Univ. u Novom Sadu
Zb. Rad. Prirod.–Mat. Fak.
Ser. Mat. 20, 1 (1990), 39-51

Review of Research
Faculty of Science
Mathematics Series

AN ALGORITHM FOR THE GENERATION OF THE MATROID ASSOCIATED TO A GIVEN MATRIX

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Abstract

An algorithm is sketched, which determines, up to an isomorphism, the matroid associated to a given matrix. A lemma which considerably shortens the corresponding computation time is used. The algorithm is given in detail for the matrices having at most 7 columns. Matroids on small ground-sets are represented by the following numerical parameters: cardinality of the ground-set, rank, number of bases and the vector of the sorted frequencies of elements w.r.t. the family of bases. It turns out that such a representation describes completely the matroids on at most 7 elements, while it is not sufficient on larger ground-sets.

AMS Mathematics Subject Classification (1980): 05B35
Key words and phrases: "Small" matroids, matrix representations, matroid invariants, matroid recognition, catalogue of matroids

1. Definitions and the problem statement

Matroid M on a finite set *(ground-set)* E is ([4]) an ordered pair $(E,\ B)$, where B is a family of subsets of E, which satisfies the following "exchange" axiom :

If B_1 and B_2 belong to B, and x is an element of $B_1 - B_2$, then there exists an element y in $B_2 - B_1$, such that $(B_1 - x) + y$ belongs to B.

The sets in family B are called *bases* of M. All the bases of M have the same cardinality, which is called the *rank* of M.

Given a matroid M on E and a subset X of E, *rank* of X (w.r.t. M) is the maximal cardinality of the intersection of X with a base of M.

It is well-known that the complements (w.r.t. E) of bases of a rank r matroid M are the bases of so-called *dual* matroid M^*, the rank of which is equal to $\mid E \mid - rank(M)$.

Two matroids are *isomorphic* if there exists a bijection between their ground-sets, which maps the bases of one matroid onto the bases of the other.

Matroid M is *representable* over a field F if and only if there exists a matrix A (the *matrix representation* of M) with entries in F, such that there exists a rank-preserving bijection bij of the set E onto the set C of columns of A (thus, if X is a rank r subset of E w.r.t. M, then $bij(X)$ is a rank r submatrix of A -the number of rows in the submatrix is equal to the number of rows in A. Given a representable matroid M with a matrix representation A, we have that rank(M) =rank(A) and bases of M correspond to the maximal linearly independent sets of columns of A.

Given a family F of subsets of a finite set E and an element e from E, the *appearance frequency* of e w.r.t. F is the number of subsets of F, which contain e.

$C(n,r)$ denotes the binomial coefficient "n over R".

We shall give an algorithm for solving the following problem: Given a real-valued $m \times n$ matrix M, determine its associated matroid $f(M)$ and identify (recognize) it in a catalogue of non-isomorphic matroids.

We distinguish two stages in calculating $f(M)$:

(1) Extraction of matroid bases from the input matrix M

(2) Recognition of the generated matroid by comparision with matroids in catalogue [1]

2. The algorithm

2.1 Stage (1)

Stage (1) can be sketched as follows:

a) Determine rank r of M and extract an $r \times r$ submatrix S of rank r

b) Let Q denote the $r \times n$ submatrix of M, consisting of exactly those rows, which intersect S.

c) Extract the bases of $f(M)$ from Q in the following way: An r-tuple R of columns of M corresponds to a base of $f(M)$ if and only if the $r \times r$ submatrix consisting of exactly those fields of M, which belong to both Q and R – is of rank r.

When the rank calculation is considered, we use the efficient algorithm described in [2]. This algorithm is based on the reduction of the input matrix M to the so-called *row echelon* form. Such a reduction is a generalization of the Gaussian elimination to the rectangular case. The rank of a matrix is equal to the number of non-zero rows in its row echelon form. The columns and rows of the rank r submatrix S are easily determined from the reduction algorithm: the columns of S are exactly those, which contain the first non-zero elements in the first r rows of the row echelon form; the rows of S can be reconstructed by memorizing the row interchanges performed during the reduction process.

