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ON A PROBLEM OF PARTIAL ALGEBRAS

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ABSTRACT

For a given language L, the problem of partial V - algebras asks whether there is a universal algorithm which for any finite partial V - algebra A, and any identity $p\approx q\in Eq(L\cup A)$ with no variables, decides whether or not $FV(A)\models p\approx q$. First, it is shown that the solution of the word problem implies the solution of the problem of partial algebras for any variety V. Second, if the problem of partial V - algebras is solvable, then, a class of finite presentations can be given for which the word problem is solvable.

1. BASIC NOTIONS

Let A be a set and $B\subseteq A^n$. Then $f: B\to A$ is called a *partial operation* on A of type n.

A *partial algebra* $\mathcal{A}$ is a pair (A,F), where A is a nonvoid set and F is a collection of partial operations on A. In our considerations F is always a finite set.

Let $\mathcal{A}$ be a partial algebra. Denote by $\Delta(\mathcal{A})$ the positive diagram of $\mathcal{A}$:

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$$\Delta(A) = \{f(a_1,\dots,a_n)=a \mid f\in F,\ a_1,\dots,a_n\in A,$$
$$f(a_1,\dots,a_n) \text{ is defined and equals } a \text{ in } A\}.$$

Of course, if A is finite, then $\Delta(A)$ is also finite.

Suppose that A and B are partial algebras. $\varphi: A\to B$ is called a *homomorphism* of A into B if, whenever $f(a_1,\dots,a_n)$ is defined, then so is $f(\varphi(a_1),\dots,\varphi(a_n))$ and

$$\varphi(f(a_1,\dots,a_n)) = f(\varphi(a_1),\dots,\varphi(a_n)).$$

A homomorphism φ is an *isomorphism* if φ is a bijection.

Let $A = (A,F)$ be a partial algebra and let $\emptyset\neq B\subseteq A$. Then

(i) B is a *subalgebra* of A if it is closed under all the operations in A i.e. if $b_1,\dots,b_n\in B$ and $f(b_1,\dots,b_n)$ is defined in A, then $f(a_1,\dots,a_n)\in B$.

(ii) B is a *relative subalgebra* of A if for all $f\in F$ and all $b_1,\dots,b_n$, $b\in B$, we have: $f(b_1,\dots,b_n)$ is defined and equals b if and only if $f(b_1,\dots,b_n)$ is defined in A and $f(b_1,\dots,b_n)=b$ in A.

It is not difficult to give an example of a partial algebra A and a set $B\subseteq A$, such that B is the carrier of some relative subalgebra of A but not the carrier of any subalgebra of A.

Let K be a class of algebras, A a nonvoid set, and F a set of partial operations on A. Then, (A,F) is a *partial K - algebra* if (A,F) is a relative subalgebra of an algebra B in K. For example, if L is the class of all the lattices, then, a partial algebra A is a partial L - algebra (or simply, partial lattice) if A is a relative subalgebra (or relative sublattice) of some lattice.

Definition 1.([3]). *Let* $\mathbf{K}$ *be a class of algebras and let* A *be a partial algebra. The algebra* $\mathbf{FK}(A)$ *is called the algebra freely generated by the partial algebra* A *over* $\mathbf{K}$ *if the following conditions are satisfied:*

(i) $\mathbf{FK}(A) \in \mathbf{K}$;

(ii) $\mathbf{FK}(A)$ *is generated by* A' *and there exists an isomorphism* $\chi: A' \to A$ *between* A' *and* A, *where* A' *is a relative subalgebra of* $\mathbf{FK}(A)$;

(iii) If φ *is a homomorphism of* A *into* $C \in \mathbf{K}$, *then, there exists a homomorphism* ψ *of* $\mathbf{FK}(A)$ *into* C *such that* ψ *is an extension of* $\chi\varphi$.

□

It is not very hard to prove that $FK(A)$ is unique up to the isomorphism and, if A is an algebra from K, then $FK(A) \cong A$. Also, it is well known that if K is an equational class, then $FK(A)$ exists if A is (isomorphic to) a relative subalgebra of an algebra B in K (see [3]). In other words, in the case of equational classes K, $FK(A)$ exists if A is a partial K - algebra.

For example, if A is a partial lattice, then, $FL(A)$ always exists. It is well known (see [3]) that these lattices (of the form $FL(A)$) are the lattices that can be described by finitely many generators and finitely many relations.

In order to establish a similar equivalence in the case of an arbitrary variety V, we need a precise definition of a finitely presented algebra in V. In the sequel, $F_V(X)$ denotes the free algebra over V, generated by a set of free generators X.

