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ASSOCIATED k-GROUPS OF n-GROUPS

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ABSTRACT

The notion of a free covering (k+1)-group of an (sk+1)-group was defined in [16] as a generalization of the notion of a free covering group (see [32]). In the paper various constructions of such (k+1)-groups are discussed and their functorial nature is emphasized. These constructions lead to the corresponding constructions of associated (k+1)-groups (the notion of an associated group is due to Post [32]) and in consequence to a new functor $As^s:Gr_{sk+1} \to Gr_{k+1}$. The main purpose of the second part of the paper is to describe some basic properties of the functor As.

1. INTRODUCTION

In investigations of the category of n-groups (abbreviated in the sequel to Gr_n) and also in investigations of properties of n-groups it is convenient to use some functors from the category Gr_n to the category Gr_m. Such functors appeared in the first papers on the theory of n-groups [7], [32] (of course implicitly since the notion of a functor was not known then). These were constructions of m-groups derived from n-groups and free

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convering groups of n-groups. In the meantime, various other constructions of m-groups from n-groups were found (cf. e.g. [2], [14], [15], [34], [10], [12], [13]), however many authors did not emphasize the functorial character of these constructions. The only exceptions are [1], [31] and a series of papers by the Macedonian school (e.g. [4]-[6]). The last ones are devoted to n-semigroups and n-groupoids rather than to n-groups. A systematic investigation of the functorial character of various constructions of n-groups was initiated in [16], [17] and is continued in [8], [9], [19]-[23], [25], [26], [28], [29]. For further bibliographic information see [10].

In this paper a new functor $As^s\colon Gr_{sk+1} \to Gr_{k+1}$ which assigns an associated (k+1)-group to an (sk+1)-group will be defined (cf. [31] for k=1). The notion of an associated group has already been introduced by Post in [32] and it is closely related to the notion of a free covering group. So, here, while considering constructions of associated (k+1)-groups, we shall say more about various constructions of free covering (k+1)--groups of (n+1)-groups.

2. PRELIMINARIES

The terminology and notation used in this paper are the same as that of [8], [9], [20], [26] (cf. also [19], [21], [23], [25]).

The symbol $\overset{(t)}{a}$ is well defined for $t \geq 0$ in each polyadic groupoid (G, f). For the purpose of our paper it is convenient to extend this definition to the case when t is a negative integer. But we can do so only under an additional condition imposed upon the polyadic groupoid (G, f):(G, f) should be a polyadic group. Now, let t be a negative integer. Then

$$f(\ldots, \overset{(t)}{a}, \ldots) = f_{(\cdot)}(\ldots, \overset{(u(n-2)+t)}{a}, \overset{(\underline{u})}{\bar{a}}, \ldots)$$

for an arbitrary u such that $u(n-2) \geq -t$. Similar notation is used when we deal with polyads. By a polyad (to be exact, an m-ad) in an n-group (G, f) we mean any sequence of elements $a_1,\dots,a_m \in G$ (possibly with repetitions). This sequence is denoted by $\langle a_1,\dots$ $\dots,a_m\rangle$ or $\langle a_1^m\rangle$. It is also convenient to consider an empty polyad.

In Section 3 we shall recall, after Post [32], a certain equivalence relation of polyads which will be denoted here by Θ (note that in [24], [27], [29] this relation is denoted by $\underset{f}{=}$ or $\underset{g}{=}$, respectively. In our investigations we do not distinguish polyads which are in the relation Θ. So in this paper the symbol $\langle a_1^m\rangle$ denotes the m-ad $\langle a_1,\dots,a_m\rangle$ up to Θ, i.e., the equivalence class of $\langle a_1^m\rangle$ with respect to Θ. According to this convention, the empty polyad can be identified with an identity n-ad of the (n+1)-group (G,f).

To avoid numerous repetitions, we shall introduce some abbreviations. Henceforth, throughout the paper we shall assume that $n=s\cdot k$. Furthermore, $\mathcal{G} = (G, f)$, $\mathcal{A} = (A, f)$, $\mathcal{B} = (B, f)$ shall always denote nonempty (n+1)-groups (f is always an (n+1)-group operation). Similarly, $\mathcal{G}^{*s} = (G^{*s}, f^*)$ shall denote a free covering (k+1)-group of an (n+1)-group $\mathcal{G}$ and (G_o, f^*) shall denote an associated (k+1)-group of $\mathcal{G}$ (when it is irrelevant how those (k+1)-groups have been constructed). In our paper we deal with various constructions of these (k+1)-groups, but by f^* we always mean a (k+1)-group operation in the free covering or in the associated (k+1)-group, irrespective of which construction we have in hand. However, the (k+1)-groups themselves will be denoted by different symbols, depending on their construction.

Let us recall briefly two important constructions of polyadic groups which will be used in this paper. Given a (k+1)-group (G, g), by $\Psi_s(G, g)$ we mean (as in [16], [8], [20], [26]) the (n+1)-group $(G, g_{(s)})$ with

$$(1)\qquad g_{(s)}(x_1^{n+1}) = \underbrace{g(g(\dots g}_{s}(g(x_1^{k+1}), x_{k+2}^{2k+1})\dots), x_{n-k+2}^{n+1}).$$

The (n+1)-group $\Psi_s(G, g)=(G, g_{(s)})$ is said to be an (n+1)-group derived from the (k+1)-group (G, g) (cf. [7], [32], [16]). This leads us to the forgetful functor $\Psi_s\colon Gr_{k+1} \to Gr_{n+1}$

