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ON A MODIFICATION OF NONLINEAR OVERRELAXATION METHODS

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Abstract

A numerical composite VAOR-Newton iteration, with a modified nonlinear accelerated overrelaxation (VAOR) as the primary iteration and the Newton method as a secondary iteration, is considered. Some sufficient conditions for the local convergence of this method are given. In the linear case these conditions describe the area of convergence of the VAOR method, and also, as a subcase, the area of convergence of the AOR method.

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1. Introduction

Let us consider the system of nonlinear equations

$$Fx = 0, \tag{1}$$

where $F : V \subset \mathbf{R}^n \to \mathbf{R}^n$, and suppose that F is F-differentiable and F' is continuous in an open neighbourhood $V_0 \subset V$ of a point x^* for which $Fx^* = 0$.

For the solution of the nonlinear system (1) one can give a direct extension of methods for solving linear systems. So, there are the following combinations: JOR-Newton, SOR-Newton, and many of their modifications, see [15] for instance. The AOR method, introduced in [11] for solving linear systems, which is a two-parameter generalization of the JOR and SOR methods, can be also extended to the nonlinear case. A modification of the AOR method is described in [2], [3], [5], [9], [12], and it is called the VAOR method. The extension of this method to the nonlinear case we have in [2], [3], [12], [14]. Our VAOR-Newton method is an extrapolated VSOR-Newton from [10], see also [4]. Hence, one can make the convergence statement using this fact in the same way for the extrapolation methods, see [15], [11]. However, it is possible to give sufficient conditions for the convergence of our method only by considering the structure of the mapping F.

Convergence studies of the extrapolation method for linear and nonlinear systems have been undertaken and reported on by various authors; see for instance, [2]-[5], [7]-[8], [11]-[14] and the books [15], [16], and references cited therein.

In this paper we considered the VAOR-Newton method in cases that $F'(x^*)$ belongs to some special subclasses of the H-matrix and give some sufficient conditions for its local convergence.

2. Notations and definitions

We denote by $\mathbf{R}^n$ the real n-dimensional linear space of column vectors $x = [x_1, x_2, \ldots, x_n]^T$ and by $L(\mathbf{R}^n)$ the linear space of real matrices $A = [a_{ij}]$ of order n with unit matrix E. We use the coordinatewise partial orderings on $L(\mathbf{R}^n)$ and $\mathbf{R}^n$; that is if $A, B \in L(\mathbf{R}^n)$ then $A \geq B (A > B)$ if and only if $a_{ij} \geq b_{ij}$ $(a_{ij} > b_{ij})$, for $i, j = 1, 2, \ldots, n$; and similarly for $\mathbf{R}^n$. Let

$$A = A_D - A_T - A_S$$

be the decomposition of $A \in L(\mathbf{R}^n)$ into its diagonal, strictly lower and strictly upper triangular parts, respectively.

Let $N = 1, 2, \ldots, n$, $N(i) = N \setminus \{i\}$ and for $A = [a_{ij}] \in L(\mathbf{R}^n)$ let

$$P_i(A) = \sum_{j \in N(i)} |a_{ij}|, \; P_i'(A) = \sum_{j=1}^{i-1} |a_{ij}|, \; Q_i(A) = \max_{j \in N(i)} |a_{ij}|, \; i \in N.$$

For any real or complex $n \times n$ matrix $A = [a_{ij}]$, we denote by

$$M(A) = [m_{ij}]$$

the $n \times n$ matrix defined by

$$m_{ij} = \begin{cases} |a_{ii}| & \text{if} \quad i = j, \\ -|a_{ij}| & \text{if} \quad i \neq j. \end{cases}$$

Definition 1. *A real matrix $A = [a_{ij}]$ is called an M-matrix if and only if $a_{ij} \leq 0$, $i \in N, j \in N(i)$, and A is nonsingular with $A^{-1} \geq 0$.*

Definition 2. *A real or complex matrix A is called an H-matrix if and only if $M(A)$ is an M-matrix.*

Definition 3. *An $n \times n$ complex matrix $A = [a_{ij}]$ is called lower semistrictly diagonally dominant if and only if*

$$|a_{ij}| \geq P_i(A), \quad i \in N,$$

$$|a_{ii}| > P_i'(A), \quad i \in N.$$

The matrix A is called semistrictly diagonally dominant if and only if there exists a permutation matrix Q such that QAQ^T is lower semistrictly diagonally dominant.

