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ON THE TOTALLY REAL MINIMAL SUBMANIFOLDS IN $HP^m(1)$

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Abstract

In this paper, we obtain some pinching theorems for compact totally real minimal submanifolds in $HP^m(1)$.

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1. Introduction

Let M be an n-dimensional compact minimal submanifold in a sphere S^{n+p} and σ be the second fundamental form of M.J. Simons [12] and Chern-Do-Carmo-Kobayashi [2] proved that: if $|\sigma|^2 \leq n/(2-1/p)$, then M is either totally geodesic or a Veronese surface in $S^4(1)$. In [9], we improved the above pinching constant to $n(3n-2)/(5n-4)$. Recently, Xu-Chen [15] improved the above pinching constant to $2n/3$.

Let M be an n-dimensional compact totally real minimal submanifold of a complex projective space $CP^n(c)$ and σ be the second fundamental form of M. Chen-Ogiue [1], Naitoh-Takeuchi [7] and Yau [16] proved that: if $|\sigma|^2 \leq n(n+1)c/4(2n-1)$, then M is either totally geodesic or a finite Riemannian covering of the unique flat torus minimally embedded in $CP^2(c)$. In [8], we improved above pinching constant to $n(n+1)(3n-2)c/4(5n-4)$. Recently, Xia [14] improved the above pinching constant to $(n+1)c/6$.

Let M be an n-dimensional compact totally real minimal submanifold of a quaternion projective space $HP^m(1)$ and σ be the second fundamental form of M. In this paper, by use of the similar way of Xu-Chen [15] and Xia [14] we establish some pinching theorems for n-dimensional compact totally real minimal submanifold of $HP^m(1)$. In fact, we have

Theorem 1. *Let M be an n-dimensional compact totally real minimal submanifold in $HP^n(1)$ and σ be the second fundamental form of M. If $|\sigma|^2 \leq (n+1)/6$, then M is either totally geodesic or a finite Riemannian covering of the unique flat torus embedded in $HP^2(1)$ with parallel second fundamental form.*

Theorem 2. *Let M be an n-dimensional compact totally real minimal submanifold in $HP^m(1)$. If $|\sigma|^2 \leq n/6$, then either M is totally geodesic or the immersion of M into $HP^m(1)$ is one of the following immersions:*

$$\psi_1 : RP^2(1/12) \to HP^4(1); \;\; \psi_5 = \psi \circ \pi,$$

where $\pi : S^2(1/12) \to RP^2(1/12)$ is the covering map.

Let $\delta(u) = |\delta(u,u)|^2$ for $u \in UM$. In [3], Coulton-Gauchman obtained the following result (c.f. [5, 6]).

Theorem 3. *Let M be an n-dimensional compact totally real minimal submanifold in $HP^m(1)$. If $\delta \leq 1/12$ for all $u \in UM$, if and only if one of the following conditions is satisfied:*

(*i*) *$\delta(u) \equiv 0$ and M is totally geodesic in $HP^m(1)$.*

(*ii*) *$\delta \equiv 1/12$ and the immersion of M into $HP^m(1)$ is one of the following immersions: $\psi_1 : RP^2(1/2) \to HP^4(1)$; $\psi_2 : CP^2(1/3) \to HP^7(1)$; $\psi_3 : HP^2(1/3) \to HP^{13}(1)$; $\psi_4 : CayP^2(1/3) \to HP^{25}(1)$ $\psi_5 : S^2(1/12) \to HP^4(1)$;*

For the definition of $\psi_i (i = 1, ..., 5)$, one can consult [3,p. 298].

In this paper, we will give a simple proof of above Coulton-Gauchman's result and prove the following:

Theorem 4. *Let M be a n-dimensional compact totally real minimal submanifold in $HP^m(1)$. Assume that n is odd. If $\delta(u) \le 1/4(3 - 2/n)$ for all $u \in UM$, then M is totally geodesic.*

