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JACOBI APPROXIMATE SOLUTION OF THE BOUNDARY LAYER PROBLEM

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Abstract

The two point selfadjoint boundary layer problem, described by the second order differential equation, is considered. Standard spectral approximation is adapted to the character of the exact solution and is constructed according to the Jacobi orthogonal basis. An error estimate is provided and theoretical results are supported by a numerical example.

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1. Introduction

We shall consider the selfadjoint boundary layer problem

$$Ly \equiv -\epsilon^2 y''(x) + g(x)y(x) = f(x), \;\; x \in [a,b] \tag{1.1}$$

$$Gy \equiv (y(a), y(b)) = (A, B), \tag{1.2}$$

where $\epsilon > 0$ is a small parameter, $f(x),\; g(x) \in C^2[a,b],\; g(x) \geq K^2 > 0$. The solution of (1.1), (1.2) describes the stationary state of the evolution equation

$$y_t' - \varepsilon^2 y'' + g^0(x,t)y = f^0(x,t),\; x \in [a,b],\; t > 0,$$

$$y(a,t) = A,\; y(b,t) = B,\; t > 0,\; y(x,0) = y^0(x),\; x \in [a,b],$$

which arises in convection-diffusion flow problems. It is well known, (see e.g. [3]), that problem (1.1), (1.2) is inverse monotone and that it has a unique solution $y(x) \in C^2[a,b]$, which represents a stable state of the evolution problem.

In this paper we want to construct the spectral approximation for the solution of problem (1.1), (1.2), adapting it to the character of boundary layer and using Jacobi orthogonal basis $\{P_k^{\alpha,\beta}(t),\ \alpha > -1,\ \beta > -1\}$ generated by the differential equation

$$(1.3)\qquad (1-t^2)\theta''(t) + (\beta - \alpha - (\alpha+\beta+2)t)\theta'(t) + \\ +k(k+\alpha+\beta+1)\theta(t) = 0, \quad t \in [-1,1]$$

or Bonnet's recurrence relation

$$(1.4)\qquad P_{k+1}^{\alpha,\beta}(t) - (\alpha_k t + \beta_k)P_k^{\alpha,\beta}(t) + \gamma_k P_{k-1}^{\alpha,\beta}(t) = 0,\ k = 0,1,\ldots, \\ P_0^{\alpha,\beta}(t) = 1,\ P_{-1}^{\alpha,\beta}(t) = 0,$$

with

$$(1.5)\qquad \begin{aligned} \alpha_k &= \frac{(2k+\alpha+\beta+1)(2k+\alpha+\beta+2)}{2(k+1)(k+\alpha+\beta+1)}, \\ \beta_k &= \frac{(2k+\alpha+\beta+1)(\alpha^2-\beta^2)}{2(k+1)(k+\alpha+\beta+1)(2k+\alpha+\beta)}, \\ \gamma_k &= \frac{(k+\alpha)(k+\beta)(2k+\alpha+\beta+2)}{(k+1)(k+\alpha+\beta+1)(2k+\alpha+\beta)}. \end{aligned}$$

Jacobi polynomials represent the largest class of classical orthogonal polynomials defined on a finite interval. As the special cases, they include: Legendre polynomials ($\alpha = \beta = 0$), Chebyshev polynomials of the first and second kind ($\alpha = \beta = -1/2$) and ($\alpha = \beta = 1/2$) and Gegenbauer polynomials ($\alpha = \beta = \sigma - 1/2,\ \sigma > -1/2$). Thus, this paper generalizes some previous works by the author, where the Chebyshev and Legendre basis were applied.

In the first part of the paper, we shall transform the original problem (1.1), (1.2) into a more convenient form, which enables the application of direct spectral method. In the second part we shall develop some reccurence relations and, finally, construct a system of linear equations for evaluating the coefficientes of the truncated orthogonal series which represents the spectral approximation. In the third part we shall estimate the error and, at the end, we shall give a numerical example which ilustrates the theoretical results.