(for more facts concerning Ψ_s see [20] and [26]). Note that the derived polyadic groups defined by (1) are a particular case of the C-derived polyadic groups defined in [16] (where they are called PE-derived ones). The functor Ψ_s is also denoted by PE--der^s or Der^s (as a particular case of the functor C-der^s, c. [29]). Next, let (G,f) be an (n+1)-group. By $ret^s_{a_1,\dots,a_{s-1}}(G,f)$ we mean (as in [2], [34], [8], [9]) the (k+1)-group (G,g) with

$$(2) \qquad g(x_1^{k+1}) = f(x_1, a_1^{s-1},\dots,x_k, a_1^{s-1}, x_{k+1}).$$

We use a special notation when the (s-1)-ads are chosen in some particular ways. Namely, if $a_1 = \dots = a_{s-1} = a$ we write briefly $Ret^s_a(G,f)$ instead of $ret^s_{a,\dots,a}(G,f)$. Similarly, if $a_1 = \dots = a_{s-2} = a$ and $a_{s-1} = \bar{a}$ we write $\bar{R}et^s_a(G,f)$ in place of $ret^s_{a,\dots,a,\bar{a}}(G,f)$. The (k+1)-group $ret^s_{a_1,\dots,a_{s-1}}(G,f)$ (also $Ret^s_a(G,f)$ and $\bar{R}et^s_a(G,f)$) is called a (k+1)-ary retract of the (n+1)-group (G,f). The assignment of the (k+1)-ary retract to an (n+1)-group is functorial (cf. [8]). So we have the functor $ret^s: Gr_{n+1} \to Gr_{k+1}$ (to be exact, the class of functors, since the (k+1)-groups $ret^s_{a_1,\dots,a_{s-1}}(G,f)$ depends on the choice of the (s-1)-ad $\langle a_1^{s-1}\rangle$) and also $Ret^s: Gr_{n+1} \to Gr_{k+1}$ and $\bar{R}et^s: Gr_{n+1} \to Gr_{k+1}$. In fact, the procedure of choice is not essential, since all functors obtained in this way are naturally equivalent (cf. [8]). The functor which is the composition of the functors $\Psi_u: Gr_{n+1} \to Gr_{un+1}$ and $ret^{su}: Gr_{un+1} \to Gr_{k+1}$ is denoted briefly by $ret^{s,u}: Gr_{n+1} \to Gr_{k+1}$ (and $Ret^{s,u}$, $\bar{R}et^{s,u}$ respectively). In particular, we have $ret^{s,u}_{a_1,\dots,a_{us-1}}(G,f) = ret^{su}_{a_1,\dots,a_{us-1}}\Psi_u(G,f)$.

The set $Z_s = \{0,1,\dots,s-1\}$ where s=2,3,... together with the (k+1)-ary operation $\varphi(x_1^{k+1}) = l_1+\dots+l_{k+1}+1 \pmod s$ is a cyclic (k+1)-group of order s (cf. [32], [16], [18], [26]). This (k+1)-group will be denoted by $C_{s,k+1}$. Additionally, by $C_{1,k+1}$ we mean the one-element (k+1)-group $(\{0\}, \varphi)$. Post has proved that the (k+1)-group $C_{s,k+1}$ has the unique element 1 of order 1 (in the sense of [32]; in the sense of [9] this is an element of order k) if and only if g.c.d. (k,s)=1. Then, as it is easy to check, $lk+1 = \varepsilon s$. This fact will be used in the constructions of some (k+1)-groups in Section 4.

By $\bar{x}$ we always mean the skew element to x in an appropriate (n+1)-group (even though polyadic groups of some other arities, especially (k+1)-groups, are actually under consideration).

To avoid numerous repetitions, we shall write f.c. (k+1)-groups for free covering (k+1)-groups and f.c. groups for free covering groups.

3. VARIOUS CONSTRUCTIONS OF FREE COVERING (k+1)-GROUPS OF (n+1)-GROUPS

The notion of a free covering (k+1)-group of an (n+1)-group was defined in [16] and investigated [18]-[20], [22]. This is a generalization of the well-known notion, succesfully exploited by many authors, of a free covering group, which was introduced by Post in [32]. Note that free covering groups have recently been called universal groups by many authors.

Recall (cf. [16], [20]) that a pair $\langle G^{*s},\tau\rangle$ where $\tau:G \to \Psi_s(G^{*s})$, $G\in Gr_{n+1}$, $G^{*s}\in Gr_{k+1}$, is said to be a free covering (k+1)-group of an (n+1)-group G if for each homomorphism $H:G \to \Psi_s(B)$, where $B\in Gr_{k+1}$, there exists a unique homomorphism $h^*: G^{*s} \to B$ such that $\Psi_s(h^*)\tau = h$. It is worth adding that by Theorem 2 of [18] (also Theorem 2 of [16]) for every f.c. (k+1)-group G^{*s}, there exists a unique homomorphism $\zeta:G^{*s} \to C_{s,k+1}$ with $\zeta^{-1}(0) = \tau(G)$ (if $G\neq\emptyset$, then ζ is an epimorphism). This is why such a (k+1)-group will be denoted also by $\langle G^{*s},\tau,\zeta\rangle$ or by $\langle G^{*s},\zeta\rangle$.

The following construction of a free covering (k+1)-group of an (n+1)-group is a generalization of the construction used in [16]. Let $G = (G,f)$ be an arbitrary nonempty (n+1)-group. Fix a sequence of polyads $p_o = \langle c_{1,1},\ldots,c_{1,n}\rangle$, $p_i = \langle c_{i,1},\ldots,c_{i,ik}\rangle$ $(i = 1,\ldots,s-1)$ in G. By p_i' $(i = 0,1,\ldots,s-1)$ we always mean a polyad inverse to the polyad p_i in G. Often such a sequence of polyads shall be briefly denoted by p. From the set $G^{*s} = G \times Z_s$ (admitting $Z_1 = \{0\}$) and define a $(k+1)$-ary operation f^* in G^{*s} in the following way:

$$(3) \qquad f^*((x_1,l_1),\dots,(x_{k+1},l_{k+1})) =$$

$$= (f_{(\cdot)}(x_1,p_{l_1},\dots,x_{k+1},p_{l_{k+1}},p'_{\varphi(l_1^{k+1})}),\ \varphi(l_1^{k+1}))$$

for $x_1,\dots,x_{k+1} \in G$ and $l_1,\dots,l_{k+1} \in Z_s$.

