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SOME COMMUTATIVITY PROPERTIES FOR RINGS

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Abstract

In this paper, the following commutativity theorem is proved: Let R be a left (resp. right) s-unital ring, and let $m > 1$, n, r and s be fixed non-negative integers. If R satisfies the polynomial identity $[x^r y \pm x^n y^m x^s, x] = 0$ (resp. $[yx^r \pm x^n y^m x^s, x] = 0$) for all $x, y \in R$, then R is commutative. Other related results are also obtained for the case $m = 1$.

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Throughout the present paper, R will represent an associative ring (with or without unity 1), $Z(R)$ the center of R, $N(R)$ the set of all nilpotent elements of R, $N'(R)$ the set of all zero-divisors of R and $C(R)$ the commutator ideal of R. By $(GF(q))_2$ we mean the ring of 2×2 matrices over the Galois field $GF(q)$ with q elements. For any $x, y \in R$, we set $[x, y] = xy - yx$ as usual.

Definition 1. *A ring R is called left (resp. right) s-unital if $x \in Rx$ (resp. $x \in xR$) for every $x \in R$. Further, R is called s-unital if it is both left and right s-unital, that is $x \in Rx \cap xR$ for all $x \in R$.*

Definition 2. *If R is s-unital (resp. left or right s-unital), then for any finite subset F of R there exists an element $e \in R$ such that $ex = xe = x$ (resp. $ex = x$ or $xe = x$) for all $x \in F$. Such an element e will be called a pseudo (resp. pseudo left or pseudo right) identity of F in R.*

The objective of the present paper is to prove the following commutativity theorems:

Theorem 1. *Let $m > 1$, n, r and s be fixed non-negative integers. If R is a left s-unital ring which satisfies the polynomial identity*

$$[x^r y \pm x^n y^m x^s, x] = 0 \text{ for all } x, y \in R, \tag{1}$$

then R is commutative.

Theorem 2. *Let $m > 1$, n, r and s be fixed non-negative integers. If R is a right s-unital ring which satisfies the polynomial identity*

$$[yx^r \pm x^n y^m x^s, x] = 0 \text{ for all } x, y \in R, \tag{2}$$

then R is commutative.

Other related results are also obtained for the case $m = 1$.

In preparation for the proof of our results, we state the following:

Lemma 1 ([7], Lemma 3). *Let R be a ring such that $[[x, y], x] = 0$ for all $x, y \in R$. Then $[x^k, y] = kx^{k-1}[x, y]$ for any positive integer k.*

Lemma 2 ([8], Lemma). *Let R be a ring with unity 1, and let $x, y \in R$. If for some integer $k \geq 1$, $x^k y = 0 = (x+1)^k y$, then $y = 0$.*

Lemma 3 ([9], Lemma 3). *Let R be a ring with unity 1. If $(1-y^k)x = 0$, then $(1-y^{km})x = 0$ for any positive integer m.*

Lemma 4 ([5], Theorem). *Let f be a polynomial in n non-commuting indeterminates x_1, x_2,..., x_n with integer coefficients. Then the following statements are equivalent:*

1. *For any ring R satisfying $f = 0$, $C(R)$ is a nil ideal.*

2. *For every prime p, $(GF(p))_2$ fails to satisfy $f = 0$.*

3. *Every semi-prime ring satisfying $f = 0$ is commutative.*

Lemma 5 (**[11], Lemma 1**). *Let R be a ring with unity 1, and let $I_0^r(x) = x^r$ for all $x \in R$. Suppose that $I_k^r(x) = I_{k-1}^r(x+1) - I_{k-1}^r(x)$ for all positive integers k. Then for all $x \in R$, we have $I_{r-1}^r(x) = (r-1)r!/2 + r!x$, $I_r^r(x) = r!$ and $I_j^r(x) = 0$ for all $j > r$.*

Theorem 3 (**[3], Theorem 18**). *Let R be a ring, and let $n > 1$ be a fixed integer. If $x^n - x \in Z(R)$ for all $x \in R$, then R is commutative.*

Now, we shall establish the following lemmas.

Lemma 6. *Let $m > 1$, n, r and s be fixed non-negative integers and let R be a left s-unital ring satisfying the polynomial identity (1). Then R is an s-unital ring.*

Proof. Let $x, y \in R$. If R is a left s-unital ring, then there exists $e = e(x,y) \in R$ such that $ex = x$ and $ey = y$. By (1), we get $e^{r+1}y \pm e^{n+1}y^m e^s = e^r ye \pm e^n y^m e^{s+1}$. Hence $y = y(e \mp y^{m-1}e^s \pm y^{m-1}e^{s+1}) \in yR$. Therefore, R is an s-unital ring.