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2. Prelimilaries

Let M be an n-dimensional compact Riemannian manifold. We denote by UM the unit tangent bundle over M and by UM_p its fiber over $p \in M$. If dp, dv and dv_p denote the canonical measures on M, UM and UM_p respectively, then for any continuous function $f : UM \to R$, we have

$$\int_{UM} f dv = \int_M [\int_{UM_p} f dv_p] dp.$$

Now, we suppose that M is isometrically immersed in an m-dimensional Riemannian manifold $\bar{M}$. We denote by $<,>$ the metric of $\bar{M}$ and metric of M. If σ domorphism associated to a normal vestor ξ, we define

$$L : T_pM \to T_pM \quad \text{and} \quad T_p^{\perp}M \times T_p^{\perp}M \to R$$

by the expressions

$$Lv = \sum_{i=1}^{n} A_{\sigma(v,e_i)} e_i \quad \text{and} \quad T(\xi, \eta) = \text{trace} A_\xi A_\eta,$$

where $T_p^{\perp}M$ is the normal space to M at p and $e_1, ..., e_n$ is an orthonormal basis of T_pM. M is called a *curvature-invariant* submanifold of $\bar{M}$ (see [11]), if $\bar{R}(X,Y)Z \in T_pM$ for all $X, Y, Z \in T_pM$, $\bar{R}$ being the curvature operator of $\bar{M}$.

Lemma 1. *[11] Let M be an n-dimensional compact minimal curvature-invariant submanifold in a m-dimensional Riemannian manifold $\bar{M}$. Then*

$$(2.1) \quad 0 = \frac{n+4}{3} \int_{UM} |(\nabla\sigma)(v,v,v)|^2 dv + (n+4) \int_{UM} |A_{\sigma(v,v)} v|^2 dv$$

$$-4\int_{UM} < Lv, A_{\sigma(v,v)}v > dv - 2\int_{UM} T(\sigma(v,v),\sigma(v,v))dv$$

$$+\int_{UM}\sum_{i=1}^{n}[\bar{R}(e_i,v,\sigma(v,e_i),\sigma(v,v)) + 2\bar{R}(e_i,v,v,A_{\sigma(v,e_i)}v)]dv.$$

$$(2.2) \qquad \int_{UM_p} < Lv, A_{\sigma(v,v)}v > dv_p = \frac{2}{n+2}\int_{UM_p} |Lv|^2 dv_p$$

for any $p \in M$.

By [3], the curvature operator of quaternion projective space $HP^{n+p}(1)$ is

$$(2.3) \qquad \bar{R}(X,Y)Z = \frac{1}{4}[\Lambda(Y,Z)X - \Lambda(X,Z)Y - 2\Gamma(X,Y)Z],$$

where

$$\Lambda(Y,Z)X = < Y,Z > X + \sum_{i=1}^{3} < J_iY,Z > J_iX,$$

$$\Gamma(X,Y)Z = \sum_{i=1}^{3} < J_iX,Y > J_iZ, \quad J_i^2 = -Id \quad (i = 1,2,3),$$

$$J_1J_2 = -J_2J_1 = J_3, \quad J_2J_3 = -J_3J_2 = J_1, \quad J_3J_1 = -J_1J_3 = J_2.$$

We say that M is a totally real submanifold of $HP^m(1)$, [4], if $\Theta(T_pM)\perp T_pM$ for any $p \in M$ and any $\Theta \in V_p$, where V_p is the fiber of $V = [J_1, J_2, J_3]$ over p (see [4]).

Let $T_i(X,Y,Z) = < \sigma(X,Y), J_iZ >$, $(i = 1,2,3)$. We have

Lemma 2. *([13] or [3])* $T_i(X,Y,Z)$ *is symmetric in all three arguments for each* $i = 1,2,3$.

3. Maximal directions

Let M be an n-dimensional compact curvature-invariant minimal submanifold in $\bar{M}$. Define $S = [(u,v) \mid u,v \in UM_p, p \in M]$ and a function f on S by

$$(3.1) \qquad f(u,v) = |\sigma(u,u) - \sigma(v,v)|^2.$$

For any $p \in M$, we can take $(\bar{u}, \bar{v}) \in UM_p \times UM_p$ with $<\bar{u}, \bar{v}>= 0$, such that $f(\bar{u}, \bar{v}) = \max_{(u,v)\in UM_p \times UM_p} f(u, v)$ (see [14, p.144]). We shall call such a pair $(\bar{u}, \bar{v})$ a *maximal direction* at p.