2. Transformation of the problem

It is well known that the solution of (1.1), (1.2), in general, has two boundary layers,one at each endpoint,and that the solution of the reduced problem is

$$y_R(x) = \frac{f(x)}{g(x)}. \tag{2.1}$$

We are going to look for the solution of (1.1), (1.2) in the form

$$y(x) = y_R(x) + u(x), \tag{2.2}$$

where $u(x)$ satisfies

$$Lu = \epsilon^2 y_R''(x),\ Gu = (A^0, B^0),\ A^0 = A - y_R(a),\ B^0 = B - y_R(b). \tag{2.3}$$

The first step is to approximate function $u(x)$ by

$$\tilde{u}(x) = \begin{cases} v(x) & x \in [a, a+\delta] \\ 0 & x \in [a+\delta, b-\delta], \\ w(x) & x \in [b-\delta, b] \end{cases} \tag{2.4}$$

where $\delta > 0$ is the appropriately chosen division point.
Functions $v(x)$ and $w(x)$ are the solutions of the following problems

$$Lv = \epsilon^2 y_R''(x),\ x \in [a, a+\delta],\ G^0 v \equiv (v(a), v(a+\delta)) = (A^0, 0) \tag{2.5}$$

$$Lw = \epsilon^2 y_R''(x),\ x \in [b-\delta, b],\ G^n w \equiv (w(b-\delta), w(b)) = (0, B^0). \tag{2.6}$$

Practically, (2.4) means that $y(x) \approx y_R(x)$ when $x \in [a+\delta, b-\delta]$ and, according to [6], the following estimate is valid

$$d(x) = |y(x) - y_R(x)| \le (M_1 + M_2)\exp(-K\delta/\epsilon) + M_3\epsilon^2, \tag{2.7}$$

where M_1, M_2, M_3 denote the constants independent of x and ϵ, which satisfy

$$M_1 \ge |A^0|,\ M_2 \ge |B^0|,\ M_3 \ge |K^{-2} y_R''(x)| \text{ while } x \in (a,b). \tag{2.8}$$

So, we are going to determine the division number δ from the request

$$d(x) \le C^2\epsilon^2, \tag{2.9}$$

where C can be any constant which satisfies $C^2 \ge 2M_3$.
This gives

$$\delta \ge \frac{\epsilon}{K}[2\ln\frac{1}{\epsilon} + \ln(|A^0|, |B^0|) - 2\ln C]. \tag{2.10}$$

For the practical use we shall take the smallest value for δ in (2.8), which will provide the intervals $[a, a+\delta]$ and $[b-\delta, b]$ to be sufficiently small and, thus, we shall be able to avoid the "stiff" solution in further procedure. The second step is to construct the spectral approximations for functions $v(x)$ and $w(x)$. As the procedure is similar in both cases, we shall develop it only for $v(x)$. First, we have to transform subinterval $[a, a+\delta]$ into $[-1,1]$ using the substitution

$$x = \frac{1}{2}\delta t + a + \frac{1}{2}\delta, \tag{2.11}$$

which (2.5) transforms into

$$L^0 V \equiv -\mu^2 V''(t) + G(t)V(t) = \epsilon^2 Y_R(t), \quad t \in [-1,1], \tag{2.12}$$

$$\Gamma^0 V \equiv (V(-1), V(1)) = (A^0, 0), \tag{2.13}$$

where

$$\begin{gathered} \mu = 2\epsilon/\delta,\ V(t) = v(\tfrac{1}{2}\delta t + a + \tfrac{1}{2}\delta),\ G(t) = g(\tfrac{1}{2}\delta t + a + \tfrac{1}{2}\delta), \\ Y_R(t) = y_R''(\tfrac{1}{2}\delta t + a + \tfrac{1}{2}\delta). \end{gathered} \tag{2.14}$$

