind);
              if ind and (inters(A[j],O[i]) <> 0) then
                ind:=false;
              l:=l+1
            end;
          j:=j+1
        end;
      i:=i+1
    end
end.
```

6. Numerical examples

Example 1. We consider the mechanism presented in Fig. 1. The obstacles are in the form of the simple polygons. Let us denote by $E_i, i = 1, 2, ..., m$ the edges of the given polygons. The mechanism position is admissible if:

$$(OA \bigcup AT) \bigcap E_i = \emptyset, i = 1, 2, ..., m \tag{14}$$

Because of the assumption that the point O is outside the interior regions of the given polygons, this statement holds. On the basis of this statement, a procedure is formed for testing the intersecting between the position of the two-link mechanism and the polygonal obstacles. The algorithmic structures were implemented in PASCAL. The input are as follows. The vertices of the firs polygon are: $(-4, 2), (-3, 3), (-3, 4), (-4, 4)$; the vertices of the second polygon are: $(0, 6), (1, 4), (5, 7)$; the vertices of the third polygon are: $(2, -2), (-2, -3), (1, -4), (3, -4)$; the lengths of the mechanism links are:

$l_1 = 4, l_2 = 2$; the initial position of the mechanism tip X is $(-1, 5)$, and the terminal point Y is $(3, 4)$. Fig. 3 gives an illustration of four different cases of this example. In the first case, there is only one point between two points X and Y, in the second case there are two points, and so on. The corresponding numerical values from Fig. 3, together with the calculated admissible path, are presented in Table 1.

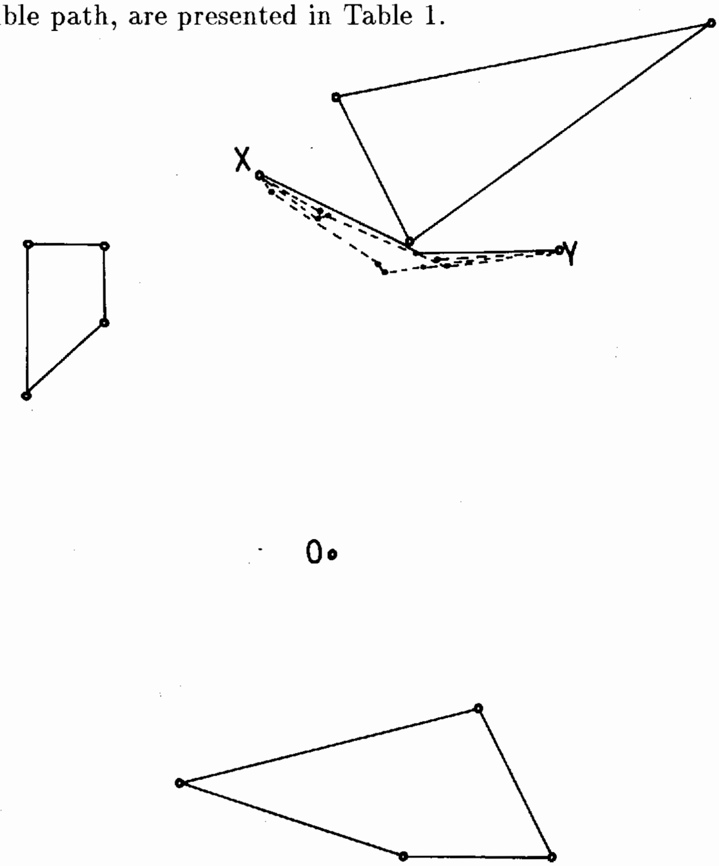

Fig. 3

Table 1.

Path	Points between X and Y	Path length
1	(1.10,3.93)	4.26
2	(-0.23,4.43),(1.37,3.82)	4.31
3	(-0.22,4.39),(-0.18,4.41),(1.5,3.81)	4.33
4	(-0.9,4.72),(0.58,3.79),(0.63,3.69),(1.19,3.72)	4.55

Example 2. In this example we shall consider the mechanism shown in Fig. 2. In this case the obstacles are given in the form of simple polyhedra. The procedure of testing the admissibility of a mechanism position is reduced to the determination of the intersection between the segment and the simple polyhedron. The algorithm for solving this problem consists of the following [9]. First, two definitions are introduced, namely:

Definition 6.1. *The intersection point between straight line r and the hull of simple polyhedron P is a piercing point if at this point r passes from the interior into the exterior domain of P, or vice versa.*

Definition 6.2. *The edge of the simple polygon F, lying on the straight line r is a piercing edge if one vertex of this edge borders upon the internal and the other on the external region of F.*

Then, the following theorem is proved:

Theorem 6.1. *Point R belongs to the interior region of F if on the same side of the point R lying on the straight line r, the sum of piercing points and piercing edges is an odd number.*

On the basis of this theorem, a procedure is formed for testing the intersection between the mechanism position and the given simple polyhedra.