The above-defined $(k+1)$-groupoid will be denoted by $fc^s_{p_o,\dots,p_{s-1}}G$ (or briefly by $fc^s_p G$). Note that if the n-ad p_o is an identity polyad in G, we get

$$f^*((x_1,l_1),\dots,(x_{i-1},l_{i-1}),\ (x_i,0),$$

$$(x_{i+1},l_{i+1}),\dots,(x_{k+1},l_{k+1})) = (f_{(\cdot)}(x_1,p_{l_1},\dots,x_{i-1},$$

$$p_{l_{i-1}},x_i,x_{i+1},p_{l_{i+1}},\dots,x_{k+1},p_{l_{k+1}},p'_{\varphi(l_1^{i-1},0,l_{i+1}^{k+1})},$$

$$\varphi(l_1^{i-1},0,l_{i+1})).$$

This formula enables us to take the empty polyad for p_o, which is equivalent to assumption that p_o is an identity polyad in G. Such a $(k+1)$-groupoid will be occasionally denoted by $fc^s_{p_1,\dots,p_{s-1}}G$ (where p_o is not at all defined). We write also $fc^s_p G$ where $p = (p_1,\dots,p_{s-1})$.

For s=1 we may identify the set G with the set $G^{*s} = G\times\{0\}$. In this case we get $fc^1_{p_o}G = ret^{1,n+1}_{p_o}G$. In particular, if p_o is the empty polyad (or equivalently, if p_o is an identity polyad in G), we have $fc^1_{p_o}G = G$.

Note that a special choice of polyads (namely, when $p_i = \overset{(ik)}{\langle c \rangle}$ for an arbitrary but fixed element $c \in G$) leads to the construction of a free covering $(k+1)$-group given in [16] (and investigated in [18], [20]). The $(k+1)$-group described above will be denoted by $Fc^s_c G$ (instead of the more complicated symbol $fc^s_{p_o,\dots,p_{s-1}}G$).

Theorem 1. *The* $(k+1)$*-groupoid* $fc_p^sG = (G\times Z_s, f^*)$ *together with the embedding* $\lambda : G \to \Psi_s fc_p^s G$ *given by* $\lambda(x) = (f(x,p_o'),0)$ *is a free covering* $(k+1)$*-group of an* $(n+1)$*-group* G.

Proof. The mapping $\rho_{p;c} : G\times Z_s \to G\times Z_s$ given by $\rho_{p;c}(x,1) = (f(x,p_1,\overset{(n-1-lk)}{c},\bar{c}),1)$ is a bijection. One can check that $\rho_{p;c} : fc_p^sG \to Fc_c^sG$ is a homomorphism of the corresponding $(k+1)$-groupoids. Thus the $(k+1)$-groupoid fc_p^sG is a $(k+1)$-group (since, by Theorem 1 of [16], Fc_c^sG is a $(k+1)$-group). Moreover, $\rho_{p;c}\lambda(x) = \rho_{p;c}(f(x,p_o'),0) = (f_{(\cdot)}(x,p_o',p_o,\overset{(n-1-lk)}{c},\bar{c}),0) = \tau(x)$ for $x \in G$. Hence $\langle fc_p^sG, \lambda\rangle$ is a f.c. $(k+1)$-group of G, which completes the proof of Theorem 1.

There is only one difference between the construction of fc_p^sG and the construction used in [16]. In [16] a single element from G was chosen, here we choose a sequence of polyads. Since in various constructions of polyadic groups (those of $(k+1)$-ary retracts of $(n+1)$-groups, $(n+1)$-groups C-derived from $(k+1)$-groups etc.) it is common to choose polyads instead of simple elements, the construction of a f.c. $(k+1)$-group given above can be more convenient for applications. Choosing polyads in some particular ways we can obtain various constructions of f.c. $(k+1)$-groups.

For example, it may be done as follows: let $p_i = \langle \overset{(ik-1)}{c}, \bar{c}\rangle$ $(i=0,\dots,s-1)$ where c is an arbitrary but fixed element of G. Then

(4) $$f^*((x_1,l_1),\dots,(x_{k+1},l_{k+1})) = (f_{(\cdot)}(x_1,\overset{(l_1k-1)}{c},\bar{c},\dots,x_{k+1},\overset{(l_{k+1}k-1)}{c},\bar{c},\overset{(n-\varphi(l_1^{k+1})k)}{c}),\ \varphi(l_1^{k+1}))$$

for $x_1,\dots,x_{k+1} \in G$ and $l_1,\dots,l_{k+1} \in Z_s$. This $(k+1)$-group will be denoted by a special symbol $\bar{F}c_c^sG$. Note that (according to the remark about fc_p^1G) for $s=1$ we may identify $\bar{F}c_c^1G$ with

$\mathrm{ret}^{1,n+1}_{c,\ldots,c,\bar{c},\bar{c}}G$.

Now we shall give another construction of a free covering (k+1)-group of an (n+1)-group (it differs from the given-above ones in forming of the set G^{*s}). This construction is founded on the original construction of an f.c. group due to Post (cf. [32]).

Post has introduced an equivalence relation θ on the set of all polyads of a given (n+1)-group $G = (G,f)$. This relation is defined as follows: $\langle a_1^m\rangle \theta \langle b_1^{m+un}\rangle$ if and only if for a certain i $(i=1,\ldots,n+1-m)$ and for some elements $c_1,\ldots,c_r \in G$ the equality

$$(5) \qquad f_{(\cdot)}(c_1^i, a_1^m, c_{i+1}^r) = f_{(\cdot)}(c_1^i, b_1^{m+un}, c_{i+1}^r)$$

holds.

We allow m=0, assuming then that $\langle a_1^0\rangle$ is the empty polyad.

