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COMMON FIXED POINT THEOREMS IN PROBABILISTIC METRIC SPACES WITH A CONVEX STRUCTURE

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ABSTRACT

Using some properties of the functions of noncompactness we prove in this paper some common fixed point theorems in probabilistic metric spaces with a convex structure.

1. INTRODUCTION

In [12] K. Menger introduced the notion of a probabilistic metric space and there are many papers and books on the theory of probabilistic metric spaces (for the bibliography, see the books [3], [15]). Some fixed point theorems in probabilistic metric spaces are proved in [2], [6], [7], [8], [9], [10], [16]. Since the space of generalized random variables is contained in the class of Menger spaces (special probabilistic metric spaces), fixed point theorems in Menger spaces are of a special interest for the stochastic analysis.

W. Takahashi introduced in [17] the notion of the convexity in metric spaces and some fixed point theorems in such spaces are proved in [4], [8], [9], [13], [14].

In this paper we shall prove common fixed point theo-

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rems in probabilistic metric spaces with a convex structure. These theorems contain, as special cases, many well known fixed point theorems.

2. PRELIMINARIES

We shall give some definitions and notations which will be used in the next text.

A triple (S,F,t) is a Menger space if S is a non-empty set, $F : S \times S \to D$, where D denotes the set of all distribution functions F and t is a T-norm [15] so that the following conditions are satisfied: $(F(p,q) = F_{p,q}$, for every $p,q \in S)$:

1. $F_{p,q}(u) = 1$, for every $u > 0$ if and only if $p = q$.
2. $F_{p,q}(0)=0$ for every $p,q \in S$.
3. $F_{p,q} = F_{q,p}$, for every $p,q \in S$.
4. $F_{p,r}(u+v) \geq t(F_{p,q}(u),F_{q,r}(v))$, for every $p,q,r \in S$ and every $u,v > 0$.

The (ε,δ)-topology is introduced by the (ε,δ)-neighbourhoods of $v \in S$:

$$U_v(\varepsilon,\delta) = \{u, u \in S, F_{u,v}(\varepsilon) > 1-\delta\},\ \varepsilon > 0, \delta \in (0,1).$$

One of the most interesting example of a Menger space is the following. Let (M,d) be a separable metric space and (Ω,Σ,P) a probability space. Further, let S be the space of all equivalence classes of measurable mappings of Ω into M, $t(u,v) = \max\{u + v - 1,0\}$ $(u,v \in [0,1])$ and for every $X,Y \in S$ and $s > 0$:

$$F_{X,Y}(s) = P\{\omega, d(X(\omega),Y(\omega)) < s,\ \omega \in \Omega\}$$

$(X = \{X(\omega)\},\ Y = \{Y(\omega)\})$. Then the triple (S,F,t) is a Menger space and the convergence in the (ε,δ)-topology and in the probability are identical.

Let us recall that a metric space (S,d) is with the convex structure in the sense of Takahashi if there exists a mapping $W : S \times S \times [0,1] \to S$ so that:

$$d(z,W(x,y,s)) \leq sd(z,x) + (1 - s)d(z,y)$$

for every $(x,y,z,s) \in S\times S\times S\times[0,1]$.

Definition. *Let* (S,F,t) *be a Menger space. A mapping* $W : S\times S\times[0,1] \to S$ *is said to be a convex structure on* S *if for every* $(x,y) \in S\times S$:

$$W(x,y,0) = y, \ W(x,y,1) = x \textit{ and for every } s \in (0,1),$$

$z \in S$ *and* $u > 0$:

$$F_{z,W(x,y,s)}(2u) \geq t(F_{z,x}(u/s),F_{z,y}(u/(1-s))).$$

It is easy to see that every metric space with the convex structure W can be considered as a Menger space (S,F,*min*) with the same function W. Every random normed space (S,F,t) is a Menger space with the convex structure W defined by:

$$W(x,y,s) = sx + (1-s)y \quad (x,y \in S, \ s \in [0,1]).$$

A nontrivial example of a Menger space with a convex structure is the following one.