The input data are:
Data structure of the simple polyhedra is given in Tables 2. - 4.

Table 2.

Ord. no. of vert.	Polyhedron P
1	(1,1,1)
2	(0,0,3.1)
3	(1,0,3.1)
4	(3.5,3.5,3.1)
5	(0,1,3.1)

Table 3.

Ord. no. of edge	Edge deter. by vertic.
1	1,2
2	1,3
3	1,4
4	1,5
5	2,3
6	3,4
7	4,5
8	5,2

Table 4.

Ord. no. of facet	Fac. deter. by ord. vert.
1	1,2,3
2	1,3,4
3	1,4,5
4	1,2,5
5	2,3,4,5

The initial point for the mechanism tip X is $(3,0,3)$, and terminal point Y is $(0,3,3)$. The link lengths are: $l_1 = 3, l_2 = 3$. The initial vector is $q = (0,0,-\pi/2)$. In the present example we consider two cases, with one and two points between X and Y. The output data are presented in Table 5.

Table 5.

Path	Points between X and Y	Path length
1	(2.08,2.30,1.75)	5.30
2	(2.13,0.48,1.99),(1.69,2.10,1.29)	5.80

7. Conclusion

The numerical approach employed may be successfully used to solve the problem of planning of movements for non-redundant mechanisms in the presence of the obstacles. The shortest path of the mechanism tip between the initial and terminal mechanism position may be adopted as optimality criterion, ψ-transform may be used for its optimisation. The algorithmic structure proposed is of general character. In concrete cases, the corresponding procedure for testing the intersection of geometrical models of the robot links and the obstacles is to be used.

References

[1] Tošić R., Surla D., The set of the admissible positions for a two-linkage in the presence of obstacles, Z. angew. Math. Mech. (to appear).

[2] Surla D., Konjović Z., Determination of the collision-free region for two-link mechanism in the presence of obstacles, Automatika 29 (1988), 1-2, A.363-A.369.

[3] Surla D., Racković M., An algorithm for determining the shortest path for two-link mechanism tip between two points in the presence of polygonal obstacles, XXXIV Yugoslav Conference of ETAN, Zagreb, (1990),IX-X-XI 119-124 (in Serbian).

[4] Divljak N., An algorithm for determining the admissible region for a two-link mechanism in the presence of polygonal obstacles, Zb. Rad. Prirod.-Mat. Univ. u Novom Sadu (to appear).

[5] Racković M., Surla D., An algorithm for determining the shortest path in a plane with polygonal obstacles, Proc. of 14th Information Technologies Conference, Sarajevo-Jahorina, (1990), 284.1-284.8 (in Serbian).

[6] Surla D., Racković M., An algorithm for determining near-optimal distance between two points in a plane containing obstacles, Automatika 31 (1990), 1-2, 55-58.

[7] Surla D., Konjović Z., Synthesis of the optimal nominal control for manipulation robots by using ψ-transformation, Inter. Sym. MIM-S, '90, Brussels, (1990), IV.B.6.1-IV.B.6.6.

[8] Chichinadze V. K., The ψ-transform for solving linear and nonlinear programming problems, Automatica, 5, (1965), 347-355.

[9] Surla D., Budimac Z., Detection of the intersection of two simple polyhedra, Informatica 1(1990), 49-54.

REZIME

PLANIRANJE TRAJEKTORIJE ZA VRH NEREDUNDANTNIH ROBOTSKIH MEHANIZAMA U PRISUSTVU PREPREKA

Formiran je algoritam za generisanje skupa dopustivih trajektorija vrha robotskih mehanizama. Na osnovu tog skupa, koristeći optimizacioni metod zasnovan na ψ-transformaciji, odredjuje se skoro optimalna dopustiva trajektorija vrha robotskog mehanizma.

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