3. Construction of the approximate solution

When speaking of the Jacobi spectral approximation for the solution $V(t)$ of the problem (2.12), (2.13), obtained applying the direct method, we, in fact, consider a truncated orthogonal series according to the Jacobi basis

$$V_n(t) = \sum_{k=0}^{n} a_k P_k^{\alpha,\beta}(t), \tag{3.1}$$

such that

$$L^0 V_n = \epsilon^2 Y_R(t),\ t \in [-1,1],\ \Gamma^0 V_n = (A^0, 0). \tag{3.2}$$

Expressing the second derivative of $V_n(t)$ as

$$V_n''(t) = \sum_{k=2}^{n} a_k (P_k^{\alpha,\beta}(t))'' \tag{3.3}$$

the equations in (3.2) become

$$-\mu^2 \sum_{k=2}^{n} a_k (P_k^{\alpha,\beta}(t))'' + G(t) \sum_{k=0}^{n} a_k P_k^{\alpha,\beta}(t) = \epsilon^2 Y_R(t) \tag{3.4}$$

$$\sum_{k=0}^{n} a_k P_k^{\alpha,\beta}(-1) = A^0, \ \sum_{k=0}^{n} a_k P_k^{\alpha,\beta}(1) = 0. \tag{3.5}$$

Further, we have to overcome two difficulties: first to express $P_k^{\alpha,\beta}(t)''$ through $P_k^{\alpha,\beta}(t)$ and second, to obtain the product $G(t)V_n(t)$ and the function $\epsilon^2 Y_R(t)$ as the truncated Jacobi series. To that purpose we shall approximate $G(t)$ by the power series

$$G(t) \approx G^n(t) = \sum_{j=o}^{n} h_j t^j \tag{3.6}$$

and $Y_R(t)$ by the finite Jacobi series

$$Y_R(t) \approx Y^n(t) = \sum_{k=0}^{n} r_k P_k^{\alpha,\beta}(t). \tag{3.7}$$

Now, we can prove the following lemmas:

Lemma 1. *For each $j, k \in N$ we have*

$$t^j P_k^{\alpha,\beta}(t) = \sum_{i=k-j}^{k+j} A_i^j P_i^{\alpha,\beta}(t), \tag{3.8}$$

with

$$A_i^j = \frac{A_{i-1}^{j-1}}{\alpha_{i-1}} - \beta_i \frac{A_i^{j-1}}{\alpha_i} + \gamma_i \frac{A_{i+1}^{j-1}}{\alpha_{i+1}} \ for\ i = k-j+2, \ldots, k+j-2,$$

$$A_{k-j+1}^j = -\beta_{k-j+1} \frac{A_{k-j+1}^{j-1}}{\alpha_{k-j+1}} + \gamma_{k-j+2} \frac{A_{k-j+2}^{j-1}}{\alpha_{k-j+2}},$$

(3.9)

$$A_{k+j-1}^j = \frac{A_{k+j-2}^{j-1}}{\alpha_{k+j-2}} - \beta_{k+j-1} \frac{A_{k+j-1}^{j-1}}{\alpha_{k+j-1}},$$

$$A_{k-j}^j = \gamma_{k-j+1} \frac{A_{k-j+1}^{j-1}}{\alpha_{k-j+1}}, \ A_{k+j}^j = \frac{A_{k+j-1}^{j-1}}{\alpha_{k+j-1}},$$

where α_i, β_i and γ_i are constants (1.5).

Proof. From Bonnet's relation (1.4) we obtain

$$tP_i^{\alpha,\beta}(t) = A_{i+1}^1 P_{i+1}^{\alpha,\beta}(t) + A_i^1 P_i^{\alpha,\beta}(t) + A_{i-1}^1 P_{i-1}^{\alpha,\beta}(t), \tag{3.10}$$

where

$$A_{i+1}^1 = \frac{1}{\alpha_i},\ A_i^1 = \frac{-\beta_i}{\alpha_i},\ A_{i-1}^1 = \frac{\gamma_i}{\alpha_i}.$$

Multiplying (3.10) by t (j-1) times, using notation (3.9) each time, we finally come to (3.8).