□

Lemma 7. *Let $m > 1$, n, r and s be fixed non-negative integers. If R satisfies the polynomial identity (1), then $C(R) \subseteq N(R)$. Furthermore, if R has unity 1, then $C(R) \subseteq Z(R)$.*

Proof. The polynomial identity (1) can be written in the form

$$x^r[x,y] = \pm x^n[y^m, x]x^s \text{ for all } x, y \in R. \tag{3}$$

Let $x = e_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $y = e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ in $(GF(p))_2$ for any prime p. Then x and y fail to satisfy (3). Thus $C(R) \subseteq N(R)$ by Lemma 4.

Let k be any positive integer. Then the polynomial identity (3) gives

$$x^{kr}[x,y] = \pm x^{(k-1)r}x^n[y^m,x]x^s = x^{(k-2)r}x^{2n}[y^{m^2},x]x^{2s} = \dots$$

So, we see that

$$x^{kr}[x,y] = \pm x^{kn}[y^{m^k},x]x^{ks} \text{ for all } x,y \in R. \tag{4}$$

If $u \in N(R)$, then for any $x \in R$, (4) yields $x^{kr}[x,u] = \pm x^{kn}[u^{m^k},x]x^{ks}$. But $u^{m^k} = 0$ for sufficiently large k. Thus $x^{kr}[x,u] = 0$ for all $x \in R$. The usual argument of replacing x by $x+1$, Lemma 2 shows that $[x,u] = 0$, and hence $N(R) \subseteq Z(R)$. Therefore,

$$C(R) \subseteq N(R) \subseteq Z(R). \tag{5}$$

□

Remark 1. Since we know that $C(R) \subseteq Z(R)$, we shall apply the conclusion of Lemma 1 without explicit mention for any ring R satisfying the hypothesis of Lemma 7.

Remark 2. By (5), identity (1) or equivalently (3) can be written in the form

$$x^r[x,y] = \mp x^{n+s}[x,y^m] \text{ for all } x,y \in R.$$

Hence R is commutative by Theorem 1 of [1] (see also Theorems 1 to 3 of [2]). For the sake of completeness, we prove the following lemma:

Lemma 8 *Let $m > 1$, n, r and s be fixed non-negative integers. If R is a ring with unity 1 which satisfies the polynomial identity 1), then R is commutative.*

Proof. Let $q = 2^{m-2}$. Then by (3), we have

$$qx^r[x,y] = 2^m x^r[x,y] - 2x^r[x,y] = \pm x^n[(2y)^m,x]x^s - x^r[x,(2y)] = 0.$$

Therefore, $qx^r[x,y] = 0 = q(x+1)^r[x,y]$. Hence $q[x,y] = 0$ by Lemma 2. So $[x^q,y] = qx^{q-1}[x,y] = 0$ for all $x,y \in R$. Thus

$$x^q \in Z(R) \text{ for all } x \in R. \tag{6}$$

Next, replace y by y^m in (3) to obtain

$$x^r[x,y^m] = \pm x^n[(y^m)^m, x]x^s \quad \text{for all } x,y \in R. \tag{7}$$

So

$$x^r[x,y^m] = [x,y^m]x^r = my^{m-1}[x,y]x^r = my^{m-1}x^r[x,y] = \\ = \pm my^{m-1}[y^m,x]x^{n+s},$$

and

$$x^n[(y^m)^m,x]x^s = m(y^m)^{m-1}[y^m,x]x^{n+s} = my^{m-1}y^{(m-1)^2}[y^m,x]x^{n+s}.$$

Thus (7) yields

$$my^{m-1}(1-y^{(m-1)^2})[y^m,x]x^{n+s} = 0 \text{ for all } x,y \in R. \tag{8}$$

By Lemma 2 and Lemma 3, (8) gives

$$my^{m-1}(1-y^{q(m-1)^2})[y^m,x] = 0 \text{ for all } x,y \in R. \tag{9}$$

Now, R is isomorphic to a subdirect sum of subdirectly irreducible rings R_i, each of which is a homomorphic image of R satisfying the polynomial identity (1). Hence we may assume that R is subdirectly irreducible. Let S be the intersection of all its non-zero ideals. Then $S \neq 0$.