Definition 2.([1]). *Let* $V = \mathrm{mod}(\Sigma)$ *be a variety in the language* $\mathcal{L}$, G *a set of new constant symbols and* R *a set of identities with no variables in the language* $\mathcal{L}\cup G$. *Then* (G,R) *is called a presentation in* V. *Let* $\hat{V} = \mathrm{mod}(\Sigma\cup R)$. *The algebra presented by* (G,R) *in* V *is the reduct of* $F_{\hat{V}}(\emptyset)$ *to the language* $\mathcal{L}$. *We denote such an algebra by* $P_V(G,R)$. *We say that* $P_V(G,R)$ *is finitely presented if* G *and* R *are finite sets.* ☐

Proposition 1. *Let* $K = \mathrm{mod}(\Sigma)$ *be a variety,* A *a partial algebra. Then,*

$$FK(A) \cong P_K(A,\Delta(A)).$$

Proof. Let $|A| = m$ and α be an ordinal with $|\alpha| = m$, and $A = \{a_\gamma | \gamma<\alpha\}$. Let $\{x_\gamma | \gamma<\alpha\}$ be a free generating set of $F_K(\alpha)$. Define the mapping

$$\hat{h}: F_K(\alpha) \to P_K(A,\Delta(A))$$

as an extension of the mapping

$$h: \{x_\gamma | \gamma<\alpha\} \to \{a_\gamma | \gamma<\alpha\},\ h(x_\gamma) = a_\gamma, \quad \text{for} \quad \gamma<\alpha,$$

to a homomorphism of the whole algebra $F_K(\alpha)$ into $P_K(A,\Delta(A))$. Such a mapping $\hat{h}$ exists (it is unique), since $F_K(\alpha)$ is the free algebra of K and $P_K(A,\Delta(A))\in K$. Further on, $\hat{h}$ is "onto" since the algebra $P_K(A,\Delta(A))$ is generated by $\{a_\gamma | \gamma<\alpha\}$. If we denote by θ the kernel of $\hat{h}$, then we have

$$F_K(\alpha)/\theta \cong P_K(A,\Delta(A)).$$

On the other hand, we can prove that $F_K(\alpha)/\theta$ satisfies conditions (i), (ii) and (iii) of Definition 1. (see [3]).

Hence, we obtain

$$FK(A) \cong P_K(A,\Delta(A)). \quad \square$$

2. SOME DECIDABILITY PROBLEMS

Denote by $Eq(\mathcal{L})$ the set of identities in language $\mathcal{L}$. Let V be a variety in language $\mathcal{L}$ and (G,R) a finite presentation in V.

The *word problem for* (G,R) *in* V asks if there is an algorithm to determine, for any identity $p\approx q\in Eq(\mathcal{L}\cup G)$ with no variables, whether or not

$$P_V(G,R)\models p\approx q.$$

Let K be a class of algebras in language $\mathcal{L}$, and let A be a partial K - algebra. The *problem of partial* K *- algebra* A asks if there is algorithm to determine for any identity $p\approx q\in Eq(\mathcal{L}\cup G)$ with no variables whether or not

$$FK(A)\models p\approx q.$$

We are going to consider the following three problems for a variety V in language $\mathcal{L}$.

I The word problem in the first level for V asks whether there is a universal algorithm to solve the word problem for all finitely presented algebras in V.

II The problem of partial V - algebras asks if there is a universal algorithm which for any finite partial V - algebra A, and any identity $p\approx q\in Eq(\mathcal{L}\cup A)$ with no variables, decides whether or not

$$FV(A)\models p\approx q.$$

III The problem of quasi-identities for V asks if there is an algorithm which for any quasi-identity q in $\mathcal{L}$ decides whether or not

$$V \models q.$$

□

It is natural to look for the relationship between the problems I, II and III. We can prove the following result

Proposition 2.([2]). *For any variety, the problem of quasi-identities and the word problem in the first level are equivalent.* □

Hence, for any variety, the problems I and III are equivalent. It is not hard to prove, using Proposition 1. that the positive solution of problem I implies the existence of an algorithm from problem II:

Proposition 3. *Let* **K** *be a variety in language* $\mathcal{L}$. *If the word problem in the first level for* **K** *is solvable, then the problem of partial* **K** *- algebras is solvable too.*

Proof. Let A be a finite partial **K** - algebra, $p \approx q \in Eq(\mathcal{L} \cup A)$, with no variables. Then, because of Proposition 1.,

$$FK(A) \cong P_K(A, \Delta(A)),$$

so that

$$FK(A) \models p \approx q \quad \text{iff} \quad P_K(A, \Delta(A)) \models p \approx q.$$

Hence, directly from the algorithm for the solution of the word problem, we obtain an algorithm for the solution of the problem of partial algebras. □

Conversely, can we construct an algorithm for the solution of the word problem knowing an algorithm which solves the problem of partial algebras?