Definition 4. *An $n \times n$ complex matrix $A = [a_{ij}]$ is called generalized diagonally dominant if and only if there exists a regular diagonal matrix $W = diag(w_1, \ldots, w_n)$ such that AW is strictly diagonally dominant, that is $|a_{ii}w_i| > P_i(AW)$, $i \in N$.*

3. On the convergence of the VAOR–Newton - method

The SOR-Newton method, cf. [15], is defined by

$$x_i^{k+1} = x_i^k - \omega \frac{f_i(x^{k,i})}{f_{ii}'(x^{k,i})},$$

where, as usual, $f_1, \ldots, f_n$ are components of F, $\omega \in \mathbf{R}\backslash\{0\}$, $f'_{ii}(x) = \frac{\partial f_i}{\partial x_i}(x)$ and $f'_{ii}(x) \neq 0$, $x \in V$, $i \in N$.

Our generalization of the SOR-Newton method is the VAOR-Newton method, see [2] - [3], [12] - [14] , which reduces to it if $\omega = \sigma$ and $\varphi_i(x) = f'_{ii}(x)$:

$$(2) \qquad \begin{aligned} x_i^{k+1} &= x_i^k - \omega \frac{f_i(z^{k,i})}{\varphi_i(z^{k,i})}, \ i \in N \\ z_i^{k+1} &= x_i^k - \sigma \frac{f_i(z^{k,i})}{\varphi_i(z^{k,i})}, \ i \in N \end{aligned}$$

where $\omega, \sigma \in \mathbf{R} \setminus \{0\}$, $\varphi_i(x) \neq 0$,and

$$z^{k,i} = [z_1^{k+1}, \ldots, z_{i-1}^{k+1}, x_i^k, \ldots, x_n^k]^T, \ i \in N.$$

Without loss of generality, we may assume that $f'_{ii}(x) > 0$, $x \in V$, $i \in N$. Because of that we assume now on that $\varphi_i(x) > 0$, $x \in V$, $i \in N$.

In case $\sigma = \omega$ our VAOR-Newton method reduces the SOR-Newton method from [10].

Let $\Phi(x) = diag(\varphi_1(x), \varphi_2, \ldots, \varphi_n(x))$. Our method (2) may be written in the form $x^{k+1} = G_{\Phi,\sigma,\omega}x^k$ although now mapping $G_{\Phi,\sigma,\omega}$ becomes rather complicated. If we denote with $G_{\Phi,\sigma}$ the iteration function of the SOR-Newton method, then for iteration function $G_{\Phi,\sigma,\omega}$ of the VAOR-Newton method we have

$$G_{\Phi,\sigma,\omega}x = \left(1 - \frac{\omega}{\sigma}\right)x + \frac{\omega}{\sigma}G_{\Phi,\sigma}x.$$

This relation shows that the VAOR-Newton method is the extrapolated VSOR-Newton method with extrapolation parameter $\frac{\omega}{\sigma}$. To prove the local convergence of the VAOR-Newton method it is sufficient to show that $G_{\Phi,\sigma,\omega}$ is differentiable at x^* and that $\rho(G'_{\Phi,\sigma,\omega}(x^*)) < 1$, see [15], the Ostrowski theorem. Here $\rho(G'_{\Phi,\sigma,\omega}(x^*))$ is the spectral radius of the matrix $G'_{\Phi,\sigma,\omega}(x^*)$.

In [15] we have $G'_{\Phi,\sigma}(x^*)$ for $\varphi_i(x) = f'_{ii}(x)$, $i \in N$, and in [10] is given

$$G'_{\Phi,\sigma}(x^*) = (\Phi(x^*) - \sigma F'_T(x^*))^{-1}(\Phi(x^*) - \sigma F'_D(x^*) + \sigma F'_S(x^*)),$$

where $F'(x^*) = F'_D(x^*) - F'_T(x^*) - F'_S(x^*)$ is the decomposition of $F'(x^*)$ into its diagonal, strictly lower and strictly upper triangular parts.