One can prove (cf. [32]) that $\langle a_1^m\rangle \theta \langle b_1^{m+un}\rangle$ if and only if for every $i=1,\ldots,n+1-m$ and for every sequence $c_1,\ldots,c_r$ equality (5) holds. Now, if a binary operation defined by concatenation of polyads is introduced in the set of all polyads (of arbitrary length) of an (n+1)-group G, then the above-mentioned relation is a congruence relation in the so-defined semigroup. Moreover, the quotient semigroup is even a group. We denote it by $\mathrm{Pfc}^n G = (G^{\sim},.)$. Note that $\mathrm{Pfc}^1 G = G$. Every element $a \in G$ may be treated as a polyad of length 1 in the (n+1)-group G. The mapping $\mu : G \to G^{\sim}$ given by $\mu(x) = \langle x\rangle$ (where, according to a remark in Section 2, by $\langle x\rangle$ we understand the equivalence class of $\langle x\rangle$ with respect to θ) is an embedding of G into $\Psi_n \mathrm{Pfc}^n G$. The group $\mathrm{Pfc}^n G$, together with the embedding μ, is a free covering group of G.

A free covering (k+1)-group of a given (n+1)-group is determined uniquely up to an isomorphism, and so $\mathrm{Pfc}^n G$ and $\mathrm{Fc}^n_c G$ are isomorphic. Moreover, there exists an isomorphism $\eta_c : \mathrm{Fc}^n G \to \mathrm{Pfc}^n G$ such that $\eta_c \tau = \mu$. By the Post Coset theorem (cf. [32]) and Theorem 1 of [18], it follows that the isomorphism η_c assigns an (l+1)-ad (of course, up to Θ) to an element (x,l); to be exact, $\eta_c(x,l) = \langle x, \overset{(l)}{c}\rangle$. In view of Corollary 1 of [18] the (k+1)-group $\mathrm{Fc}_c^s G$ is isomorphic to the sub-(k+1)-group of $\Psi_k \mathrm{Fc}_c^n G$ consisting of all elements of the form (x,lk) where l=0,...,s-1. The embedding $\xi:\mathrm{Fc}_c^s G \to \Psi_k \mathrm{Fc}_c^n G$ is then defined by $\xi(x,l)=(x,lk)$. The (k+1)-groups $\mathrm{Fc}_c^s G$ and $\eta_c\xi(\mathrm{Fc}_c^s G)$ are isomorphic. Denote the latter one by $\mathrm{Pfc}^s G$. As is easy to see, the (k+1)-group $\mathrm{Pfc}^s G$ is a sub-(k+1)-group of $\Psi_k \mathrm{Pf}^n G$ consisting of all polyads of length lk+1, i.e., of all (lk+1)-ads (up to Θ). For s=1 we have $\mathrm{Pfc}^1 G = G$.

Proposition 1. *The* (k+1)*-group* $\mathrm{Pfc}^s G$ *together with the embedding* $\mu: G \to \Psi_s \mathrm{Pfc}^s G$ *is a free covering* (k+1)*-group of an* (n+1)*-group* G.

Above, we have given several from many possible constructions of free covering (k+1)-groups. Note that in a sense the constructions of the (k+1)-groups $\mathrm{fc}_p^s G$ (depending on the choice of polyads) are derived from the construction of $\mathrm{Pfc}^s G$. Namely, Post has considered equivalence classes of polyads, whereas our constructions are based on some representatives of these classes. It happens that in various specific situations it is easier to deal with representatives rather than with equivalence classes. Corollary 1 of [18] enables us to give an appropriate construction of an f.c. (k+1)-group for any construction of an f.c. group (cf. [3], [4], [30], [33]).

As was noticed and discussed in [20] (cf. also [16] and [26]) the assignment of free covering (k+1)-groups to (n+1)-groups is of a functorial character. Note that in investigations of the category Gr_{n+1} it is convenient to add the empty (n+1)-group. However, all the above-described constructions of an f.c. (k+1)-group require the assumption that the (n+1)-group under consideration is nonempty. Therefore, when we define these functors, it is necessary to assume additionally that the free covering (k+1)-group of the empty (n+1)-group is the empty (k+1)-group (for k>1) or the one-element group (for k=1).

Various constructions of an f.c. (k+1)-group and even various choices of polyads or elements give different functors. But all of these are naturally equivalent. So, in investigations of this functor (denoted by Φ_s in [16], [17], [19]-[21], [23], [25], [26]), we can use an appropriate construction of the f.c. (k+1)-group.

Now, we shall give, in an explicit form, some of the natural equivalences mentioned above:

$$\alpha: fc^s \to fc^s \quad \text{where} \quad \alpha_{p;q}(x,l) = (f(x,p_1,q_1'),l);$$

$$\beta: fc^s \to Pfc^s \quad \text{where} \quad \beta_p(x,l) = \langle x, p_1 \rangle;$$

$$\gamma: Pfc^s \to fc^s \quad \text{where} \quad \gamma_p(\langle x_1^{lk+1} \rangle) = (f(x_1^{lk+1}, p_1'),l).$$

Choosing polyads in appropriate ways and substituting them for p and q in the above formulas we get various other natural equivalences which have already appeared in our considerations or will appear in the next sections. So we have:

$$\rho: fc^s \to Fc^s \quad \text{where} \quad \rho_{p;c}(x,l) = (f(x,p_1, \overset{(n-1-lk)}{c}, \bar{c}),1);$$

$$\eta: Fc^s \to Pfc^s \quad \text{where} \quad \eta_c(x,l) = \langle x, \overset{(l)}{c} \rangle;$$

$$\delta: \bar{F}c^s \to Pfc^s \quad \text{where} \quad \delta_c(x,l) = \langle x, \overset{(lk-1)}{c}, \bar{c} \rangle;$$

$$\omega: Pfc^s \to \bar{F}c^s \quad \text{where} \quad \omega_c(\langle x_1^{lk+1} \rangle) = (f(x_1^{lk+1}, \overset{(n-lk)}{c}),l).$$