Let (M,d) be a separable metric space with a convex structure W which has the property that for every $s \in [0,1]$ the mapping $(x,y) \to W(x,y,s)$ is continuous. Let (Ω,Σ,P) be a probability space. We shall prove that the Menger space of all equivalence classes of measurable mappings from Ω into M is a probabilistic metric space with a convex structure if $t(u,v) = \max\{u + v - 1,0\}$ and for every $X,Y \in S$: $F_{X,Y}(u) = P\{\omega,d(X(\omega),Y(\omega)) < u\}$, $u \in R$.

Let $\bar{W}(X,Y,s)(\omega) = W(X(\omega),Y(\omega),s)$, for every $\omega \in \Omega$, every $X,Y \in S$ and every $s \in [0,1]$. It is easy to see that

$\bar{W}: S\times S\times[0,1] \to S$ and that for every $X,Y,U \in S$, $s \in (0,1)$ and every $u > 0$:

$$F_{U,\bar{W}(X,Y,s)}(2u) \geq F_{U,X}(u/s) + F_{U,Y}(u/(1-s)) - 1.$$

Since $\bar{W}(X,Y,0)(\omega) = W(X(\omega),Y(\omega),0) = Y(\omega)$ and $\bar{W}(X,Y,1)(\omega) = W(X(\omega),Y(\omega),1) = X(\omega)$, for every $\omega \in \Omega$ it follows that the mapping $\bar{W}$ is a convex structure on the probabilistic metric space (S,F,t).

In this paper we shall suppose that a convex structure W on a Menger space (S,F,t) satisfies the condition:

$$F_{W(x,z,s),W(y,z,s)}(us) \geq F_{x,y}(u),$$

for every $(x,y,z) \in S\times S\times S$.

A similar condition for metric spaces with a convex structure is introduced in [3].

The notion of the Kuratowski function is introduced in [1] as a probabilistic generalization of the notion of the Kuratowski measure of noncompactness.

Suppose that (S,F,t) be a Menger space and A a nonempty subset of S. The function $D_A(\cdot)$, defined on the set $[0,\infty) = R^+$, by $D_A(u) = \sup_{s<u} \inf_{p,q\in A} F_{p,q}(s)$, $u \in R^+$ is called the probabilistic diameter of the set A and the set A is probabilistic bounded if and only if $\sup_{u>0} D_A(u) = 1$.

Let A be a probabilistic bounded subset of S. The Kuratowski function $\alpha_A(u)$, $u \in R = (-\infty,\infty)$, of the set A is defined by:

$$\alpha_A(u) = \sup\{r > 0,\ \text{there is a finite family } A_j (j\in J) \text{ such that } A = \bigcup_{j\in J} A_j \text{ and } D_{A_j}(u) \geq r, \text{ for every } j \in J\}$$

The Kuratowski function has the following properties:

1) $\alpha_A \in D$.

2) $\alpha_A(u) \geq D_A(u)$, for every $u \in R^+$.

3) $\emptyset \neq A \subset B \subset S \Rightarrow \alpha_A(u) \geq \alpha_B(u)$, for every $u \in R^+$.

4) $\alpha_{A\cup B}(u) = \min\{\alpha_A(u), \alpha_B(u)\}$, for every $u \in R^+$.

5) $\alpha_A(u) = \alpha_{\bar{A}}(u)$, $(u \in R)$, where $\bar{A}$ is the closure of A.

6) $\alpha_A = H \Leftrightarrow A$ is precompact, where $H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0. \end{cases}$

In [3] the function $\beta_A(u)$ is defined in the following way:

$$\beta_A(u) = \sup\{r > 0, \text{ there exists a finite subset } A_f \text{ of } S \text{ such that } \tilde{F}_{A,A_f}(u) \geq r\}$$

where:

$$\tilde{F}_{A,B}(u) = \sup_{s<u} \inf_{x\in A} \sup_{y\in B} F_{x,y}(s)$$

and A and B are two probabilistic bounded subsets of S. The function β satisfies 1) - 6) and the following inequalities:

$$\beta_A(u) \geq \alpha_A(u) \geq \beta_A(u/2), \text{ for every } u > 0, \text{ if } t = \min.$$