Thus, if $G_{\Phi,\sigma}$ is F-differentiable at $x^*, G_{\Phi,\sigma,\omega}$ is also F-differentiable at the same point and

$$(3) \qquad \begin{aligned} G'_{\Phi,\sigma,\omega}(x^*) = \ & (\Phi(x^*) - \sigma F'_T(x^*))^{-1} (\Phi(x^*) - \omega F'_D(x^*) + \\ & + (\omega - \sigma)F'_T(x^*) + \omega F'_S(x^*)). \end{aligned}$$

Theorem 1. *Let $F : V \subset \mathbf{R}^n \to \mathbf{R}^n$ be F-differentiable in an open neigbourhood $V_0 \subset V$ of a point $x^* \in V$ at which F' is continuous and $Fx^* = 0$. If $F'(X^*)$ is an H-matrix and $\omega \in (o, q]$, $\sigma \in [o, q]$, where*

$$q = \min_{i \in N} \frac{\varphi_i(x^*)}{f'_{ii}(x^*)},$$

then the VAOR-Newton method is locally convergent.

Proof of this Theorem follows immediately from the corresponding theorem for the VAOR method from [3]. For technique see [2]- [5] , [12]- [14].

Theorem 2. *If $F'(x^*) = F' = [f_{ij}]$ and*

(4) *F' is lower semistrictly diagonally dominant, or*

(5) $f_{ii} f_{jj} > P_i(F') P_j(F'),\ i \in N,\ j \in N(i)$, *or*

(6) *there exists $i \in N$ such that*

$$f_{ii}(f_{jj} - P_j(F') + |f_{ji}|) > P_i(F')|f_{ji}|,\ j \in N(i),\ or$$

(7) $$f_{ii} > min\{P_i(F'), Q_i(F')\},\ i \in N, and \quad f_{ii} + f_{jj} > P_i(F') + P_j(F'),\ i \in N,\ j \in N(i),$$

then $F'(x^*)$ is an H-matrix.

Proof. If $F'(x^*)$ satisfies (4), then the statement follows immediately from [1] . In other cases proof is given in [6].

□

One can easily test conditions (4) - (7) and if $F'(x^*)$ satisfies one of them there follows the local convergence of the VAOR-Newton method for $\omega \in (0, q], \sigma \in [0, q]$.

It is known that a matrix A is an H-matrix if and only if it is generalized diagonally dominant, [3], [6], that is, there exists a regular diagonal matrix W such that AW is strictly diagonally dominant. In case that the matrix W is known, one can obtain a new area of convergence of the VAOR-Newton method using techniques from [2]- [5], [7]- [9], [12]- [14], for strictly diagonally dominant matrices and the following theorem, [3], [6].

Theorem 3. *Let $F'(x^*) = F' = [f_{ij}]$ be strictly diagonally dominant matrix. If $\varphi_i(x^*) > 0$, $i \in N$ and*

$$0 \le \sigma < t,\ 0 < \omega < \max\{t, \frac{2\sigma}{1+\rho(G'_{\Phi\sigma,\sigma})}\}, \quad or$$

$$\max_{i \in N} \frac{-\omega[f_{ii} - P_i(F')] + 2\max\{0, \omega f_{ii} - \varphi_i(x^*)\}}{2P_i(F'_T)} < \sigma < 0,\ 0 < \omega < t, \quad or$$

$$t \le \sigma < \min_{i \in N} \frac{\omega[f_{ii} + P_i(F'_T) - P_i(F'_S)] + 2\min\{0, \varphi_i(x^*) - \omega f_{ii}\}}{2P_i(F'_T)},\ 0 < \omega < t,$$

where

$$t = \min_{i \in N} \frac{2\varphi_i(x^*)}{f_{ii} + P_i(F')},$$

then $\rho(G'_{\Phi,\sigma,\omega}) < 1$, that is, the VAOR-Newton method converges locally.

Our aim is to obtain in each of the cases (4) - (7), Theorem 2, a corresponding matrix $W = diag(w_1, w_2, w_3, \ldots, w_n)$ such that the matrix $F'(x^*)W$ is strictly diagonally dominant and $\varphi_i(x^*)w_i > 0$, $i \in N$. Then by using Theorem 3 we find wider than $\sigma \in [0,q], \omega \in (0,q]$ intervals of convergence for both σ and ω.