Lemma 3. *[14] Let $p \in M$ and assume that $\max_{(u,v)\in UM_p\times U\,M_p} f(u,v) \neq 0$. Take an orthonormal basis $e_1, ..., e_n$ of T_pM such that (e_1, e_n) is a maximal direction at $p, e_1, ..., e_n$ diagonalizes A_ξ, $\xi = [\sigma(e_1, e_1) - \sigma(e_n, e_n)]/|\sigma(e_1, e_1) - \sigma(e_n, e_n)|$ and that $\lambda_1 =: <\sigma(e_1, e_1), \xi> \geq \lambda_2 =: <\sigma(e_2, e_2), \xi> \geq ... \geq \lambda_n =: <\sigma(e_n, e_n), \xi>$. Then, at the point p, it holds*

$$(3.2)\sum_{i=1}^{n} < \sigma(e_1, e_1) - \sigma(e_n, e_n), \nabla^2\sigma(e_i, e_i, e_1, e_1) - \nabla^2\sigma(e_i, e_i, e_n, e_n) >$$

$$\geq |\sigma(e_1, e_1) - \sigma(e_n, e_n)| \sum_{i=1}^{n}[\bar{R}(e_i, e_1, \sigma(e_1, e_i), \xi) - \bar{R}(e_i, e_n, \sigma(e_i, e_n), \xi)$$
$$+(\lambda_1 - \lambda_i)\bar{R}(e_i, e_1, e_1, e_i) - (\lambda_n - \lambda_i)\bar{R}(e_i, e_n, e_n, e_i)]$$
$$-\frac{3}{2}|\sigma(e_1, e_1) - \sigma(e_n, e_n)|^2 \cdot |\sigma|^2.$$

Proof of Theorem 1. Let L be a function on M defined by

$$L(x) = \max_{(u,v)\in UM_\infty \times UM_\infty} f(u, v).$$

Following an idea in [10] we prove that L is a constant function on M by using the maximal principle. It suffices to show that L is subharmonic in the generalized sense. Fix $p \in M$, let (e_1, e_n) be a maximal direction at p and $e_1, ..., e_n$ be an orthonormal basis of T_pM as stated in Lemma 3. From (2.3) and Lemma 2, we have

$$(3.3) \qquad |\sigma(e_1, e_1) - \sigma(e_n, e_n)| \sum_{i=1}^{n}[\bar{R}(e_i, e_1, \sigma(e_1, e_i), \xi)$$

$$-\bar{R}(e_i, e_n, \sigma(e_i, e_n), \xi) + (\lambda_1 - \lambda_i)\bar{R}(e_i, e_1, e_1, e_i) - (\lambda_n - \lambda_i)\bar{R}(e_i, e_n, e_n, e_i)]$$

$$= \frac{1}{4}\sum_{i=1}^{n}\sum_{k=1}^{3} < \sigma(e_1, e_1) - \sigma(e_n, e_n), J_k e_i > + \frac{n}{4}(\lambda_1 - \lambda_n)|\sigma(e_1, e_1) - \sigma(e_n, e_n)|$$

$$= \frac{n+1}{4}|\sigma(e_1, e_1) - \sigma(e_n, e_n)|^2.$$

In an open neighborhood U_p of p within the cut-locus of p we shall denote by $E_1(x)$ (resp. $E_n(x)$) the tangent vectors to M obtained by parallel transport of $e_1 = E_1(p)$ (resp. $e_n = E_n(p)$) along the unique geodesic joining x to p. Define $g_p(x) = |\sigma(E_1(x), E_1(x)) - \sigma(E_n(x), E_n(x))|^2$. Then

$$(3.4) \qquad \frac{1}{2}\Delta g_p(p) = \sum_{i=1}^{n}[|(\nabla\sigma)(e_i, e_1, e_1) - (\nabla\sigma)(e_i, e_n, e_n)|^2$$

$$+ < \sigma(e_1, e_1) - \sigma(e_n, e_n), (\nabla^2\sigma)(e_i, e_i, e_1, e_1) - (\nabla^2\sigma)(e_i, e_i, e_n, e_n) > .$$

If $|\sigma(e_1, e_1) - \sigma(e_n, e_n)| = 0$, then $\Delta g_p(p) \geq 0$ by (3.4). If $|\sigma(e_1, e_1) - \sigma(e_n, e_n)| \neq 0$, then, by (3.3), (3.4), Lemma 3 and hypothesis on $|\sigma|^2$, we have

$$\frac{1}{2}\Delta g_p(p) \geq |\sigma(e_1, e_1) - \sigma(e_n, e_n)|^2(\frac{n+1}{4} - \frac{3}{2}|\sigma|^2) \geq 0.$$