It has been proved in [18] that if an (n+1)-group $G = (G,f)$ is derived from a (k+1)-group (G,g), then the (k+1)-group $(G,g)\times C_{s,k+1}$ is also a f.c. (k+1)-group of G. In Theorem 6 of [18] we gave the explicit form of the natural equivalence $\nu_c: Fc^s_c(G,f) \to (G,g)\times C_{s,k+1}$. Namely, $\nu_c(x,l) = (f(x, \overset{(n+l-s)}{c}, \overset{(s-1)}{d}),l)$ (where d denotes the skew element to c in (G,g)). However, in this case the (k+1)-group $Fc^s G$ itself was not of an especially simple form. The situation changes if we use the construction of Theorem 1 and choose an appropriate polyad.

Proposition 2. *Given an* $(n+1)$*-group* (G,f) *and a* $(k+1)$*-group* (G,g)*, let* $\langle c_1^k\rangle$ *be an identity* k*-ad in* (G,g)*. Put* $p_i = \langle c_1^k, \ldots, c_1^k\rangle$ *for* $i=1,\ldots,s-1$*. Then* $(G,f) = \Psi_s(G,g)$*, if and only if* $fc_p^s(G,f)=(G,g)\times C_{s,k+1}$.

$$\underbrace{\qquad\qquad}_{i}$$

Proof. Suppose that $(G,f) = \Psi_x(G,g)$. Then we have $f^*=g$.

Conversely, let $fc_p^s(G,f)=(G,g)\times C_{s,k+1}$. Then $f^*_{(s)} = g_{(s)}\times\varphi_{(s)}$, i.e., $f^*_{(s)}((x_1,l_1),\ldots,(x_{n+1},l_{n+1}))=(g_{(s)}(x_1^{n+1}), \varphi_{(s)}(l_1^{n+1}))$. Putting $l_i=0$ for $i=1,\ldots,n+1$ we obtain $(f(x_1^{n+1}),0) = (g_{(s)}(x_1^{n+1}),0)$, i.e., $f = g_{(s)}$.

4. ASSOCIATED (k+1)-GROUPS OF (n+1)-GROUPS

Together with the notion of a free covering group in [32] Post introduced the notion of an associated group of an n-group. By the Post Coset theorem every f.c. group $G^{*n} = (G^{*n},f^*)$ of an $(n+1)$-group $G = (G,f)$ contains an invariant subgroup (G_o,f) such that G^{*n}/G_o is a cyclic group of order n. Post called this group (G_o,f^*) the associated group of an $(n+1)$-group G. The construction of such a group is founded on the construction of a free covering group. Basing the construction of an associated group on the Post construction we define an associated group as the set of all the equivalence classes of n-ads with concatenation as the group operation. Hence, the associated group of a group is identical to the group itself.

The construction in the previous section enables us to describe the associated group of an $(n+1)$-group $G = (G,f)$ as the set of elements of the form $(x,n-1)$ with an appropriate operation f^*, i.e., (when we identify the set G with the set $G_o = G\times\{n-1\}$) as a binary retract of the $(n+1)$-group (G,f). According to Theorem 1 of [8], every binary retract is isomorphic to an associated group. Moreover, choosing an appropriate n-ad in G one can show that every binary retract of G is an associated group of G, provided an appropriate construction of an f.c. group is used.

In Theorem 2 of [18] a characterization of free covering (k+1)-groups of (n+1)-groups is given, which is a generalization of the above-mentioned Coset theorem. However, in contrast to the Post theorem, when we deal with the case of $k=1$, in general the f.c. (k+1)-group G^{*s} need not contain an invariant sub-(k+1)-group $(G_o f^*)$ such that G^{*s}/G_o is a cyclic (k+1)-group of order s. The (k+1)-group G_o exists only in the case when g.c.d. $(k,s)=1$ (cf. Theorem 3 of [18]). And only in this case it makes sense to define an associated (k+1)-group of an (n+1)-group. Then, in view of Theorem 3 of [18], the f.c. (k+1)-group $\langle G^{*s},\tau,\zeta\rangle$ contains a sub-(k+1)-group (G_o,f^*) such that $G^{*s}/G_o = C_{s,k+1}$. Namely, $G_o = \zeta^{-1}(1)$, where $1 \in C_{s,k+1}$ is the unique element of order 1. Therefore, henceforth 1 shall always denote the so-defined element of $C_{s,k+1}$.

Let σ_G be an embedding of the (k+1)-group (G_o,f^*) into G^{*s}. From Theorem 4 of [18] (cf. also [1], [4], [32] for k=1), it follows that the assignment of the associated (k+1)-group (G_o,f^*) to an (n+1)-group G is functorial (just as for f.c. (k+1)-groups of (n+1)-groups) and σ is a natural transformation of these functors. Similarly to the case of f.c. (k+1)-groups, we assume that the associated (k+1)-group of the empty (n+1)-group is the empty one if $k>1$ or the one-element group for $k=1$. Since free covering (k+1)-groups play a central role in the construction of associated (k+1)-groups, this construction depends on the choice of a construction of f.c. (k+1)-groups. Namely, using the Post construction, by the associated (k+1)-group, we mean the set of equivalence classes of (lk+1)-ads (where l is defined as above, i.e., $l \in C_{s,k+1}$ is the only element with $\varphi(\overset{(k+1)}{l})=1$) together with a (k+1)-group operation defined just as in the f.c. (k+1)-group $\mathrm{Pfc}^s G$ (i.e., the concatenation of polyads). The associated (k+1)-group of an (n+1)-group G described above will be denoted by $\mathrm{As}^s G$ and the corresponding functor by $\mathrm{As}^s:\mathrm{Gr}_{n+1} \to \mathrm{Gr}_{k+1}$ (or briefly As). Note that, according to this definition, we have $As^1G = G$ and the functor $As^1: Gr_{n+1} \to Gr_{n+1}$ is simply the identity functor. The functor As^s can be embedded into the functor Pfc^s. To be exact, $\sigma : As^s \to Pfc^s$ is a natural transformation of these functors (cf. [31] for k=1).