Let $a \in N'(R)$. Then by (6) $a^{q(m-1)^2} \in N'(R) \cap Z(R)$ and $Sa^{q(m-1)^2} = 0$. By (9), we get

$$ma^{m-1}[a^m,x](1-a^{q(m-1)^2}) = 0 \text{ for all } x \in R.$$

If $ma^{m-1}[a^m,x] \neq 0$, then $(1-a^{q(m-1)^2}) \in N'(R)$ and $S = S(1-a^{q(m-1)^2}) = 0$, which is a contradiction. Thus

$$ma^{m-1}[a^m,x] = 0 \text{ for all } x \in R.$$

By (4) and Lemma 1, $x^{2r}[x,a] = \pm[a^{m^2},x]x^{2(n+s)} = \pm ma^{m(m-1)}[a^m,x] x^{2(n+s)} = \pm ma^{m-1}a^{(m-1)^2}[a^m,x]x^{2(n+s)} = \pm ma^{m-1}[a^m,x]a^{(m-1)^2}x^{2(n+s)} = 0$, and hence $[x,a]=0$ for all $x \in R$ by Lemma 2. Therefore, $N'(R) \subseteq Z(R)$.

Now, since $x^q \in Z(R)$ and $x^{qm} \in Z(R)$ for any $x \in R$, then by (3), we have $(x^q - x^{qm})x^r[x,y] = x^q(x^r[x,y]) - x^{qm}(x^r[x,y]) = x^r(x^q[x,y]) \mp x^{qm}x^n[y^m,x]x^s = x^r[x,(x^qy)] \mp x^n[(x^qy)^m,x]x^s = 0$ for all $y \in R$. Therefore,

$$(x - x^{qm-q+1})x^{r+q-1}[x,y] = 0 \text{ for all } x,y \in R. \tag{10}$$

If R is not commutative, then by Theorem 3, there exists an element $x \in R$ such that $x - x^{qm-q+1} \notin Z(R)$ and thus $x \notin Z(R)$. Hence neither x nor $x - x^{qm-q+1}$ is a zero-divisor. Thus $(x - x^{qm-q+1})x^{r+q-1} \notin N'(R)$. Hence (10) forces that $[x, y] = 0$ for all $y \in R$ and thus $x \in Z(R)$, which is a contradiction. Therefore, R is commutative.

□

Now, we complete the proof of Theorem 1.

Proof of Theorem 1. Let R be a left s-unital ring satisfying the polynomial identity (1). Then by Lemma 6, R is s-unital. In view of Proposition 1 of [4], R is commutative, if R with unity 1 is commutative and this is guaranteed by Lemma 8.

□

In preparation for proving Theorem 2, we prove the following lemmas:

Lemma 9. *Let $m > 1$, n, r and s be fixed non-negative integers, and let R be a right s-unital ring. If R satisfies the polynomial identity (2), then R is s-unital.*

Proof. If $x, y \in R$, then there exists $f = f(x, y) \in R$ such that $xf = x$ and $yf = y$. Thus $y = (f \pm f^{n+1}y^{m-1} \mp f^n y^{m-1})y \in Ry$, for $m > 1$. Therefore, R is an s-unital ring.

□

Lemma 10. *Let $m > 1$, n, r and s be fixed non-negative integers. If R satisfies the polynomial identity (2), then $C(R) \subseteq N(R)$. Further, if R has unity 1, then $C(R) \subseteq Z(R)$.*

Proof. The polynomial identity (2) can be rewritten in the form

(11) $$[x, y]x^r = \pm x^n [y^m, x]x^s \text{ for all } x, y \in R.$$

If $x = e_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ and $y = e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ in $(GF(p))_2$ for any prime p, then x and y fail to satisfy (11). By Lemma 4, $C(R) \subseteq N(R)$.

Also, if k is any positive integer, then (11) implies that

$$[x,y]x^{kr} = \pm x^{kn}[y^{m^k},x]x^{ks} \text{ for all } x,y \in R. \tag{12}$$

Following the proof of Lemma 7, we prove that $N(R) \subseteq Z(R)$. Therefore,

$$C(R) \subseteq N(R) \subseteq Z(R). \tag{13}$$

□

Proof of Theorem 2. In view of Lemma 9, R is s-unital. Hence we can assume that R has unity 1 as suggested by Proposition 1 of [4]. By (13), the polynomial identity (11) is equivalent to

$$x^r[x,y] = \pm x^n[y^m,x]x^s \text{ for all } x,y \in R.$$

Therefore, R is commutative by Lemma 8.