Theorem 4. *Let matrix $F'(x^*) = F' = [f_{ij}]$ be lower semistrictly diagonally dominant, and let $W = diag(w_1, w_2, \ldots, w_n)$, where*

$$1 > w_n > \frac{P'_n(F')}{f_{nn}}, \tag{8}$$

$$1 > w_i > \frac{P'_i(F') + \sum_{j=i+1}^{n} w_j |f_{ij}|}{f_{ii}},\ i = n-1, n-2, \ldots, 1. \tag{9}$$

Then the matrix $F'(x^*)W$ is strictly diagonally dominant.

Proof. Our assumptions are: $f_{ii} > 0$, $i \in N$, and

$$f_{ii} > P'_i(F') = \sum_{j=1}^{i-1} |f_{ij}|,\ i \in N,$$

$$f_{ii} \ge P'_i(F') = \sum_{j \in N} |f_{ij}|,\ i \in N.$$

So, we have that there exists w_n such that (8) is satisfied. Let us assume that there exists $w_{n-1}, w_{n-2}, \ldots, w_{i+1} \in (0,1)$, such that (9) holds. Then

$$P_i(F') = P_i'(F') + \sum_{j=i+1}^{n} |f_{ij}| \geq P_i'(F') + \sum_{j=i+1}^{n} w_j |f_{ij}|,$$

where equality holds only if $\sum_{j=i+1}^{n} |f_{ij}| = 0$, that is, if $P_i'(F') = P_i(F')$. Therefore,

$$f_{ii} > P_i'(F') + \sum_{j=i+1}^{n} w_j |f_{ij}|,$$

and it follows that all w_i, $i \in N$ are well defined.

Let us consider the matrix AW and prove that it is strictly diagonally dominant, that is

$$f_{ii} w_i > P_i(F'W), \; i \in N.$$

From (8), (9) we have

$$f_{ii} w_i > P_i'(F'W) + \sum_{j=i+1}^{n} w_j |f_{ij}|,$$

and, since,

$$P_i'(F'W) \geq \sum_{j=1}^{i-1} w_j |f_{ij}|$$

we obtain

$$f_{ii} w_i > \sum_{j \in N(i)} w_j |f_{ij}| = P_i(F'W), \; i \in N.$$

□

Theorem 5. *Let the matrix $F'(x^*) = F' = [f_{ij}]$ be not strictly diagonally dominant and let it satisfy*

$$(10) \qquad f_{ii} > 0, \; f_{ii} f_{jj} > P_i(F') P_j(F'), \; i \in N, \; j \in N(i).$$

Then there exists exactly one $p \in N$ such that $f_{pp} \leq P_p(F')$.
Let $W = diag(w_1, w_2, \ldots, w_n)$, where $w_i = 1$, $i \in N(p)$, and $w_p > \frac{P_p(F')}{f_{pp}}$, if $f_{ip} = 0$, $i \in N(p)$, or

$$w_p \in \left(\frac{P_p(F')}{f_{pp}}, \; 1 + \min \left\{ \frac{f_{ii} - P_i(F')}{|f_{ip}|}; \; i \in N(p), \; f_{ip} \neq 0 \right\} \right).$$

Then the matrix $F'(x^*)W$ is strictly diagonally dominant.

Proof. From (10) it is obviously that there is the most one $p \in N$ such that

$$f_{ii} > P_i(F'),\ i \in N(p)$$
$$f_{pp} \leq P_p(F').$$

Since F' is not a strictly diagonally dominant matrix, it follows that there exists exactly one p with this property. If $f_{ip} = 0$ for all $i \in N(p)$, then

$$f_{ii}w_i = f_{ii} > P_i(F') = P_i(F'W),$$

and the matrix $F'W$ is strictly diagonally dominant.