The next construction of an associated (k+1)-group of an (n+1)-group G is founded on the construction of the (k+1)-group fc_p^sG. In this case we may identify the associated (k+1)-group of an (n+1)-group G with some retracts of G. Namely, every (k+1)-ary retract of the form $ret_{p_1}^{s,\varepsilon}$, where p_1 is an lk-ad and ε has the same meaning as in Section 2 (i.e., $lk+1 = \varepsilon s$), may be identified with the associated (k+1)-group. So, we have the natural transformations of the functors:

$$\sigma_p : ret_{p_1}^{s,\varepsilon}G \to fc_p^sG \;;$$

$$\sigma_c : Ret_c^{s,\varepsilon}G \to Fc_c^sG \;;$$

$$\sigma_c : \overline{R}et_c^{s,\varepsilon}G \to \overline{F}c_c^sG \;;$$

where σ is given by the formula $\sigma(x) = (x,1)$. Note that the retract and the corresponding (k+1)-group is always considered for the same choice of the lk-ad (or the element). All these constructions (As^s, $ret^{s,\varepsilon}$, $Ret^{s,\varepsilon}$, $\overline{R}et^{s,\varepsilon}$) lead to naturally equivalent functors, which simplifies the investigation of its properties. Since the associated (k+1)-group is a sub-(k+1)-group of the free covering (k+1)-group and the embedding σ is a natural transformation of the corresponding functors, the natural equivalences of the functors fc^s, Fc^s, $\overline{F}c^s$, Pfc^s given in the previous section determine the natural equivalences of the functors $ret^{s,\varepsilon}$, $Ret^{s,\varepsilon}$, $\overline{R}et^{s,\varepsilon}$, As^s. These equivalences are given by the same formulas. They differ only from the former equivalences by domains and codomains. That is why, to simplify notation, they will be denoted (though it is informal) by the same symbols. And so we write

$$\alpha_{p_1;q_1}: \mathrm{ret}^{s,\varepsilon}_{p_1} G \to \mathrm{ret}^{s,\varepsilon}_{q_1} G ;$$

$$\beta_{p_1}: \mathrm{ret}^{s,\varepsilon}_{p_1} G \to \mathrm{As}^s G ;$$

$$\gamma_{p_1}: \mathrm{As}^s G \to \mathrm{ret}^{s,\varepsilon}_{p_1} G$$

and so on.

5. SOME PROPERTIES OF THE FUNCTOR As

In the description of the functor As^s it is convenient to use the fact that As^s is naturally equivalent to the functors $\mathrm{ret}^{s,\varepsilon}$, $\mathrm{Ret}^{s,\varepsilon}$, $\bar{\mathrm{R}}\mathrm{et}^{s,\varepsilon}$.

Let $h:(A,f) \to (B,f)$ be a homomorphism of $(n+1)$-groups. The morphism $\bar{\mathrm{R}}\mathrm{et}^{s,\varepsilon}_{a;b} h: \bar{\mathrm{R}}\mathrm{et}^{s,\varepsilon}_a (A,f) \to \bar{\mathrm{R}}\mathrm{et}^{s,\varepsilon}_b (B,f)$ is given by $\bar{\mathrm{R}}\mathrm{et}^{s,\varepsilon}_{a;b} h(x) = f(h(x), \overset{(1k-1)}{h(a)}, h(\bar{a}), \overset{(n-1k)}{b})$ (cf. [8], [9]). Hence

PROPOSITION 3. *The functor* As *preserves and reflects monomorphisms and epimorphisms.*

Consider the following example. Let $\langle d_1^n \rangle$ be a central non-identity n-ad of order s in an $(n+1)$-group (A,f). The mapping $h:A \to A$ given by $h(x) = f(x, d_1^n)$ is an endomorphism of (A,f). As is easy to check,

$$\mathrm{Ret}^{s,\varepsilon}_{a;a} h(x) = f(h(x), \overset{(1k-1)}{h(a)}, h(\bar{a}), \overset{(n-1k)}{a}) = x.$$

On the other hand, it is evident that $\mathrm{Ret}^{s,\varepsilon}_{a;a} \mathrm{id}_A(x) = x$. Hence

Proposition 4. *The functor* As *is not faithful.*

Given a $(k+1)$-group (A,g), let $\langle e_1^k \rangle$ be an identity k-ad in (A,g). Observe that $\mathrm{ret}^{s,\varepsilon}_{e_1,\dots,e_k} \Psi_s(A,g) = (A,g)$. So we get

Proposition 5. *Every* $(k+1)$*-group is isomorphic to an associated* $(k+1)$*-group of an* $(n+1)$*-group.*

Since not every $(n+1)$-group is derived from a $(k+1)$-group (cf. [7], [32], [18]), it follows that non-isomorphic $(n+1)$-groups may have isomorphic associated $(k+1)$-groups. Hence, by Proposition 5 we get

Proposition 6. *The functor* As *is not full.*

Now we are going to give a criterion which enables us to check if the homomorphism $\tilde{h}:As^s(A,f) \to As^s(B,f)$ is of the form $\tilde{h} = As^s h$ (here $h:(A,f) \to (B,f)$). Using Theorem 2 of [9] and Proposition 6 of [9] we can give the required conditions for associated $(k+1)$-groups treated as $(k+1)$-ary retracts. Below we shall use Proposition 6 of [9] to formulate an appropriate condition for the functor As.

