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REMARK ON THE PAPER OF CHITRA AND SUBRAHMANYAM ABOUT THE SECTION THEOREM OF KY FAN

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Abstract

In 1961 Ky Fan proved in [2] an interesting result with important applications to the variational inequality theory and the minimax theory.

This theorem is known now as "the Section Theorem" and it has been the subject of several papers [4], [5] etc.

Recently in paper [1] Chitra and Subrahmanyam proved a generalization of this theorem and proof is based essentially on Brouwe's fixed point theorem.

Now, we show in tis note that, this generalization is, in fact, a direct consequence of Section Theorem.

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We recall that in 1972, in paper [3, pg. 105], Ky Fan reformulated the Section Theorem as the following result.

Theorem 1. *[Ky Fan][3]. Let X be a nonempty compact convex set in a Hausdorff topological vector space. Let D be subset of $X \times X$ having the following properties:*

(a) *for every* $x \in X, (x,x) \in D$,

(b) *for each fixed* $y \in X$, *the set* $D_y = \{x \in X | (x,y) \notin D\}$ *is convex (possibly empty),*

(c) *for each fixed* $x \in X$, *the set* $D_x = \{y \in X | (x,y) \in D\}$ *is closed.*

Then there exists a point $y_0 \in X$ *such that* $X \times \{y_0\} \subset D$.

The generalization of Section Theorem proved in [1] by Chitra and Subramanyam is the following.

Theorem 2. *Let* K *be a nonempty compact convex subsets of Hausdorff topological vector space* E.

Let $A \subset K \times K$ *and* $g : K \to K$ *such that the following conditions are satisfied:*

i. *for every* $y \in K, \{x \in K | (g(x),y) \in A\}$ *is closed,*

ii. *for every* $x \in K, (g(x),x) \in A$,

iii. *for each* $x \in K, \{y \in K | (g(x),y) \notin A\}$ *is empty or convex.*

Then there exists $x_0 \in K$ *such that* $\{g(x_0\} \times K \subset A$.

We prove now that Theorem 2 is an immediate consequence of Theorem 1.

Indeed, if we denote $X = K$ and $D = \{(x_1,x_2) \in K \times K | (g(x_2),x_1) \in A\}$ we constat that the all assumptions of Theorem 1 are satisfied.

Obviously, $D \subseteq X \times X$ and we have:

(a) for every $x \in X, (x,x) \in D$ by assumption (ii),

(b) for every $y \in X$ the set
$D_y = \{x \in X | (x,y) \notin D\}$ is empty or convex since,
$D_y = \{x \in X | (g(y),x) \notin A\}$ and the second part is an empty or convex set by assuption (iii),

(c) for each fixed $x \in X$ the set $D_x = \{y \in X | (x,y) \in D\}$ is closed since, $D_x = \{y \in X | (g(y),x) \in A\}$ and the second part is a closed set by assumption (i).

Hence from Theorem 1 there exists $x_0 \in X$ such that $(y, x_0) \in D$, for every $y \in X$, that is $(g(x_0), y) \in A$ for every $y \in K$ and Theorem 2 is proved. □

In consequence Theorem 2 is a corollary of Section Theorem.

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REZIME

PRIMEDBE NA RAD CHITRE I SUBRAHMANYAMA O TEOREMI O SEČENJU KY FANA

1961. godine Ky Fan [2] je dokazao zanimljiv rezultat sa važnim primenama na varijacione nejednakosti i minimaks teoriju.

Ova je teorema poznata kao "Teorema o sečenju" i bila je predmet mnogih radova [4],[5].

Nedavno su u [1] Chitra i Subrahmanyam dokazali uopštenje ove teoreme i dokaz je u osnovi zasnovan na Brauerovoj teoremi o nepokretnoj tački.

U ovoj noti je pokazano da je ovo uopštenje zapravo direktna posledica teoreme o sečenju.

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