□

Lemma 2. *The second derivative of the Jacobi polynomial $P_k^{\alpha,\beta}(t)$ may be represented as*

$$(P_k^{\alpha,\beta}(t))'' = \sum_{i=0}^{k-2} b_i^{(2)} P_i^{\alpha,\beta}(t) \quad k = 2, 3, \ldots, \tag{3.11}$$

where the coefficients $b_i^{(2)}$, $i = 0, \ldots, k-2$ are determined recursively by the relations (3.15), (3.16).

Proof. The first derivative of $P_k^{\alpha,\beta}(t)$ is a polynomial of $k-1$ degree, which means that it can be represented exactly as a linear combination of the elements of any basis of space π_{k-1} (the space of all the real polynomials of degree up to $k-1$). Thus, using the Jacobi basis, we get

$$(P_k^{\alpha,\beta}(t))' = \sum_{i=0}^{k-1} b_i^{(1)} P_i^{\alpha,\beta}(t). \tag{3.12}$$

On the other hand, for all the classical orthogonal polynomials $\theta_k(t)$ we have

$$A(t)\theta_k'(t) = (u_k t + v_k)\theta_k(t) - w_k\theta_{k-1}(t) \tag{3.13}$$

(see [4]), where, for the Jacobi polynomials,

$$A(t) = 1 - t^2,\ u_k = -k,\ v_k = \frac{k(\alpha-\beta)}{(2k+\alpha+\beta)},$$

(3.14)

$$w_k = -\frac{2(k+\alpha)(k+\beta)}{(2k+\alpha+\beta)}.$$

Introducing (3.12) in (3.13) and making use of (3.8) for $j = 2$, after equating the coefficients at $P_i^{\alpha,\beta}(t)$, we come to

$$b_{k-1}^{(1)} = -\frac{u_k A_{k+1}^1}{A_{k+1}^2},\ b_{k-2}^{(1)} = -\frac{u_k A_k^1}{A_k^2} - \frac{v_k}{A_k^2} - b_{k-1}^{(1)},$$

$$b_{k-3}^{(1)} = -\frac{u_k A_{k-1}^1}{A_{k-1}^2} + \frac{w_k}{A_{k-1}^2} - b_{k-2}^{(1)} - b_{k-1}^{(1)}(1 - \frac{1}{A_{k-1}^2}), \tag{3.15}$$

$$b_{i-2}^{(1)} = -b_{i-1}^{(1)} - b_i^{(1)}(1 - \frac{1}{A_i^2}) - b_{i+1}^{(1)} - b_{i+2}^{(1)} \quad i = k-2, \ldots, 2.$$

Further on, we are going to substitute (3.11) and (3.12) into (1.3) and, using the same technique as above, we shall obtain

$$b_{k-2}^{(2)} = k(k+\alpha+\beta+1) - (\alpha+\beta+2)b_{k-1}^{(1)},$$

$$b_{k-3}^{(2)} = -b_{k-2}^{(2)} + b_{k-1}^{(1)}\left(-(\alpha+\beta+2)\frac{A_{k-1}^1}{A_{k-1}^2} + \frac{(\beta-\alpha)}{A_{k-1}^2}\right) - b_{k-1}^{(1)}\frac{(\alpha+\beta+2)}{A_{k-1}^2},$$

(3.16)

$$b_{i-2}^{(2)} = -b_{i-1}^{(2)} + b_i^{(2)}(\frac{1}{A_i^2} - 1) - b_{i+1}^{(2)} - b_{i+2}^{(2)} - b_{i+1}^{(1)}(\alpha+\beta+2)\frac{A_i^1}{A_i^2} +$$

$$+ b_i^{(2)}\frac{(\beta-\alpha-(\alpha+\beta+2)A_i^1)}{A_i^2} - b_{i-1}^{(1)}(\alpha+\beta+2)\frac{A_i^1}{A_i^2}, \quad i = k-2, \ldots, 2.$$

□

Now, we can state the main theorem.