For the Laplacian of continuous functions, we have the generalized definition

$$\Delta L = a \lim_{r\to 0}\frac{1}{r}((\int_{B(r,p)} L/\int_{B(p,r)} 1) - L(p)),$$

where a is positive constant and $B(p, r)$ denotes the geodesic ball of radius r with center p. With this definition L is subharmonic on M if and only if $\Delta L(p) \geq 0$ at each point $p \in M$. Since $g_p(p) = L(p)$ and $g_p \leq L$ on U_p, $\Delta L(p) \geq \Delta g_p(p) \geq 0$. Thus, L is subharmonic and hence $L = b$ =constant on M. When $b = 0$, M is totally geodesic. When $b \neq 0$, it is easy to see that $|\sigma|^2 \equiv (n+1)/6$ on M and that for any $p \in M$, by the fact that equality holds in (3.2), the orthonormal bases $e_1, ..., e_n$ of T_pM further satisfies (c.f. [14])

$$(3.5) \qquad \sigma(e_1, e_i) = \sigma(e_n, e_i) = \sigma(e_i, e_j) = 0, \;\; 2 \leq i, j \leq n-1,$$

$$(3.6) \qquad |\sigma(e_1, e_1)|^2 = |\sigma(e_n, e_n)|^2 = |\sigma(e_1, e_n)|^2 = \frac{n+1}{24},$$

$$(3.7) \qquad \sigma(e_1, e_1) = -\sigma(e_n, e_n).$$

Marking the similar discussion as proof of Theorem 1 of [14], we can conclude that M has parallel second fundamental form by use of (2.3) and (3.5)-(3.7). Theorem 1 now follows from the classification of n-dimensional totally real minimal submanifolds in $HP^n(1)$ with parallel second fundamental from by K.Tsukada in [13].

Proof of Theorem 2. As in the proof of Theorem 1, we show that the function $L(p) = \max_{(u,v)\in UM_p\times U M_p} f(u,v)$ is subharmonic in the generalized sense. For any $p \in M$, let $e_1, ..., e_n$ be an orthonormal basis of T_pM as in Lemma 3 such that (e_1, e_n) is a maximal direction at p. Then

$$(3.8) \qquad |\sigma(e_1,e_1)-\sigma(e_n,e_n)|\sum_{i=1}^{n}[\bar{R}(e_i,e_1,\sigma(e_1,e_i),\xi)$$

$$-\bar{R}(e_i,e_n,\sigma(e_i,e_n),\xi)+(\lambda_1-\lambda_i)\bar{R}(e_i,e_1,e_1,e_i)-(\lambda_n-\lambda_i)\bar{R}(e_i,e_n,e_n,e_i)]$$

$$=\frac{1}{4}\sum_{i=1}^{n}\sum_{k=1}^{3}<\sigma(e_1,e_1)-\sigma(e_n,e_n),J_ke_i>+\frac{n}{4}(\lambda_1-\lambda_n)|\sigma(e_1,e_1)-\sigma(e_n,e_n)|$$

$$\geq \frac{n}{4}|\sigma(e_1,e_1)-\sigma(e_n,e_n)|^2.$$

Let g_p be the function defined as in the proof of Theorem 1. Then from (3.8), Lemma 3 and $|\sigma|^2 \leq n/6$, we have $\Delta g_p(p) \geq 0$. By the same arguments as in the proof of Theorem 1, we know that L is subharmonic (and so L =constant on M) and that either $|\sigma|^2 \equiv 0$ or $|\sigma|^2 \equiv n/6$. When $|\sigma|^2 \equiv n/6$, the orthonormal basis $e_1, ..., e_n$ of T_pM satisfies

$$(3.9) \qquad \sigma(e_1,e_i)=\sigma(e_n,e_i)=\sigma(e_i,e_j)=0, \ \ 2\leq i,j\leq n-1,$$

$$(3.10) \qquad |\sigma(e_1,e_1)|^2=|\sigma(e_n,e_n)|^2=|\sigma(e_1,e_n)|^2=\frac{n}{24},$$

$$(3.11) \qquad \sigma(e_1,e_1)=-\sigma(e_n,e_n).$$

$$(3.12) \qquad <\sigma(X,Y),J_iZ>=0,$$

for $i = 1,2,3$ and $X, Y, Z \in T_pM$, $p \in M$.