After the submatrix S is extracted, we should further calculate only $C(n,r)$ determinants corresponding to the $r \times r$ submatrices of Q (more precisely, we should just distinguish between zero- and non-zero- determinants among them). The reduction of search for bases of $f(M)$ to the columns of the "horizontal" band Q (instead of considering the whole columns of M) – is justified by the following Lemma:

Lemma 1. *Let an $m \times n$ matrix M of rank r be given and let K and L be two $r \times n$ submatrices of M, such that $rank(K) = rank(L) = r$. Let J denote an arbitrary $m \times r$ submatrix of M. Finally, let A and B be the $r \times r$ submatrices of M, which are composed of fields belonging both to K and J, respectively to both L and J. Then*

$$rank(A) = r \iff rank(B) = r$$

Proof. ([3]) Assume that $\text{rank}(A) = r$ and $\text{rank}(B) < r$ (the opposite assumption is treated in the same way). Since $\text{rank}(L) = r$, there exists an $r \times r$ submatrix C in the "horizontal band" L, such that $\text{rank}(C) = r$.

Consider a row R of M, which belongs to $L - K$. The assumption $\text{rank}(B) < r$ implies the linear dependence of rows of B. In particular, the subrow $R \bigcap B$ may be expressed as a linear combination of the other rows of B. It follows that there exists a linear transformation T of M, which consists of multiplying the rows of $L - R$ by the appropriate coefficients and adding these products to R, and which makes the fields of $B \bigcap R$ equal to zero. It is obvious that at least one field F of $C \bigcap R$ is not equal to zero after applying T (otherwise we would have a contradiction with $\text{rank}(C) = r$).

Now, consider the submatrix W of size $(r+1) \times (r+1)$ of the matrix $T(M)$, which consists of the matrix A and the additional row and column, which intersect in F (this row and column are extracted from M and all their elements, apart from F, lie in those columns, respectively rows, which intersect A). Since all the fields in the additional row, apart from F, are equal to zero, it follows that

$$\det(W) = \det(A) \cdot \det(F) \neq 0.$$

This implies that $\text{rank}(M) = \text{rank}(T(M)) \geq r + 1$, a contradiction. □

Consequence of Lemma. Arbitrary rank r $r-$tuple of rows is sufficient to determine the matroid bases. Namely, according to the Lemma, an $r-$tuple J of columns corresponds (or does not correspond) to a matroid base, without regard to the rank r $r-$tuple of rows, which is used.

The easiest way to determine matroid bases is to start from the first submatrix S of rank r (we assume that such a submatrix is detected in the process of rank calculating). Then the matroid bases associated to M are the same as the matroid bases associated to $H(S)$, where $H(S)$ denotes the "horizontal band" associated to S, i.e., the submatrix of M, which consists of exactly those rows, which intersect S.

2.2 Stage (2)

The comparison between families F_1 and F_2 of bases, which respectively correspond to the matroid $M_1 = f(M)$ and a matroid M_2 from the catalogue, is preceded by comparison of the following four types of numerical parameters in turn:

a) cardinalities of the ground-sets of M_1 and M_2 (this cardinality should be equal to n with the matroid M_2; note that there may be elements (loops) which do not belong to any base – therefore such a cardinality should be given in a catalogue independently from the family of bases). If the ground-sets of M_1 and M_2 are equicardinal, then they may be considered to coincide.

b) ranks of M_1 and M_2

c) the numbers of bases in F_1 and F_2

d) sorted (non-increasing) sequences of appearance frequencies of elements of the common ground-set w.r.t. the families F_1 and F_2 respectively.