Let (S,F,t) be a Menger space, K a probabilistic bounded subset of S and $T : K \to S$ so that $T(K)$ is probabilistic bounded. If for every $B \subset K$:

$$\gamma_{T(B)}(u) \leq \gamma_B(u), \text{ for every } u > 0 \Rightarrow B \text{ is precompact}$$

$(\gamma_A \in \{\alpha_A, \beta_A\})$ we say that the mapping T is *densifying on the set K in respect to the function* γ or probabilistic γ-densifying. The map $T : K \to S$ $(K \subset S)$ is said to be *a probabilistic q-contraction* if there exists $q \in (0,1)$ so that for every $x,y \in K$:

$$F_{Tx,Ty}(qu) \geq F_{x,y}(u), \text{ for every } u > 0.$$

If $T : K \to K$ is a probabilistic q-contraction with a bounded set $O_T(x) = \{T^n(x), n \in N\}$ $(x \in K)$ it is known [7] that there exists one and only one element $x \in K$ such that $x = Tx$, under the assumption that T-norm t is continuous.

If $q = 1$ the mapping T is said to be nonexpansive. A mapping $T : K \to S$ is (W,x_0)-convex $(x_0 \in K)$ if for every $x \in K$, $TW(x,x_0,s) = W(Tx,x_0,s)$ $(s \in [0,1])$.

2. COMMON FIXED POINT THEOREMS

If (S,F,t) is a Menger space with a convex structure W and $K \subset S$ we say that K is *starshaped* if there exists $x_0 \in K$ (the star center of K) if for every $x \in K$ and every $s \in (0,1)$ $W(x,x_0,s) \in K$. For nonexpansive mappings defined on a starshaped subset of S we shall prove the following fixed point theorem.

Theorem 1. *Let (S,F,t) be a complete Menger space with a convex structure W and continuous T-norm t, K a closed and starshaped subset of S and $f : K \to K$ so that $f(K)$ is probabilistic bounded. If f is nonexpansive and such that there exists $m \in N$ so that f^m is densifying on the set $\{W(x,x_0,s), x \in f(K), s \in (0,1)\}$ in respect to γ $(\gamma \in \{\alpha,\beta\})$ where x_0 is the star center of K then there exists $x \in K$ so that $x = fx$.*

Proof. First, we shall prove that there exists a sequence $\{x_n\}_{n \in N}$ from the set $\{W(x,x_0,s), x \in f(K), s \in (0,1)\}$ such that for every $u > 0$:

$$\lim_{n\to\infty} F_{x_n, f^m x_n}(u) = 1.$$

Let $\{r_n\}_{n \in N}$ be a sequence of numbers from $(0,1)$ and $\lim_{n\to\infty} r_n = 1$. For every $n \in N$ and every $x \in K$, let:

$$f_n x = W(fx, x_0, r_n).$$

It is obvious that f_n is a probabilistic r_n-contraction since:

$$F_{W(fx,x_0,r_n),W(fy,x_0,r_n)}(u) =$$

$$= F_{W(fx,x_0,r_n),W(fy,x_0,r_n)}(r_n(u/r_n))$$

$$\geq F_{fx,fy}(u/r_n), \text{ for every } y,x \in K \text{ and every } u > 0.$$

Further for every $u > 0$ we have that:

$$\sup_{s<u} \inf_{x,y\in K} F_{W(fx,x_0,r_n),W(fy,x_0,r_n)}(s) \geq$$

$$\geq \sup_{\frac{s}{r_n}<\frac{u}{r_n}} \inf_{x,y\in K} F_{fx,fy}(s/r_n)$$

which implies that:

$$D_{f_n(K)}(u) \geq D_{f(K)}(u/r_n)$$

and hence $f_n(K)$ is probabilistic bounded for every $n \in N$. Then the set $O_{f_n}(x) = \{f_n^m x, m \in N\}$ is probabilistic bounded and hence, there exist, for every $n \in N$, $x_n \in K$ so that $f_n x_n = x_n$. Then for every $n \in N$ and every $u > 0$:

$$F_{x_n,fx_n}(2u) = F_{W(fx_n,x_0,r_n),fx_n}(2u) \geq$$

$$\geq t(F_{fx_n,fx_n}(u/r_n),F_{fx_n,x_0}(u/(1-r_n))) =$$

$$= t(1,F_{fx_n,x_0}(u/(1-r_n))) = F_{fx_n,x_0}(u/(1-r_n))$$

and since $f(K)$ is probabilistic bounded it follows that:

(1) $$\lim_{n\to\infty} F_{x_n,fx_n}(u) = 1, \text{ for every } u > 0.$$

Further, from the relation $x_n = W(fx_n,x_0,r_n)$, $(n \in N)$ it follows that $\{x_n\}_{n\in N} \subset W(f(K),x_0,(0,1))$. From the inequality:

$$F_{x_n,f^m x_n}(u) \geq t(F_{x_n,fx_n}(u/2), t(F_{x_n,fx_n}(u/2^2), \ldots$$

$$\ldots, F_{x_n,fx_n}(u/(2^{m-1})) \ \ldots)$$

the continuity of t and (1) we obtain that:

(2) $$\lim_{n\to\infty} F_{x_n,f^m x_n}(u) = 1, \text{ for every } u > 0.$$

We shall show, using (2), that there exists a convergent subsequence $\{x_{n_k}\}_{k\in N}$. First, we shall prove that:

(3) $$\gamma_{\{x_n,n\in N\}}(u) = \gamma_{\{f^m x_n,n\in N\}}(u), \text{ for every } u > 0.$$

Let us suppose that $\gamma = \beta$. In order to prove (3) we shall prove that for every $s \in (0,u)$:

(4) $$\beta_{\{f^m x_n,n\in N\}}(u - s) \leq \beta_{\{x_n,n\in N\}}(u).$$

Then from the left continuity of β it follows that:

$$\beta_{\{f^m x_n,n\in N\}}(u) \leq \beta_{\{x_n,n\in N\}}(u).$$

Similarly we can prove that:

$$\beta_{\{f^m x_n,n\in N\}}(u) \geq \beta_{\{x_n,n\in N\}}(u)$$

for every $u > 0$ which implies that (3) is satisfied. Hence, we shall prove that (4) is satisfied. If $\beta_{\{f^m x_n,n\in N\}}(u - s) = 0$ for some u and $s \in (0,u)$ then (4) holds. So, we shall suppose that $\beta_{\{f^m x_n,n\in N\}}(u - s) > 0$ for every $u > 0$ and every $s \in (0,u)$. Let $r > 0$ be such that:

$$r < \beta_{\{f^m x_n,n\in N\}}(u - s).$$

Then there exists a finite set $A_f \subset S$ such that:

$$\inf_{n\in N} \max_{z\in A_f} F_{f^m x_n, z}(u - s) > r$$

and hence there exist, for every $n \in N$, $z_n \in A_f$ such that $F_{f^m x_n, z_n}(u - s) > r$. From the relation $t(1,r) = r$ and the continuity of t it follows that for every $d \in (0,r)$ there exists $d' \in (0,1)$ so that:

$$1 \geq h > 1-d' \Rightarrow t(h,r) > r-d.$$

From (2) it follows that there exists $n_0(s,d') \in N$ so that:

$$F_{f^m x_n, x_n}(s/2) > 1-d', \text{ for every } n \geq n_0(s,d')$$

which implies that:

$$(5) \qquad F_{x_n, z_n}(u - s/2) \geq t(F_{x_n, f^m x_n}(s/2), F_{f^m x_n, z_n}(u - s)) >$$
$$> r - d,$$

for every $n \geq n_0(s,d')$.