We can conclude that M has the parallel second fundamental form in the similar discussion as proof of Theorem 1 (c.f. [14]). All totally real minimal submanifolds in $HP^m(1)$ with parallel second fundamental form were classified by K.Tsukada [13]. There are two possible types of such immersions, which are denoted as (R - R)-type and (R - C)- type (Proposition 3.2, [13]). It follows from (3.12) that our immersion is not of (R - C)-type. On the other hand, we can deduce by using a similar argument as in [2,p.70] that $n = 2$, it is easy to see from (3.9)-(3.11) that M is $\sqrt{1/12}$–isotropic. Theorem 2 follows from the classification of (R - R)-type totally real isotropic minimal surfaces with parallel second fundamental form in $HP^m(1)$ by K.Tsukada in [13].

Proof of Theorem 3. Let $p \in M$ and $e_1, ..., e_n$ be an orthonormal basis of T_pM, from (2.3), we have

$$(3.13) \qquad \sum_{i=1}^{n}[\bar{R}(e_i, v, \sigma(v, e_i), \sigma(v, v)) + 2\bar{R}(e_i, v, v, A_{\sigma(v,e_i)}v)]$$

$$= \frac{1}{2} < Lv, v > -\frac{1}{2}|\sigma(v, v)|^2 + \frac{1}{4}\sum_{i=1}^{n}\sum_{k=1}^{3} < \sigma(v, v), J_k e_i >^2 .$$

From (2.2) and Holder's inequality,

$$\frac{2}{n+2}\int_{UM_p} |Lv|^2 dv_p \leq [\int_{UM_p} |Lv|^2 dv_p]^{1/2}.$$

$$(3.14) \qquad \cdot [\int_{UM_p} |A_{\sigma(v,v)}v|^2 dv_p]^{1/2}, \quad \text{or}$$

$$(3.15) \qquad \int_{UM_p} |A_{\sigma(v,v)}v|^2 dv_p \geq \frac{2}{n+2}\int_{UM_p} < Lv, A_{\sigma(v,v)}v > dv_p.$$

Substituting (3.13) and (3.15) into (2.1), we obtain

$$(3.16) \quad 0 = \frac{n+4}{3}\int_{UM} |(\nabla\sigma)(v, v, v)|^2 dv + (n+4)\int_{UM} |A_{\sigma(v,v)}v|^2 dv$$

$$-4\int_{UM} < Lv, A_{\sigma(v,v)}v > dv - 2\int_{UM} T(\sigma(v, v), \sigma(v, v))dv$$

$$+ \int_{UM}[\frac{1}{2} < Lv, v > -\frac{1}{2}|\sigma(v, v)|^2 + \frac{1}{4}\sum_{i=1}^{n}\sum_{k=1}^{3} < \sigma(v, v), J_k e_i >^2]dv$$

$$\geq \frac{n+4}{3}\int_{UM} |(\nabla\sigma)(v, v, v)|^2 dv + \frac{n}{4}\int_{UM} |\sigma(v, v)|^2 dv$$

$$-n\int_{UM} |A_{\sigma(v,v)}v|^2 dv - 2\int_{UM} T(\sigma(v, v), \sigma(v, v))dv.$$

For any $v \in UM$, we can put $\sigma(v, v) = |\sigma(v, v)|\xi$ for some unit vector ξ normal to M. Since $|\sigma(v, v)|^2 \leq 1/12$ for any $v \in UM$, we have by Schwartz's inequality

$$(3.17) \qquad |A_\xi u|^2 \leq (\text{maximum eigenvalue of } A_\xi)^2 \leq 1/12$$

for any $u \in UM$. Hence

$$\frac{n}{4}|\sigma(v,v)|^2 - n|A_{\sigma(v,v)}v|^2 - 2T(\sigma(v,v),\sigma(v,v)) \tag{3.18}$$

$$= |\sigma(v,v)|^2\left(\frac{n}{4} - n|A_\xi v|^2 - 2\sum_{i=1}^{n} < A_\xi e_i, A_\xi e_i >\right) \geq 0,$$