Given a matrix M, the parameter a) (=the number of matrix columns) is known in advance, the parameters b) and c) are obtained in Stage (1), while the parameter d) is easily derived from the family of bases. On the other hand, when the family F_2 is considered, the parameters a), b) and c) are given with each matroid in catalogue [1]. The values of the parameter d) were derived from that catalogue, in order to be applied in Section 4.

Our hierarchy of numerical features associated to a matroid can be incirporated in a nested sequence of IF-statements in the following way:

```
IF the paramaters  a)  coincide THEN
  IF the paramaters  b)  coincide THEN
    IF we are not sure that M1 = M2 THEN
      IF the paramaters  c)  coincide THEN
        IF we are not sure that M1 = M2 THEN
          IF the paramaters  d)  coincide THEN
            IF we are not sure that M1 = M2 THEN
                Compare directly  F1  and  F2.
```

The three lines "IF we are not sure that $M_1 = M_2$ THEN" are inserted because we may stop searching for a matroid isomorphic to M_1 in the catalogue, in the cases when there is a unique matroid in the catalogue with the given numerical parameters.

The first stopping possibility works only for the matroids of rank 0 and n (on the ground-set of cardinality n). The second one works for matroids of ranks 1 and $n-1$, but also for some matroids of "interior" ranks as well. Finally, an inspection of catalogue [1] assures us that the third stopping possibility will work with all the matroids on ≤ 7 elements; thus we need not compare the families themselves for $n \leq 7$. This is not the case for higher values of n, as is shown by the counterexample in the next section.

3. The auxiliary list of matroids

We shall proceed with a list which enables the calculation of each matroid on ≤ 7 elements, provided that the corresponding numbers n (cardinality of the ground-set), r (rank), b (number of bases), as well as the vector of sorted appearance frequencies w.r.t. the family of bases are given. Each triple (n, r, b) is followed by the symbol ":", each matroid is followed by the symbol ";". If there exists only one matroid (up to an isomorphism) with some given parameters n, r, b, then it is represented by "one" ; otherwise it is represented by the vector v.

There exists only one (empty) rank O matroid on n elements. There exist exactly n non-isomorphic rank 1 matroids, which respectively have i bases for each i between 1 and n. If $n > r/2$, then the matroids on ≤ 7 elements are uniquely represented by the sorted vectors of appearance frequencies, which are associated to their dual matroids. Therefore, we may conclude that it suffices to give a list for (n, r) belonging to the set $\{(4,2),(5,2),(6,2),(6,3),(7,2),(7,3)\}$.

3.1 List

$n = 4, \quad r = 2$

$b = 1:$ one ; $b = 2:$ one ; $b = 3:$ $(2,2,2,0)$; $(3,1,1,1)$;
$b = 4:$ one ; $b = 5:$ one ; $b = 6:$ one ;

$\underline{n = 5, \quad r = 2}$

$b = 1:$ one ; $b = 2:$ one ; $b = 3:$ $(2,2,2,0,0)$; $(3,1,1,1,0)$;
$b = 4:$ $(2,2,2,2,0)$; $(4,1,1,1,1)$; $b = 5:$ one ;
$b = 6:$ $(3,3,3,3,0)$; $(3,3,2,2,2)$; $b = 7:$ one ;
$b = 8:$ one ; $b = 9:$ one ; $b = 10:$ one ;

$\underline{n = 6, \quad r = 2}$

$b = 1:$ one ; $b = 2:$ one ;
$b = 3:$ $(2,2,2,0,0,0)$; $(3,1,1,1,0,0)$;
$b = 4:$ $(2,2,2,2,0,0)$; $(3,3,2,2,0,0)$;
$b = 5:$ $(3,3,2,2,0,0)$; $(5,1,1,1,1,1)$;
$b = 6:$ $(3,3,3,3,0,0)$; $(3,3,2,2,2,0)$; $b = 7:$ one ;
$b = 8:$ $(4,3,3,3,3,0)$; $(4,4,2,2,2,2)$;
$b = 9:$ $(4,4,4,3,3,0)$; $(3,3,3,3,3,3)$;
$b = 10:$ one ; $b = 11:$ one ; $b = 12:$ one ;
$b = 13:$ $(5,5,5,3,3,3)$; $(5,5,4,4,4,4)$;
$b = 14:$ one ; $b = 15:$ one ;