From (5) we obtain that:

$$r-d \leq \beta_{\{x_n, n \geq n_0(s,d')\}}(u) = \beta_{\{x_n, n\in N\}}(u).$$

Since d is an arbitrary element of the interval $(0,r)$ we obtain that $\beta_{\{x_n, n\in N\}}(u) \geq r$ which implies (4). The mapping f^m is densifying on $W(f(K), x_0, (0,1))$ in respect to β and since

$$\beta_{\{f^m x_n, n\in N\}}(u) \leq \beta_{\{x_n, n\in N\}}(u)$$

we obtain that the set $\{x_n, n\in N\}$ is precompact. This means that there exists a convergent subsequence $\{x_{n_k}\}_{k\in N}$. Suppose that $\gamma = \alpha$. Similarly as in the case $\gamma = \beta$ we shall prove that:

$$\alpha_{\{f^m x_n, n\in N\}}(u - s) \leq \alpha_{\{x_n, n\in N\}}(u)$$

for every $u > 0$ and every $s \in (0,u)$. Let $r > 0$ be such that $0 < r < \alpha_{\{f^m x_n, n\in N\}}(u - s)$. From the definition of the function α it follows that there exist $A_1, A_2, \ldots, A_n \subset S$ so that:

$$(6) \qquad \{f^m x_n, n \in N\} = \bigcup_{j=1}^{n} A_j, D_{A_j}(u - s) \geq r, j \in \{1,2,\ldots,n\}.$$

Relation (6) implies that $F_{x,y}(u - s) \geq r, j \in \{1,2,\ldots,n\}$ for every $x,y \in A_j$. Let $d \in (0,r)$ and $d' \in (0,1)$ be such that

$$1 \geq h_1, h_2 > 1-d' \Rightarrow t(h_1, t(r,h_2)) > r-d.$$

Let $B_j = \{z, F_{z,y}(s/4) > 1-d'$, for some $y \in A_j\}$, $j \in \{1,2,\ldots,n\}$ and $n_1(s,d') \in N$ so that:

$$F_{x_n, f^m x_n}(s/4) > 1-d', \text{ for every } n \geq n_1(s,d').$$

Then we can prove that $\{x_n, n \geq n_1(s,d')\} \subset \bigcup_{j=1}^{n} B_j$ where $D_{B_j}(u) \geq r-d$, for every $j \in \{1,2,\ldots,n\}$. This implies that $\alpha_{\{x_n,n\in N\}}(u) \geq r-d$, where d is an arbitrary element from $(0,r)$. Hence:

$$\alpha_{\{x_n,n\in N\}}(u) \geq \alpha_{\{f^m x_n,n\in N\}}(u)$$

which implies that the set $\{x_n, n\in N\}$ is precompact. The space S is complete and so there exists a convergent subsequence $\{x_{n_k}\}_{k\in N}$. Let $\lim_{k\to\infty} x_{n_k} = x$. Then from (1) and the continuity of f we obtain that $fx = x$.

Theorem 2. *Let* K *be a closed, starshaped subset of* S *where* (S,F,t) *is a complete Menger space with a convex structure* W *and continuous* T*-norm* t. *Suppose that the mappings* $f,g,S,T : K \to K$ *are such that* S *and* T *commute with* f *(or* g*)*, $f(K)$ *(or* $g(K)$*) is probabilistic bounded and the following conditions are satisfied:*

(i) There exists $m \in N$ *such that* f^m *(or* g^m*) is probabilistic* γ*-densifying* $(\gamma \in \{\alpha,\beta\})$ *on the set* $\{W(x,x_0,s), x \in f(K), s \in (0,1)\}$ *(or* $\{W(x,x_0,s), x \in g(K), s \in (0,1)\}$*, where* x_0

is the star center for K, *and for all* x,y ∈ K:

$$F_{fx,gy}(s) \geq F_{Sx,Ty}(s), \text{ for every } s \geq 0.$$

(ii) S *and* T *are continuous and* (W,x_0)*-convex.*
Then there exists x∈K *such that* x = fx = gx = Sx = Tx.

Proof. It is easy to see that $x_0 \in Fix(S,T) = \{x, x \in K, x = Sx = Tx\}$ and that $Fix(S,T)$ is a closed and starshaped subset of S. Further for every $x,y \in Fix(S,T)$:

$$F_{fx,fy}(s) \geq F_{x,y}(s), \quad \text{for all } s > 0$$

and fx = gx, for every $x \in Fix(S,T)$. Let us prove that $f(Fix(S,T)) \subseteq Fix(S,T)$. Suppose that $x \in Fix(S,T)$. Then:

$$fx = fSx = Sfx, \; fx = fTx = Tfx$$

and hence $fx \in Fix(S,T)$. This means that $f(Fix(S,T)) \subset Fix(S,T)$. So, we can apply Theorem 1 taking for the set K the set $Fix(S,T)$.