where $e_1, ..., e_n$ is a locally orthonormal basis of TM. It follows from (3.16) and (3.18) that M has parallel second fundamental form,

$$< \sigma(X,Y), J_k Z >= 0, \tag{3.19}$$

for $k = 1,2,3$ and any $X, Y, Z \in TM$, and equalities hold in (3.15) and (3.18). Hence, we have

$$|A_{\sigma(v,v)}v|^2 = \frac{1}{12}|\sigma(v,v)|^2, \quad Lv = \frac{n+2}{2}A_{\sigma(v,v)}v. \tag{3.20}$$

From (3.19), we know that M is of type (R - R) ([13]). Now given $p \in M$, let ω be the 1-form on UM_p defined by

$$\omega_v(e) =< \sigma(v,v), \sigma(v,e) > |\sigma(v,v)|^2$$

for all $v \in UM_p$, $e \in T_v UM_p$. Integrating on UM_p the codifferential of ω, we have (also see [14])

$$(n+6)\int_{UM_p} |\sigma(v,v)|^4 dv_p = 4\int_{UM_p} |A_{\sigma(v,v)}v|^2 dv_p + \tag{3.21}$$

$$+2\int_{UM_p} < Lv, v > |\sigma(v,v)|^2 dv_p.$$

Substituting (3.20) into (3.21), we find

$$\int_{UM} |\sigma(v,v)|^2(1/12 - |\sigma(v,v)|^2)dv = 0. \tag{3.22}$$

Since $|\sigma(v,v)|^2 \leq 1/12$ for any $v \in UM$, we derive from (3.22) that either $|\sigma(v,v)| \equiv 0$ (i.e., M is totally geodesic) or $|\sigma|^2 \equiv 1/12$. When $|\sigma|^2 \equiv 1/12$, we conclude from the classifications of isotropic (R - R)-type totally real minimal submanifolds with parallel second fundamental from in $HP^m(1)$ ([13]) that the immersion of M into $HP^m(1)$ is one of the following immersions:

$\psi_1 : RP^2(1/2) \rightarrow HP^4(1)$; $\psi_2 : CP^2(1/3) \rightarrow HP^7(1)$; $\psi_3 : HP^2(1/3) \rightarrow HP^{13}(1)$; $\psi_4 : CayP^2(1/3) \rightarrow HP^{25}(1)$; $\psi_5 : S^2(1/12) \rightarrow HP^4(1)$; This complete the proof of Theorem 3.

Proof of Theorem 4. Let $v \in UM_p$ and $\sigma(v,v) = |\sigma(v,v)|\xi$. Take an orthonormal basis $e_1, ..., e_n$ of T_pM such that $A_\xi e_i = \lambda_i e_i, \ \ i = 1, ..., n$. Then

$$\sum_{i=1}^{n} \lambda_i = 0. \tag{3.23}$$

Denote by $K = \max_i \lambda_i^2$. Since n is odd, it follows from [6,p.256] that

$$\sum_{i=1}^{n} < A_\xi e_i, A_\xi e_i >= \sum_{i=1}^{n} \lambda_i^2 \leq (n-1)K \leq \frac{n-1}{4(3-2/n)}. \tag{3.24}$$

Using the same arguments as in the proof of Theorem 3 and the hypothesis: $|\sigma(v,v)|^2 \leq 1/4(3-2/n)$, we conclude that M is (R - R)-type totally real minimal submanifolds with parallel second fundamental form and either $|\sigma(v,v)|^2 \equiv 0$ or $|\sigma(v,v)|^2 \equiv 1/4(3-2/n)$ on UM. Using the classifications of the isotropic (R - R)-type totally real minimal submanifolds with parallel second fundamental form in a quaternion projective $HP^m(1)$ by K.Tsukada ([13]), we know that the case $|\sigma|^2 = 1/4(3-2/n)$ can not occur. Thus M is totally geodesic. This complete the proof of Theorem 4.

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REZIME

O TOTALNO REALNOJ MINIMALNOJ
PODMNOGOSTRUKOSTI U $HP^m(1)$

U ovom radu su dokazane neke granične teoreme za kompaktne totalno realne minimalne podmnogostrukosti u $HP^m(1)$.

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