Let us suppose that $f_{ip} \neq 0$ for some $i \in N(p)$, and let

$$\frac{f_{jj} - P_j(F')}{|f_{jp}|} = \min\left\{\frac{f_{ii} - P_i(F')}{|f_{ip}|} : i \in N(p),\ f_{ip} \neq 0\right\}.$$

Then $f_{jj} > P_j(F') \geq |f_{jp}| > 0$ and there follows $(f_{jj} - P_j(F'))(P_j(F') - |f_{jp}|) \geq 0$, that is,

$$\frac{f_{jj} - P_j(F')}{|f_{jp}|} + 1 \geq \frac{f_{jj}}{P_j(F')}.$$

Since, from (10), $\frac{f_{jj}}{P_j(F')} > \frac{P_p(F')}{f_{pp}}$, we now have that w_p is well defined.

Let us now consider the rows of the matrix $F'W$ for which $f_{ip} \neq 0$. Then we obtain

$$P_i(F'W) = P_i(F') + (w_p - 1)|f_{ip}| < P_i(F') + (f_{ii} - P_i(F')) \leq f_{ii},$$

$$P_p(F'W) = P_p(F') < w_p f_{pp},$$

and conclude that $F'W$ is a strictly diagonally dominant matrix.

□

The proof of the following Theorem 6 is analogous to the proof of Theorem 5.

Theorem 6. *Let the matrix $F'(x^*) = F' = [f_{ij}]$ satisfy (6) and let $W = diag(w_1, w_2, \ldots, w_n)$, where $w_j = 1$, $j \in N(i)$ and*

$$w_i > \frac{P_i(F')}{f_{ii}} \text{ if } f_{ji} = 0,\ j \in N(i) \text{ or}$$

$$(11) \qquad w_i \in \left(\frac{P_i(F')}{f_{ii}},\ 1 + \min\{ \frac{f_{jj} - P_j(F')}{|f_{ji}|} : \ j \in N(i),\ f_{ji} \neq 0\} \right).$$

Then the matrix $F'W$ is strictly diagonally dominant.

Proof. Let condition (6) be satisfied for some fixed $i \in N$. If $P_i(F') \geq f_{ii}$, then $P_j(F') < f_{jj}$, $j \in N(i)$, and the reduces to the proof of the previous Theorem. Indeed, from (6) we have $f_{jj} - P_j(F') > 0$ if $f_{ji} = 0$, and

$$f_{jj} - P_j > |f_{ji}| \left(\frac{P_i(F')}{f_{ii}} - 1 \right) \geq 0, \text{ if } f_{ji} \neq 0,\ j \in N(i).$$

If $P_i(F') < f_{ii}$, then

$$f_{jj} - P_j(F') > 0, \text{ if } f_{ji} = 0$$

and

$$\frac{f_{jj} - P_j(F')}{|f_{ji}|} + 1 > \frac{P_i(F)}{f_{ii}}, \text{ if } f_{ji} \neq 0.$$

So, w_i is well defined. Let us consider now the matrix $F'W$:

$$(F'W)_{jj} = \begin{cases} f_{jj}, & j \in N(i), \\ w_i f_{ii}, & j = i, \end{cases}$$

$$P_j(F'W) = \begin{cases} P_j(F') - |f_{ji}| + w_i|f_{ji}|, & j \in N(i), \\ P_i(F'), & j = i. \end{cases}$$

From (11) it follows that

$$(F'W)_{ii} = w_i f_{ii} > P_i(F') = P_i(F'W),$$

$$(F'W)_{jj} = f_{jj} > P_j(F') - |f_{ji}| + w_i|f_{ji}| = P_j(F'W),\ j \in N(i),$$

and we can conclude that $F'W$ is a strictly diagonally dominant matrix.

□

Theorem 7. *Let the matrix $F'(x^*) = F' = [f_{ij}]$ be not strictly diagonally dominant and let it satisfy (7).*

Then there exists exactly one $p \in N$ such that $f_{pp} \leq P_p(F')$.

Let $W = diag(w_1, w_2, \ldots, w_n)$, where $w_i = 1$, $i \in N(p)$, and

$$w_p > \frac{P_p(F')}{f_{pp}} \text{ if } Q_p(F') = 0, \text{ or}$$

$$w_p \in \left(\frac{P_p(F')}{f_{pp}}, 1 + \min\{\frac{f_{ii} - P_i(F')}{Q_p(F')} : i \in N(p)\} \right) \text{ if } Q_p(F') \neq 0.$$

Then the matrix $F'(x^*)W$ is strictly diagonally dominant.