Theorem 1. *The coeffcients a_k of the approximate solution (3.1) satisfy the following system of equations*

$$-\mu^2 b_i^{(2)} \sum_{k=0}^{i+2} a_k + \sum_{j=0}^{n} (h_j A_i^j \sum_{k=M}^{m} a_k) = \epsilon^2 r_i \quad i = 0, \ldots, n-2, \tag{3.17}$$

$$\sum_{k=0}^{n} (-1)^k \binom{k+\alpha}{k} a_k = A^0, \quad \sum_{k=0}^{n} \binom{k+\alpha}{k} a_k = 0, \tag{3.18}$$

where $M = \max(0, i-j)$, $m = \min(n, i+j)$.

Proof. In order to obtain equations (3.17), we start from (3.4), make use of (3.6),(3.7) and(3.11) and come to

$$-\mu^2\sum_{k=2}^{n}(\sum_{i=0}^{k-2} b_i^{(2)} P_i^{\alpha,\beta}(t)) + \sum_{j=0}^{n} h_j(\sum_{k=0}^{n} a_k t^j P_k^{\alpha,\beta}(t)) = \epsilon^2\sum_{i=0}^{n} r_i P_i^{\alpha,\beta}(t).$$

With account of (3.8) the above equation will give

$$-\mu^2\sum_{k=2}^{n}(\sum_{i=0}^{k-2} b_i^{(2)} P_i^{\alpha,\beta}(t)) + \sum_{j=0}^{n} h_j(\sum_{k=0}^{n} a_k(\sum_{i=k-j}^{k+j} A_i^j P_i^{\alpha,\beta}(t))) = \epsilon^2\sum_{i=0}^{n} r_i P_i^{\alpha,\beta}(t).$$

After changing the order of summation, equating terms at $P_i^{\alpha,\beta}(t)$, $i = 0,\dots,n-2$ we come to (3.17). The last two equations (3.18) are obtained from boundary conditions (3.5) directly, using that for the Jacobi polynomials, we have

$$P_k^{\alpha,\beta}(1) = \binom{k+\alpha}{k} \quad \text{and } P_k^{\alpha,\beta}(-t) = (-1)^k P_k^{\alpha,\beta}(t).$$

□

Once the coefficients a_k are evaluated from the system (3.17), (3.18) the Jacobi spectral approximation (3.1) may be, using substitution (2.11), determined at each point $x \in [a, a+\delta]$ as

$$v_n(x) = V_n(\frac{2(x-a)}{\delta} - 1) \tag{3.19}$$

and the approximate solution to (1.1), (1.2), thus, becomes

$$y_n(x) = y_R(x) + v_n(x), \quad x \in [a, a+\delta]. \tag{3.20}$$

4. The error estimate

The error estimate, apart from the boundary layers, for $x \in [a+\delta, b-\delta]$, is already given by (2.7). With account of (2.10) this gives

$$d(x) = |y(x) - y_R(x)| \le C^2\epsilon^2 \text{ for } x \in [a+\delta, b-\delta]. \tag{4.1}$$

In order to estimate the error on the subinterval $[a, a+\delta]$, we shall represent the error function $d(x)$ in the form

$$(4.2)\qquad \begin{aligned} d(x) &= |y(x) - y_n(x)| = |u(x) - v_n(x)| \\ &\leq |u(x) - v(x)| + |v(x) - v_n(x)|. \end{aligned}$$

To provide the estimate for these two terms we have to prove the following lemmas:

Lemma 3. *For $x \in [a, a+\delta]$ we have*

$$(4.3)\qquad d^0(x) = |u(x) - v(x)| \leq |\delta^0|,$$

where

$$(4.4)\qquad \delta^0 = y(a+\delta) - y_R(a+\delta).$$

Proof. From (2.3), using notation (4.4), we can see that $u(x)$ satisfies

$$(4.5)\qquad Lu = \epsilon^2 y_R(x),\ x \in [a, a+\delta],\ \ G^0 u = (A^0, \delta^0).$$