$\underline{n = 6, \quad r = 3}$

$b = 1:$ one ; $b = 2:$ one ;
$b = 3:$ $(3,2,2,2,0,0)$; $(3,3,1,1,1,0)$;
$b = 4:$ $(3,3,3,3,0,0)$; $(4,2,2,2,2,0)$; $(4,4,1,1,1,1)$;
$b = 5:$ one ;
$b = 6:$ $(4,4,4,3,3,0)$; $(6,3,3,2,2,2)$; $(6,3,3,3,3,0)$;
$b = 7:$ $(5,5,5,3,3,0)$; $(7,4,4,2,2,2)$;
$b = 8:$ $(5,5,5,5,4,0)$; $(8,4,3,3,3,3)$; $(4,4,4,4,4,4)$;
$b = 9:$ $(6,6,5,5,5,0)$; $(6,6,6,3,3,3)$; $(9,4,4,4,3,3)$;
$b = 10:$ $(6,6,5,5,4,4)$; $(7,7,7,3,3,3)$; $(10,4,4,4,4,4)$;
$b = 12:$ $(7,7,7,7,4,4)$; $(8,8,5,5,5,5)$; $(6,6,6,6,6,6)$;
$b = 13:$ one ; $b = 14:$ one ;
$b = 15:$ $(9,8,8,8,6,6)$; $(9,9,7,7,7,6)$;
$b = 16:$ $(9,9,9,9,6,6)$; $(8,8,8,8,8,8)$; $(10,10,7,7,7,7)$;
$b = 17:$ one ; $b = 18:$ $(9,9,9,9,9,9)$; $(10,9,9,9,9,8)$;
$b = 19:$ one ; $b = 20:$ one ;

$\underline{n = 7, \quad r = 2}$

$b = 1:$ one ; $b = 2:$ one ;
$b = 3:$ $(2,2,2,0,0,0,0)$; $(2,1,1,1,0,0,0)$;

$b=4:$ $(2,2,2,2,0,0,0)$; $(4,1,1,1,1,0,0)$;
$b=5:$ $(3,3,2,2,0,0,0)$; $(5,1,1,1,1,1,0)$;
$b=6:$ $(3,3,2,2,2,0,0)$; $(6,1,1,1,1,1,1)$; $b=7:$ one ;
$b=8:$ $(4,3,3,3,3,0,0)$; $(4,4,2,2,2,2,0)$;
$b=9:$ $(4,4,4,3,3,0,0)$; $(3,3,3,3,3,3,0)$; $(5,5,2,2,2,2,0)$;
$b=10:$ $(4,4,4,4,4,0,0)$; $(5,5,2,2,2,2,2)$;
$b=11:$ $(5,4,4,3,3,3,0)$; $(6,6,2,2,2,2,2)$;
$b=12:$ $(4,4,4,4,4,4,0)$; $(5,5,5,3,3,3,0)$; $(4,4,4,3,3,3,3)$;
$b=13:$ one ; $b=14:$ $(5,5,5,5,4,4,0)$; $(6,5,5,3,3,3,3)$;
$b=15:$ $(5,5,5,5,5,5,0)$; $(6,4,4,4,4,4,4)$; $(6,6,6,3,3,3,3)$;
$b=16:$ one ; $b=17:$ one ;
$b=18:$ $(6,5,5,5,5,5,5)$; $(6,6,6,6,4,4,4)$;
$b=19:$ one ; $b=20:$ one ; $b=21:$ one ;