Proof. It is easy to see that there is exactly one $p \in N$ such that $f_{pp} \leq P_p(F')$. $Q_p(F') = 0$ implies $f_{ip} = 0$ for $i \in N(p)$, and, as in the proof of Theorem 5, we conclude that $F'(x)W$ is strictly diagonally dominant. If $Q_p(F') \neq 0$ we have $f_{pp} > Q_p(F')$ and

$$1 + \frac{f_{ii} - P_i(F')}{Q_p(F')} > 1 + \frac{f_{ii} - P_i(F')}{f_{pp}} > \frac{P_p(F')}{f_{pp}}, \; i \in N(p)$$

because $f_{ii} - P_i(F') > P_p(F') - f_{pp}$, which follows from (7). So, w_p is well defined. As in the proof of Theorem 5, we can now obtain $P_i(F'W) < f_{ii}w_i$, $i \in N$, that is, $F'W$ is strictly diagonally dominant.

□

Now, we can easily prove the following theorem.

Theorem 8. *Let $F'(x^*) = F' = [f_{ij}]$ satisfy the conditions from one of the theorems 4,5,6,7 and let W be the diagonal matrix defined in the same theorem. If*

$$0 \leq \sigma < t, \; 0 < \omega < \max\{t, \frac{2\sigma}{1 + \rho_{\sigma,\sigma}}\}, \text{or}$$

$$\max_{i \in N} \frac{-\omega[f_{ii}w_i - P_i(F'W) + 2\max\{0, \omega f_{ii} - \varphi_i(x^*)\}w_i]}{2P_i(F_T'W)} < \sigma < 0,$$

$$0 < \omega < t, \text{or}$$

$$t \leq \sigma \leq \min \frac{\omega[f_{ii}w_i + P_i(F_T'W) - P_i(F_S'W)] + 2\min\{0, \varphi_i(x^*) - \omega f_{ii}\}w_i}{2P_i(F_T'W)}$$

$$0 < \omega < t,$$

where $\rho_{\sigma,\sigma} = \rho(H(\sigma,\sigma))$,

$$H(\sigma,\omega) = (\Phi(x^*)W - \sigma F_T'W)^{-1} (\Phi(x^*)W - \omega F_D'W + +(\omega - \sigma)F_T'W + \omega F_S'W),$$

and

$$t = \min \frac{2\varphi_i(x^*)w_i}{f_{ii}w_i + P_i(F'W)},$$

then the VAOR-Newton method converges locally.

Proof. Under the conditions of Theorem 8, $F'W$ is strictly diagonally dominant, $\varphi_i(x^*)w_i > 0$, $i \in N$ and we can apply Theorem 3. Thus, we conclude that

$$\rho(H(\sigma,\omega)) < 1.$$

Since,

$$W^{-1}G'_{\Phi,\sigma,\omega}W = H,$$

the spectral radius of $G'_{\Phi,\sigma,\omega}$ is less than 1, and the VAOR-Newton method converges locally.

□

From the definition of t, we have $t \geq q$ because of $f_{ii}w_i > P_i(F'W)$. So, we have obtained new intervals for convergence σ and ω, which are wider than $[0, q]$ and $(0, q]$. At the same time matrix W in each of the considered cases is very simple and it can be calculated during test conditions (4)-(7).

If $\Phi = F_D$, we have the AOR-Newton method and Theorem 8 gives the area of convergence of this method. If F is a linear mapping and $\Phi = F_D$, the VAOR-Newton method reduces to the linear AOR method, and Theorem 8 describes the area of convergence for this method.

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REZIME

O MODIFIKACIJI NELINEARNIH METODA GORNJE RELAKSACIJE

Posmatra se komponovani VAOR-Njutnov iterativni postupak sa modifikovanom relaksacijom (VAOR) kao primarnim i Njutnovim postupkom kao sekundarnim postupkom. Dati su neki dovoljni uslovi za lokalnu konvergenciju ovog postupka. U linearnom slučaju ovi uslovi opisuju oblast konvergencije VAOR postupka i takodje, kao podslučaj, oblast konvergencije AOR postupka.

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