Subtracting (2.5) from (4.5) we come to

$$(4.6)\qquad L(u - v) = 0 \text{ for } x \in [a, a+\delta],\ G^0(u - v) = (0, \delta^0).$$

By the principle of inverse monotonicity we can easily conclude that $u(x) - v(x) \geq 0$ for $\delta^0 \geq 0$,and $u(x) - v(x) \leq 0$ for $\delta^0 \leq 0$. In the first case, we have to construct function $\Omega(x) = u(x) - v(x) - \delta^0$ and apply operator (L, G^0) to it, which will give

$$(4.7)\qquad L\Omega = -\delta^0 g(x) \leq 0,\ \Omega(a) = -\delta^0 \leq 0,\ \Omega(a+\delta) = 0.$$

Using inverse monotonicity, again, we conclude from (4.7) that $\Omega(x) \leq 0$, which implies

$$(4.8)\qquad u(x) - v(x) \leq \delta^0 \text{ when } \delta^0 \geq 0,$$

In the second case, we have to apply the same technique to the function $-\Omega(x)$ which will give

$$(4.9)\qquad u(x) - v(x) \geq \delta^0 \text{ when } \delta^0 \leq 0,$$

and (4.8), (4.9) together give (4.3).

□

The following lemma is, in fact, a generalization of Oliver's estimate for the one dimensional case, given in [5] for the Chebyshev basis.

Lemma 4. *For the error function*

$$Z(t) = |V(t) - V_n(t)|,\ t \in [-1,1], \tag{4.10}$$

the following approximate estimate is valid

$$Z(t) \approx Z_q(t) = \sum_{j=n+1}^{n+q} S_j(t)a_j,\ q \in N, \tag{4.11}$$

with

$$S_j(t) = \sum_{i=0}^{n} \sigma_{i,j} P_i^{\alpha,\beta}(t) - P_j^{\alpha,\beta}(t),\ j = n+1,\dots, \tag{4.12}$$

where $\sigma_{i,j}, j \geq n+1,\ i = 0,\dots,n$, are defined as solutions of the system

$$\sum_{i=0}^{n} F_i \sigma_{i,j} = F_j. \tag{4.13}$$

(F_i and F_j are column matrices of the coefficients in the system (3.17), (3,18).)

Proof. As the exact solution $V(t)$ of (2.12), (2.13) is a function from the space $C^2[-1,1]$ and we can represent it by its Jacobi series

$$V(t) = \sum_{k=0}^{\infty} A_k P_k^{\alpha,\beta}(t), \tag{4.14}$$

so we have

$$Z(t) = |\sum_{i=0}^{n}(a_i - A_i)P_i^{\alpha,\beta}(t) - \sum_{i=n+1}^{\infty} A_i P_i^{\alpha,\beta}(t)|. \tag{4.15}$$

If we write down the system (3.17), (3.18) in the form

$$\sum_{i=0}^{n} F_i a_i = R,$$

where F_i and R are column matrices, then A_i, $i = 0,\dots$ are solutions of the analogue infinite system. Using $\sigma_{i,j}$, defined by (4.13), (4.15) becomes

$$Z(t) = |\sum_{j=n+1}^{\infty} S_j(t)A_j|, \tag{4.16}$$

where $S_j(t)$ are defined in (4.12). Taking into account that the sum (4.16) is dominantly determined by the first few terms and that the magnitude of A_j is approximately the same as a_j (evaluated for a larger degree, i.e. $n+q$), when n is large enough, we obtain (4.11).

□

These two lemmas provide the approximate estimate for the error (4.2), which is stated in the following theorem.