<u>$n=7, \quad r=3$</u>

$b=1:$ one ; $b=2:$ one ;
$b=3:$ $(3,2,2,2,0,0,0)$; $(3,3,1,1,1,0,0)$;
$b=4:$ $(3,3,3,3,0,0,0)$; $(4,2,2,2,2,0,0)$; $(4,4,1,1,1,1,0)$;
$b=5:$ $(5,3,3,2,2,0,0)$; $(5,5,1,1,1,1,1)$;
$b=6:$ $(4,4,4,3,3,0,0)$; $(6,3,3,2,2,2,0)$; $(6,3,3,3,3,0,0)$;
$b=7:$ $(5,5,5,3,3,0,0)$; $(7,4,4,2,2,2,0)$;
$b=8:$ $(5,5,5,5,4,0,0)$; $(4,4,4,4,4,4,0)$; $(8,4,3,3,3,3,0)$;
$(8,4,4,2,2,2,2)$;
$b=9:$ $(6,6,5,5,5,0,0)$; $(9,4,4,4,3,3,0)$; $(6,6,6,3,3,3,0)$;
$(9,5,5,2,2,2,2)$; $(9,3,3,3,3,3,3)$;
$b=10:$ $(10,4,4,4,4,4,0)$; $(7,7,7,3,3,3,0)$; $(6,6,6,6,6,0,0)$;
$(6,6,5,5,4,4,0)$; $b=11:$ one ;
$b=12:$ $(6,6,6,6,6,6,0)$; $(8,8,5,5,5,5,0)$; $(7,7,7,7,4,4,0)$;
$(6,6,6,6,4,4,4)$; $(8,8,8,3,3,3,3)$; $(12,4,4,4,4,4,4)$;
$(12,5,5,5,3,3,3)$;
$b=13:$ $(8,8,7,6,5,5,0)$; $(9,9,9,3,3,3,3)$; $(13,5,5,4,4,4,4)$;
$b=14:$ $(9,9,7,7,5,5,0)$; $(8,8,7,7,4,4,4)$; $(14,5,5,5,5,4,4)$;
$b=15:$ $(9,8,8,8,6,6,0)$; $(9,9,7,7,7,6,0)$; $(9,9,6,6,5,5,5)$;
$(15,5,5,5,5,5,5)$;
$b=16:$ $(9,9,9,9,6,6,0)$; $(8,8,8,8,8,8,0)$; $(10,10,7,7,7,7)$;
$(8,8,8,6,6,6,6)$; $(9,9,9,9,4,4,4)$;
$b=17:$ $(9,9,9,8,8,8,0)$; $(11,11,7,7,5,5,5)$;
$b=18:$ $(9,9,9,9,9,9,0)$; $(10,9,9,9,9,8,0)$; $(9,9,8,8,8,6,6)$;