□

Remark 3. Let $r = n = 0$ (resp. $r = s = 0$) in (1) (resp. (2)). Then

$$[x,y] = \pm[y^m,x]x^s \text{ for all } x,y \in R \tag{14}$$

(resp.

$$[x,y] = \pm x^n[y^m,x] \text{ for all } x,y \in R). \tag{15}$$

If $m > 1$ or $s \geq 1$ in (14) (resp. $m > 1$ or $n \geq 1$ in (15)), then R is a $(\mathbf{Z},\bar{\beta})$-ring in the sense of Streb ([10]), hence R is commutative even if R is not assumed to be a left (resp. right) s-unital ring (ring with unity 1 (cf. Lemma 8)).

Now, we relax the condition $m > 1$ in (1) and (2). The following results are analogous to the results proved in [1] (see Theorems 2 to 7). Now, we present this results.

Theorem 4 *Let r be a fixed non-negative integer. If R is a left (resp. right) s-unital ring satisfies*

$$x^r[x,y] = 0 \text{ for all } x,y \in R \tag{16}$$

(resp.

$$[x,y]x^r = 0 \text{ for all } x,y \in R,) \tag{17}$$

then R is commutative.

Proof. If $x, y \in R$, then there exists $e = e(x,y) \in R$ (resp. $f = f(x,y) \in R$) such that $ex = x$ and $ey = y$ (resp. $xf = x$ and $yf = y$). Thus $y = ye$ (resp. $y = fy$). Similarly, $x = xe$ (resp. $x = fx$). Therefore, R is s-unital. Thus R is commutative by Theorem 3 (b) of [1].

□

Remark 4. In case $r > 0$ Theorem 3 need not be true for right (resp. left) s-unital ring. Indeed, we have the following:

Example 1. Let K be any field. Then the non-commutative ring $R = \begin{pmatrix} K & 0 \\ K & 0 \end{pmatrix}$ (resp. $R^* = \begin{pmatrix} 0 & K \\ 0 & K \end{pmatrix}$ has a right (resp. left) identity element and satisfies the polynomial identity $x[x,y] = 0$ (resp. $[x,y]x = 0$) for all $x, y \in R$. Also R is not s-unital ring.

Example 2. If we drop the restriction that R is left or right s-unital (the unity 1), then the ring R may be non-commutative. Let D_k be the ring of all $k \times k$ matrices over a division ring D, and let

$$A_k = \{ (a_{ij}) \in D_k \mid a_{ij} = 0,\ j \geq i \}.$$

Then A_k is a non-commutative nilpotent ring of index k, for any positive integer $k > 2$. Clearly, A_3 satisfies (1) and (2).

In the remaining case we suppose that $m = 1$ in (1) and (2).

Theorem 5. *Let n, r and s be fixed non-negative integers, and let R be an s-unital ring satisfying*

$$x^r[x,y] = \pm x^n[y,x]x^s \text{ for all } x, y \in R. \tag{18}$$

Then R is commutative in any of the following:

(i) R satisfies $[x,y] = [y,x]$, and R is 2-torsion free.

(ii) $0 = s = n < r$.

(iii) $0 < s < r$, $n = 0$ and R is $r!$-torsion free.

(iv) $0 < n < r$, $s = 0$ and R is $r!$-torsion free.

(v) $r = 0$ and $n > 0$ or $s > 0$.

Proof. According to Proposition 1 of [4], we may assume that R has unity 1.

(i) By hypothesis, $2[x,y] = 0$. Therefore, R is commutative, since R is 2-torsion free.

(ii) The identity (18) becomes $x^r[x,y] = \pm[y,x]$ for all $x, y \in R$. Therefore R is commutative by [6], Theorem, or by [10], Hauptsatz.

(iii) Let $I_0^r(x) = x^r$ and $I_0^s(x) = x^s$. Then the polynomial identity (18) gives

$$x^r[x,y] = \pm[y,x]x^s,$$

and hence

$$I_0^r(x)[x,y] = \pm[y,x]I_0^s(x) \text{ for all } x, y \in R.$$

Replace x by $x+1$ in the last identity to get

$$I_0^r(x+1)[x,y] = \pm[y,x]I_0^s(x+1).$$

By Lemma 5, we have

$$I_1^r(x)[x,y] = \pm[y,x]I_1^s(x).$$

Again, replace x by $x+1$ and apply Lemma 5 to obtain

$$I_2^r(x)[x,y] = \pm[y,x]I_2^s(x).$$

Now iterating the last identity r times we finally get

$$I_r^r(x)[x,y] = \pm[y,x]I_r^s(x) \text{ for all } x, y \in R. \tag{19}$$

Since by Lemma 5, $I_r^r(x) = r!$ and $I_r^s(x) = 0$ for $r > s$, the identity (19) reduces to $r![x,y] = 0$. As every commutator in R is $r!$-torsion free, we get $[x,y] = 0$ for all $x, y \in R$. Therefore R is commutative.