Theorem 2. *For the approximate solution (3.20) of the problem (1.1), (1.2) it holds that*

$$d(x) = |y(x) - y_n(x)| \approx C^0\epsilon^2 + z_q(x), \; x \in [a, a+\delta], \tag{4.17}$$

with

$$z_q(x) = Z_q(\frac{2(x-a)}{\delta} - 1), \tag{4.18}$$

where $Z_q(t)$ is given by (4.11).

Proof. Using (4.3) from Lemma 3, with respect to (4.4) and (4.1) we directly obtain that for the first term in (4.2) one has

$$|u(x) - v(x)| \le C^0\epsilon^2. \tag{4.19}$$

As for the second term, we first have to remark that, according to (2.9),

$$|v(x) - v_n(x)| = |V(t) - V_n(t)| = Z(t) \approx Z_q(t) = z_q(x), \tag{4.20}$$

where (4.11) and notation (4.18) were used. Starting from (4.2), relations (4.19) and (4.20) imply (4.17).

□

Remark 1. The error estimate for $x \in [b-\delta, b]$ could be provided in the same manner.

5. Numerical example

The following tables give results for the boundary value problem due to [2]

$$Ly \equiv -\epsilon^2 y''(x) + \frac{1-\epsilon}{(2-x)^2} y(x) = \frac{(1-\epsilon)(x-1)}{(2-x)^2}, \quad Gy \equiv (y(0), y(1)) = (0,0)$$

The solution of the reduced problem is $y_R(x) = x - 1$, so that we have the boundary layer only at $x = 0$. Here $K^2 = (1-\epsilon)/4$, which we need in (2.10) to determine δ. We can choose $C = 1$ as, from (2.8) we only have $M_3 \geq 0$. As the orthogonal basis in (3.1) we have used the Jacobi polynomials with $\alpha = \beta = -\frac{1}{2}$, i.e. Chebyshev basis.

$\epsilon = 0.001$	$\delta = 0.0276$		d(x)	
x	$y(x)$	$n = 5$	$n = 10$	$n = 15$
0.0001	-0.05	-8.3(-3)	-7.1(-5)	-4.4(-5)
0.0003	-0.14	-2.1(-2)	-1.5(-4)	-1.2(-4)
0.0008	-0.33	-3.7(-2)	-1.6(-4)	-2.7(-4)
0.0015	-0.53	-3.5(-2)	-1.6(-4)	-3.9(-4)
0.0025	-0.71	-6.4(-3)	-3.9(-4)	-4.4(-4)
0.005	-0.91	5.4(-2)	-4.7(-4)	-3.2(-4)
0.01	-0.99	-2.2(-3)	-1.4(-4)	-7.3(-5)

Table 1.

$\epsilon = 0.00001$	$\delta = 0.00046$			$d(x)$	
x	$y(x)$	$n = 5$	$n = 10$	$n = 15$	$n = 20$
0.000001	-0.05	-1.8(-2)	-4.3(-4)	8.8(-6)	1.3(-5)
0.000003	-0.14	-4.9(-2)	-8.3(-4)	7.3(-6)	9.5(-6)
0.000008	-0.33	-1.0(-1)	-1.3(-4)	7.9(-6)	-1.8(-6)
0.000015	-0.53	-1.3(-1)	2.2(-3)	1.1(-5)	8.1(-7)
0.000025	-0.71	-1.1(-1)	3.8(-3)	-1.5(-5)	-2.2(-6)
0.00005	-0.91	1.2(-2)	-1.3(-3)	9.3(-6)	-2.5(-6)
0.0001	-0.99	1.1(-1)	7.0(-4)	-2.0(-5)	-6.8(-7)

Table 2.

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REZIME

JAKOBJIEVA APROKSIMACIJA REŠENJA KONTURNOG PROBLEMA

Posmatra se samokonjugovani konturni problem opisan diferencijalnom jednačinom drugog reda. Standardna spektralna aproksimacija je prilagodjena karakteru tačnog rešenja i konstruisana je u odnosu na Jakobijevu ortogonalnu bazu. Data je ocena greške, a teoretski rezultati su potkrepljeni numeričkim primerom.

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