Theorem 2. *Given* $(n+1)$*-groups* (A,f) *and* (B,f), *let* $\tilde{h}:As^s(A,f) \to As^s(B,f)$ *be a homomorphism of the corresponding* $k+1$ *-groups. Then the following conditions are equivalent:*

i) $\tilde{h} = As^s h$ *where* $h:(A,f) \to (B,f)$;

ii) for every element a A *there exists an element* $b \in B$ *such that*

$$\tilde{h}(\langle \overset{(lk)}{a}, \bar{a}\rangle) = \langle \overset{(lk)}{b}, \bar{b}\rangle, \tag{6}$$

$$\tilde{h}(\langle \overset{(n(n+k-\varepsilon s-\varepsilon)+\varepsilon s)}{a} \rangle) = \langle \overset{(n(n+k-\varepsilon s-\varepsilon)+\varepsilon s)}{b} \rangle, \tag{7}$$

$$\tilde{h}(\langle \overset{(n-lk)}{a}, x_1^{lk+1}, \overset{(lk-1)}{a}, \bar{a}\rangle) = \tag{8}$$

$$= \langle \overset{(n-lk)}{b}, \tilde{h}(\langle x_1^{lk+1}\rangle), \overset{(lk-1)}{b}, \bar{b}\rangle;$$

iii) for some element $a\in A$ *there exists an element* $b\in B$ *such that equalities (6), (7) and (8) hold,*

Proof. Let $h:(A,f) \to (B,f)$ be a homomorphism of $(n+1)$-groups and let $\tilde{h} = As^s h$. Taking into consideration the natural equivalences given in the previous section and using the notation given there we can write an explicite form of the homomorphism $\tilde{h}$. Take any element $a\in A$, let $b = h(a)$. Then, by Lemma 2 of [9] $\bar{R}et^{s,\varepsilon}_{a;b}h(x) = h(x)$ for every $x\in A$, i.e., $h:\bar{R}et^{s,\varepsilon}_{a}(A,f) \to \bar{R}et^{s,\varepsilon}_{b}(B,f)$ is also a homomorphism of $(k+1)$-groups. As is easy to see, we have $\tilde{h} = \delta_b h \omega_a$, where $\omega_a : As^s(A,f) \to \bar{R}et^{s,\varepsilon}_{a}(A,f)$, and $\delta_b : \bar{R}et^{s,\varepsilon}_{b}(B,f) \to As^s(B,f)$ are isomorphisms defined in the previous section, i.e., $\omega_a(\langle x_1^{lk+1}\rangle) = f(x_1^{lk+1}, \overset{(n-lk)}{a})$ and $\delta_b(x) = \langle x, \overset{(lk-1)}{b}, \bar{b}\rangle$. Hence, $\tilde{h}(\langle x_1^{lk+1}\rangle) = \delta_b h \omega_a(\langle x_1^{lk+1}\rangle) = \delta_b(f(h(x_1),\ldots,h(x_{lk+1}), \overset{(n-lk)}{b})) = \langle f(h(x_1),\ldots,h(x_{lk+1}), \overset{(n-lk)}{b}), \overset{(lk-1)}{b}, \bar{b}\rangle = \langle h(x_1),\ldots,h(x_{lk+1})\rangle$. We show that $\tilde{h}$ satisfies (6), (7), (8). In fact

$$h(\langle \overset{(lk)}{a}, \bar{a}\rangle) = \delta_b h(f(\overset{(lk)}{a}, \bar{a}, \overset{(n-lk)}{a})) =$$

$$= \delta_b h(a) = \delta_b(b) = \langle \overset{(lk)}{b}, \bar{b}\rangle .$$

In a similar way one can check that condition (7) is satisfied. Now, we have

$$\tilde{h}(\langle \overset{(n-lk)}{a}, x_1^{lk+1}, \overset{(lk-1)}{a}, \bar{a}\rangle) =$$

$$= \delta_b h(f(\overset{(n-lk)}{a}, x_1^{lk+1}, \overset{(lk-1)}{a}, \bar{a}, \overset{(n-lk)}{a})) =$$

$$= \delta_b(f(\overset{(n-lk)}{b}, h(x_1),\ldots,h(x_{lk+1}), \overset{(lk-1)}{b}, \bar{b}, \overset{(n-lk)}{b})) =$$

$$= \langle \overset{(n-lk)}{b}, h(\langle x_1^{lk+1}\rangle), \overset{(lk-1)}{b}, \bar{b}\rangle ,$$

which shows that (8) is also satisfied. Thus (i) implies (ii).

Obviously, (ii) implies (iii).

Now, let $\tilde{h}:As^{s}(A,f) \to As^{s}(B,f)$ be a homomorphism of $(k+1)$-groups. Assume that for some $a\in A$ there exists $b\in B$ such that $\tilde{h}$ satisfies conditions (6), (7), (8). Let $h:A \to B$ be a mapping given by $h = \omega_b\tilde{h}\delta_a$. Thus h, as the composition of homomorphisms, is a homomorphism of $(k+1)$-groups, i.e., $h:\bar{R}et_a^{s,\varepsilon}(A,f) \to \bar{R}et_b^{s,\varepsilon}(B,f)$. Note that

$$h(a) = \omega_b\tilde{h}(\langle a, \overset{(lk-1)}{a}, \bar{a}\rangle) = \omega_b(\langle \overset{(lk)}{b}, \bar{b}\rangle) =$$

$$= f(\overset{(lk)}{b}, \bar{b}, \overset{(n-lk)}{b}) = b.$$

In similar way one can verify that

$$h(f_{(\cdot)}(\overset{(n(n+k-\varepsilon s-\varepsilon+1)+1)}{a})) = f_{(\cdot)}(\overset{(n(n+k-\varepsilon s-\varepsilon+1)+1)}{b}),\text{ i.e.,}$$