From Theorem 2 we obtain as a Corollary the following result obtained by Xie Ping Ding in [4].

Corollary. *Let* K *be a closed, starshaped subset of* X *where* (X,F,t) *is a complete Menger space with a convex structure* W *and continuous* T*-norm* t. *Suppose that the mappings* f,g, S,T : K → K *are such that* S *and* T *commute with* f *(or* g*)*, f(K) *is probabilistic bounded and the following conditions are satisfied:*

(i) There exists m ∈ N *such that* f^m *(or* g^m*) is precompact on the set* $\{W(x,x_0,s), x \in f(K), s \in (0,1)\}$ *(or* $\{W(x,x_0,s), x \in g(K), s \in (0,1)\}$ *where* x_0 *is the star center for* K, *and for all* x,y ∈ K:

$$F_{fx,gy}(s) \geq F_{Sx,Ty}(s), \text{ for every } s > 0.$$

(ii) S *and* T *are continuous and* (W,x_0)*-convex.*

Then there exists $x \in K$ *such that* $x = fx = gx = Sx = Tx$.

Proof. It remains to be proved that the condition (i) of Theorem 2 is satisfied. Since f^m (or g^m) is precompact on the set $\{W(x,x_0,s),\ x \in f(K),\ s \in (0,1)\}$ (or on the set $\{W(x,x_0,s),\ x \in g(K),\ s \in (0,1)\}$) it follows from the property 6) of the Kuratowski function α and from the same property of the function β that:

$$\gamma_{f^m(W(f(K),x_0,(0,1)))} = H,\ \gamma \in \{\alpha,\beta\}.$$

Suppose now that for some $B \subset W(f(K),x_0,(0,1))$:

$$\gamma_{f^m(B)}(u) \leq \gamma_B(u), \text{ for every } u > 0.$$

We have to prove that the set B is precompact. Using property 3) we obtain that for every $u>0$:

$$1 = \gamma_{f^m(W(f(K),x_0,(0,1)))}(u) \leq \gamma_{f^m(B)}(u) \leq \gamma_B(u)$$

and so $\gamma_B(u) = 1$. This implies that the set B is precompact. This means that f^m is probabilistic γ-densifying. The following well known result of D. Göhde is also a Corollary of Theorem 1.

Corollary 2. *Let* $(X,\|\cdot\|)$ *be a Banach space,* K *a closed, starshaped subset of* $X, f : K \to K$ *a nonexpansive mapping so that there exists* $m \in N$ *such that* $f^m(K)$ *is relatively compact. Then there exists* $x \in K$ *such that* $x = fx$.

Proof. Every Banach space is a random normed space $(X,F,\min)$ where the mapping $F : X \to D$ is defined by:

$$F_x(u) = \begin{cases} 1, & \|x\| < u \\ 0, & \|x\| \geq u \end{cases}$$

It is easy to see that $F_{fx,fy}(u) \geq F_{x,y}(u)$, for every $x,y \in K$

and every u > 0. Further, the set $f^m(K)$ is relatively compact if and only if $\gamma_{f^m(K)} = H$ and so the mapping f^m is densifying on K in respect to γ ($\gamma \in \{\alpha,\beta\}$). Hence, if we take that $W(x,y,s) = sx + (1 - s)y$ ($x,y \in X$, $s \in [0,1]$) all the conditions of Theorem 1 are satisfied.

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REZIME

TEOREME O ZAJEDNIČKOJ NEPOKRETNOJ TAČKI U VEROVATNOSNIM METRIČKIM PROSTORIMA SA KONVEKSNOM STRUKTUROM

Korišćenjem osobina funkcija nekompaktnosti u ovom radu su dokazane neke teoreme o zajedničkoj nepokretnoj tački u verovatnosnim metričkim prostorima sa konveksnom strukturom.

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