(iv) Similar to the proof of case (iii).

(v) Without loss of generality suppose that $n > 0$. Then we have

$$[x,y] = \pm x^n[y,x]x^s \text{ for all } x, y \in R, \tag{20}$$

and thus, R is commutative by [10], Hauptsatz.

□

Remark 5. In Theorem 4 (i), (ii) and (v), R is not necessarily to be an s-unital ring (ring with unity 1).

Theorem 6. *Let r, n and s be fixed non-negative integers, and let R be an s-unital ring satisfying*

$$(21)\qquad [x,y]x^r = \pm x^n[y,x]x^s \textit{ for all } x,y \in R.$$

Then R is commutative in any of the following:

(i) $0 = s = n < r$.

(ii) $0 < s < r$, $n = 0$ *and R is $r!$-torsion free.*

(iii) $0 < n < r$, $s = 0$ *and R is $r!$-torsion free.*

(iv) $r = 0$ *and* $n > 0$ *or* $s > 0$.

Theorem 7. *Let r, n and s be fixed non-negative integers such that $r \neq n+s$. Suppose that R is an s-unital ring satisfying the polynomial identity (18). Further, if every commutator in R is $|p^{r+1} - p^{n+s+1}|$-torsion free for an integer $p > 1$, then R is commutative.*

Proof. According to Proposition 1 of [4], we can assume that R has unity 1. Thus

$$(22)\qquad (px)^r[(px),y] = \pm(px)^n[y,(px)](px)^s \text{ for all } x,y \in R.$$

By using (18) and (22), we obtain

$$(23)\qquad |p^{r+1} - p^{n+s+1}|\, x^r[x,y] = 0 \text{ for all } x,y \in R.$$

By Lemma 2 and the hypothesis, (23) yields $[x,y] = 0$ for all $x,y \in R$. Therefore R is commutative.

□

Theorem 8. *Let r, n and s be fixed non-negative integers such that $r \neq n+s$. Suppose that R is an s-unital ring satisfying the polynomial identity (21). Further, if every commutator in R is $|p^{r+1} - p^{n+s+1}|$-torsion free for an integer $p > 1$, then R is commutative.*

Next, we suppose that $r > 0$, $n > 0$ and $s > 0$ in (18) and (21). Indeed we prove the following:

Theorem 9. *Let r, n and s be fixed positive integers and let R be an s-unital ring satisfying the polynomial identity (18). If, further, $N(R) \subseteq Z(R)$, then R is commutative provided that $r \neq n + s$ and every commutator in R is $r!$ resp. $(n + s)!$-torsion free for $r > n + s$, resp. $r < n + s$.*

Proof. It is easy to see that $C(R) \subseteq Z(R)$. Thus $x^r[x, y] = \pm[y, x]x^{n+s}$ for all $x, y \in R$. Therefore, R is commutative by Theorem 5 (iii).

□

Theorem 10. *Let r, n and s be fixed positive integers and let R be an s-unital ring satisfying the polynomial identity (21). If, further, $N(R) \subseteq Z(R)$, then R is commutative provided that $r \neq n + s$ and every commutator in R is $r!$ resp. $(n + s)!$-torsion free for $r > n + s$, resp. $r < n + s$.*

Finally, we present the following commutativity theorem for semi-prime rings.

Theorem 11. *Let m, n, r and s be fixed non-negative integers such that $(m, n, r, s) \neq (0, 0, 0, 0)$. Suppose that R is a semi-prime ring satisfying the polynomial identity (1) or (2). Then R is commutative.*

Proof. If R satisfies (1), then $x = e_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $y = e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ in $(GF(p))_2$ for any prime p, fail to satisfy (1). By Lemma 4, R is commutative.

Let R satisfying (2). Then $x = e_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ and $y = e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ in $(GF(p))_2$ for any prime p, fail to satisfy (2). Therefore, R is commutative by Lemma 4.

□

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REZIME

NEKE OSOBINE KOMUTATIVNOSTI ZA PRSTENE

U ovom radu dokazana je sledeća teorema o komutativnosti: Neka je R levi (respektivno desni) s-unitalan prsten i neka je $m > 1, n, r, s$ fiksirani nenegativni celi brojevi. Ako za R važi polinomijalni identitet $[x^r y \pm x^n y^m x^s, x] = 0$ (resp. $[yx^r \pm x^n y^m x^s, x] = 0$) za sve $x, y \in R$, tada je R komutativan. Dobijeni su i drugi rezulatati za slučaj $m = 1$.

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