h satisfies the condition (4) of Proposition 6 of [9]. Observe that $\tilde{h}(\langle x, \overset{(lk-1)}{a}, \bar{a}\rangle) = \delta_b h\omega_a(\langle x, \overset{(lk-1)}{a}, \bar{a}\rangle) = \delta_b h(f(x, \overset{(lk-1)}{a}, \bar{a}, \overset{(n-k)}{a})) = \langle h(x), \overset{(lk-1)}{b}, \bar{b}\rangle$. Using this fact we get

$$h(f(\overset{(n-lk)}{a}, x, \overset{(lk-1)}{a}, \bar{a})) =$$

$$= \omega_b\tilde{h}(\langle f(\overset{(n-lk)}{a}, x, \overset{(lk-1)}{a}, \bar{a}), \overset{(lk-1)}{a}, \bar{a}\rangle) =$$

$$= \omega_b(\langle \overset{(n-lk)}{b}, \tilde{h}(\langle x, \overset{(lk-1)}{a}, \bar{a}\rangle), \overset{(lk-1)}{b}, \bar{b}\rangle) =$$

$$= \omega_b(\langle \overset{(n-lk)}{b}, h(x), \overset{(lk-1)}{b}, \bar{b}, \overset{(lk-1)}{b}, \bar{b}\rangle) =$$

$$= f(\overset{(n-lk)}{b}, h(x), \overset{(lk-1)}{b}, \bar{b}),$$

i.e., condition (3) of Proposition 6 of [9] is also satisfied. Thus $h:(A,f) \to (B,f)$ is a homomorphism of (n+1)-groups. Moreover, $\bar{R}et^{s,\varepsilon}_{a;b}h(x) = h(x)$ for every $x \in A$. On the other hand we have $\bar{R}et^{s,\varepsilon}_{a;b}h = \omega_b As^s h\delta_a$, whence $\omega_b \tilde{h}\delta_a = \omega_b As^s h\delta_a$. Then $\tilde{h} = As^s h$, which shows that (iii) implies (i). This completes the proof.

Note that if k=1, then we have s=n, l=n−1, ε=1. In this case the condition (6) is always satisfied, whenever $h:As^n(A,f) \to As^n(B,f)$ is a homomorphism of groups. So Theorem 2 for k=1 has a simpler formulation.

Corollary 1 (cf. [32]). *Given* (n+1)*-groups* (A,f), (B,f), *let* $h:As^n(A,f) \to As^n(B,f)$ *be a homomorphism of the corresponding groups. Then the following conditions are equivalent:*

i) $\tilde{h} = As^n h$ *where* $h:(A,f) \to (B,f)$;

ii) for every element $a \in A$ *there exists an element* $b \in B$ *such that*

$$\tilde{h}(\langle \overset{(n)}{a} \rangle) = \langle \overset{(n)}{b} \rangle, \tag{9}$$

$$\tilde{h}(\langle a, x_1^n, \overset{(n-2)}{a}, \bar{a} \rangle) = \langle b, \tilde{h}(\langle x_1^n \rangle), \overset{(n-2)}{b}, \bar{b} \rangle ; \tag{10}$$

iii) for some element $a \in A$ *there exists an element* $b \in B$ *such that equalities (9) and (10) hold.*

Associated (k+1)-groups of isomorphic (n+1)-groups are isomorphic. As was shown above, the converse statement is not always true; there are non-isomorphic (n+1)-groups with isomorphic associated (k+1)-groups. Already in [32] a condition was given which decides when the existence of an isomorphism of (n+1)-groups follows from the existence of an isomorphism of associated groups. Corollaries 5, 6, 7 and 8 of [9] give the required criteria for (k+1)-ary retracts. Now we shall give a criterion for associated (k+1)-groups treated as sets of appropriate polyads, i.e., for the functor As^s. Immediately from Theorem 2 we obtain

Corollary 2. *Given* (n+1)*-groups* (A,f) *and* (B,f), *let* $(A,g) = As^s(A,f)$ *and* $As^s(B,f) = (B,g)$. *Then the following conditions are equivalent:*

i) *the* (n+1)*-groups* (A,f) *and* (B,f) *are isomorphic;*

ii) *for every element* $a \in A$ *there exists an element* $b \in B$ *and an isomorphism* $h:(A,g) \to (B,g)$ *satisfying the equalities (6), (7) and (8);*

iii) *for some element* $a \in A$ *there exists an element* $b \in B$ *and an isomorphism* $h:(A,g) \to (B,g)$ *satisfying the equalities (6), (7) and (8).*

Note that for k=1 Corollary 2 takes form of the well-known Post condition (cf. [32], also [9]).

Corollary 3. *Given* (n+1)*-groups* (A,f) *and* (B,f), *let* $(A,\cdot) = As^n(A,f)$ *and* $(B,\cdot) = As^n(B,f)$. *Then the following conditions are equivalent:*

i) *the* (n+1)-groups (A,f) *and* (B,f) *are isomorphic;*

ii) *for every element* $a \in A$ *there exists an element* $b \in B$ *and an isomorphism* $h:(A,\cdot) \to (B,\cdot)$ *satisfying the equalities (9) and (10);*

iii) *for some element* $a \in A$ *there exists an element* $b \in B$ *and an isomorphism* $h:(A,\cdot) \to (B,\cdot)$ *satisfying the equalities (9) and (10).*

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REZIME

ASOCIRANE k-GRUPE n-GRUPA

Pojam slobodne pokrivajuće (k+1)-grupe (sk+1)-grupe je definisan u [16] kao uopštenje pojma slobodne pokrivajuće grupe (videti [32]). U ovom radu diskutovane su različite konstrukcije ovih (k+1)-grupa i njihova funkcionalna veza je naglašena. Ove konstrukcije dovode do odgovarajuće konstrukcije asociranih (k+1)-grupa (pojam asociranih grupa potiče od Posta [32]) i, kao posledica, do novog funktora $As^s:Gr_{sk+1} \to Gr_{k+1}$. Osnovni cilj drugog dela rada je opisivanje osnovnih osobina funktora As.

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