	(9,9,9,9,6,6,6) ; (11,11,9,8,5,5,5) ;
$b = 19$:	(10,10,10,9,9,9,0) ; (11,11,9,7,7,6,6) ; (12,12,9,9,5,5,5) ;
$b = 20$:	(10,10,10,10,10,10,0) ; (12,12,8,7,7,7,7) ;
	(12,8,8,8,8,8,8) ; (10,10,8,8,8,8,8) ;
$b = 21$:	(11,11,9,8,8,8,8) ; (12,11,11,11,6,6,6) ;
	(13,13,9,7,7,7,7) ; (12,12,9,9,9,6,6) ;
$b = 22$:	(12,12,12,12,6,6,6) ; (13,13,9,9,8,7,7) ;
$b = 23$:	(13,11,11,10,10,8,8) ; (14,14,9,9,9,7,7) ;
$b = 24$:	(12,12,12,9,9,9,9) ; (12,11,11,11,11,8,8) ;
	(12,11,11,11,9,9,9) ; (14,14,9,9,9,9,8) ;
$b = 25$:	(13,13,13,9,9,9,9) ; (13,12,12,11,11,8,8) ;
	(15,15,9,9,9,9,9) ;
$b = 26$:	(13,13,12,11,11,9,9) ; (14,11,11,11,11,10,10) ;
	(14,14,11,11,11,8,8) ;
$b = 27$:	(13,13,13,12,12,9,9) ; (14,13,13,12,11,9,9) ;
	(12,12,12,12,12,12,9) ;
$b = 28$:	(13,13,13,13,12,10,10) ; (14,14,14,12,12,9,9) ;
	(13,13,13,12,11,11,11) ; (12,12,12,12,12,12,12) ;
$b = 29$:	(14,14,13,13,13,10,10) ; (13,13,13,12,12,12,12) ;
	(14,14,13,12,12,11,11) ;
$b = 30$:	(14,13,13,13,13,12,12) ; (14,14,14,14,14,10,10) ;
	(13,13,13,13,13,13,12) ; (15,14,14,12,12,12,11) ;
	(14,14,14,12,12,12,12) ;
$b = 31$:	(15,13,13,13,13,13,13) ; (14,14,13,13,13,13,13) ;
	(14,14,14,13,13,13,12) ; (15,15,15,12,12,12,12) ;
$b = 32$:	(15,14,14,14,13,13,13) ; (14,14,14,14,14,13,13) ;
	(14,14,14,14,14,14,12) ;
$b = 33$:	(15,14,14,14,14,14,14) ; (15,15,14,14,14,14,13) ;
$b = 34$:	one ; $b = 35$: one ;

3.2 A counterexample

The following counterexample (also derived from catalogue [1]) shows that the parameters n, r, b and the vector of sorted appearance frequencies are not sufficient to determine matroids on 8 elements up to an isomorphism:

The non-isomorphic rank 4 matroids M_1 and M_2 on 8 elements have 63 bases each, which are all the 4-subsets of the ground-set $\{A, B, C, D, E, F, G, H\}$, except for the subsets in the families

$\{ABCD, ABEF, ABGH, ACEG, BDFH, CDEF, CDGH\}$ and

$\{AEFG, ABCD, BEFH, ACGH, BDEG, CDFH, ADEH\}$
respectively.

The corresponding vectors of appearance frequencies of elements A, B, C, D, E, F, G, H (w.r.t. the families of bases of the matroids M_1 and M_2) are (31, 31, 31, 31, 32, 32, 32, 32) and (31, 32, 32, 31, 31, 32, 32, 31) respectively. After sorting, these two vectors become equal. This means that there exist two non-isomorphic matroids on 8 elements, with which the parameters b), c) and d) are coincident.

A further elaboration of this counterexample leads to a more fascinating conclusion that there exist even 22 non-isomorphic matroids on 8 elements with coincident a), b), c), d), although it was not possible to find only two matroids with all the four coinciding parameters on 7 elements. We shall give some further relevant details:

There exist 71 non-isomorphic rank 4 matroids on 8 elements with exactly 63 bases each ([1]). The distribution of these matroids with respect to the vectors of sorted appearance frequencies looks as follows:

$n = 8, \;\; r = 4, \;\; b = 63$

(32, 32, 32, 32, 31, 31, 31, 31) – 22 matroids
(32, 32, 32, 32, 32, 31, 31, 30) – 8 matroids
(33, 32, 32, 32, 31, 31, 31, 30) – 8 matroids
(33, 32, 32, 31, 31, 31, 31, 31) – 8 matroids
(33, 33, 31, 31, 31, 31, 31, 31) – 3 matroids
(32, 32, 32, 32, 32, 32, 30, 30) – 3 matroids

The following 19 vectors of sorted appearance frequencies have just one corresponding matroid each:

(32, 32, 32, 32, 32, 32, 32, 28) , (33, 33, 33, 31, 31, 31, 31, 29) ,
(33, 32, 32, 32, 32, 31, 31, 29) , (32, 32, 32, 32, 32, 32, 31, 29) ,
(34, 32, 32, 32, 32, 30, 30, 30) , (33, 33, 32, 32, 31, 31, 30, 30) ,
(34, 32, 32, 31, 31, 31, 31, 30) , (33, 33, 32, 31, 31, 31, 31, 30) ,
(35, 31, 31, 31, 31, 31, 31, 31) , (34, 32, 31, 31, 31, 31, 31, 31) ,
(33, 32, 32, 32, 32, 31, 30, 30) , (33, 33, 33, 32, 32, 30, 30, 29) ,
(33, 33, 33, 33, 32, 30, 30, 28) , (34, 33, 33, 32, 32, 30, 29, 29) ,
(33, 33, 33, 33, 32, 30, 29, 29) , (34, 33, 33, 31, 31, 30, 30, 30) ,

(34, 34, 33, 31, 31, 30, 30, 29) , (34, 34, 33, 31, 30, 30, 30, 30) ,
(35, 33, 33, 31, 30, 30, 30, 30)

4. An illustrative example

As an illustration, we shall give the results of the application of our algorithm, for $m = 2$ and $n = 4$, to a class $M(x, y)$ of 2×4 matrices, the entries of which are defined by using two independent real variables x and y in the following way:

$$
\begin{array}{ll}
a[1,1] = y - x - 3 \; ; & a[1,2] = 0 \; ; \\
a[1,3] = 0 \text{ if abs(sqr}(x) + \text{sqr}(y) - 49) < 7 \; , & \\
a[1,3] = 1 \text{ otherwise} \; ; & a[1,4] = x - 2 \; ; \\
a[2,1] = 0 \; ; & a[2,2] := y + 2 \cdot x + 5 \; ; \\
a[2,3] := x \cdot (y - 3) \; ; & a[2,4] := -2 + \text{abs}(y - 3) \; ;
\end{array}
$$

Let the seven non-isomorphic matroids for $(n, r) = (4, 2)$, listed in the above order, be denoted by 1, 2, 3, 4, 5, 6, 7 respectively. Then the list of the values of $f(M(x, y))$ for $-10 \leq x, y \leq 10$ looks as in Table 1. below.

It can be easily verified (by computer) that all the considered 441 matroids are of rank 2. All the seven rank 2 matroids on 4 elements can be found among them. The seven "lines" corresponding to the matrix fields $a[1,1]$, $a[1,4]$, $a[2,2]$, $a[2,3]$, $a[2,4]$ (two "lines" in each of the last two cases), as well the "circle" associated to the matrix field $a[1,3]$ can be recognized in the table below.

References

[1] Acketa, D., M., The catalogue of all the non-isomorphic matroids on at most 8 elements, Inst. of Math. Novi Sad, spec. issue, 1983, XVIII + 157.

[2] Chua, L. O., Lin, P. - M., Computer – aided Analysis of Electronic Circuits (Algorithms and Computational Techniques), Prentice-Hall, 1975.

[3] Stojaković, Z., private comunication

[4] Welsh, D. J. A., Matroid Theory, Academic Press, 1973.

$y\backslash x$	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
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-10	7	7	7	7	7	7	7	7	7	7	6	7	6	7	7	7	7	7	7	7	7

Table 1.

REZIME

JEDAN ALGORITAM ZA GENERISANJE MATROIDA PRIDRUŽENOG DATOJ MATRICI

U radu je opisan jedan algoritam, koji sa tačnošću do na izomorfizam odredjuje matroid pridružen datoj matrici. Algoritam je detaljno opisan za matrice, koje nemaju više od 7 kolona. Matroidi na malim nosačima su reprezentovani pomoću sledećih parametara: kardinalnost nosača, rang, broj baza, i vektor sortiranih učestanosti javljanja pojedinih elemenata nosača u familiji baza. Ovi parametri su dovoljni da opišu matroid sa najviše 7 elemenata, a nisu dovoljni da opišu matroide na većim nosačima.

Received by the editors October 23, 1989.