In this paper we shall give a class of finite presentations for which it is possible to construct an algorithm for the solution of the word problem.

3. SOME AUXILIARY CONSTRUCTIONS

Definition 3. *Let* $\mathbf{K}$ *be a variety in language* $\mathcal{L}$ *and* (A,R) *some finite presentation in* $\mathbf{K}$. *Then,*

(1) If t *is a term in* $\mathcal{L}$, *then by* Sub(t) *we denote the set of all the subterms of* t;

(2) $\mathrm{Sub}(R) = \cup\{\mathrm{Sub}(t) \mid (\exists s)(s\approx t\in R \vee t\approx s\in R\}$;

(3) $A' = \{C_\sigma \mid \sigma\in \mathrm{Sub}(R)\}\cup A$.

□

Note, that the elements of Sub(R) are terms in $\mathcal{L}\cup A$, with no variables. Denote by $|t|$ the length of a term t (i.e. the number of symbols in t).

Definition 4. *Let* $\mathbf{K}$ *be a variety in* $\mathcal{L}$ *and* (A,R) *a finite presentation in* $\mathbf{K}$.

(1) Define the mapping

$$\varphi: \mathrm{Sub}(\mathbf{R}) \to \mathrm{Eq}(\mathcal{L}\cup A')$$

in the following way:

(i) *If* $|t|=1$, *then* $\varphi(t)$ *is* $t\approx C_t$;

(ii) *If* $t = f(t_1,\dots,t_n)$ *where* f *is an* n-*ary function symbol and* $t_1,\dots,t_n$ *are terms, then* $\varphi(t)$ *is* $f(C_{t_1},\dots$ $\dots,C_{t_n}) \approx C_t$.

(2) Define the set $\mathbf{R}'$ *as*

$$R' = \varphi[\mathrm{Sub}(R)]\cup\{C_p\approx C_q \mid |p|=1 \text{ and } p\approx q\in R\}\cup$$

$$\cup\{f(C_{t_1},\dots,C_{t_n})\approx C_q \mid p=f(t_1,\dots,t_n) \text{ and } p\approx q\in R\},$$

where $\varphi[\mathrm{Sub}(R)] = \{\varphi(t) \mid t\in \mathrm{Sub}(R)\}$.

□

Note that if $t\in\mathrm{Sub}(R)$ and $|t| = 1$, then $t\in\mathbf{A}$ or t is a constant in $\mathcal{L}$ and the set R' is a set of identities, in the language $\mathcal{L}\cup A'$, with no variables.

Example 1. Let L be the variety of all the lattices in the language $\{\wedge,\vee\}$, $A = \{a, b, d\}$ and

$$R = \{a\wedge b\approx d,\ (a\vee d)\wedge b\approx d\wedge a\}.$$

Then,

$$\mathrm{Sub}(R) = \{a,\ b,\ d,\ a\wedge b,\ (a\vee d)\wedge b,\ a\vee d,\ d\wedge a\},$$

$$A' = \{C_a,\ C_b,\ C_d,\ C_{a\wedge b},\ C_{(a\vee d)\wedge b},\ C_{a\vee d},\ C_{d\wedge a},\ a,\ b,\ d\},$$

$$R' = \{a\approx C_a,\ b\approx C_b,\ d\approx C_d,\ C_a\wedge C_b\approx C_{a\wedge b},$$

$$C_{a\vee d}\wedge C_b\approx C_{(a\vee d)\wedge b},\ C_a\vee C_d\approx C_{a\vee d},$$

$$C_d\wedge C_a\approx C_{d\wedge a},\ C_a\wedge C_b\approx C_d,\ C_{a\vee d}\wedge C_b\approx C_{d\wedge a}\}.$$

□

Recall, now, the rules of inference in the equational logic.

Let τ, σ, θ, ρ,... be arbitrary terms in $\mathcal{L}$. Then,

(1) $\tau\approx\tau$ is an axiom;

(2) From $\sigma\approx\tau$ infer $\tau\approx\sigma$;

(3) From $\sigma\approx\tau$ and $\tau\approx\theta$ infer $\sigma\approx\theta$;

(4) If $\sigma_i\approx\tau_i$ for $i\in\{1,\dots,n\}$, then for any n-ary function symbol $f\in\mathcal{L}$ infer

$$f(\sigma_1,\dots,\sigma_n) \approx f(\tau_1,\dots,\tau_n);$$

(5) If $\sigma(x_1,\dots,x_n) \approx \tau(x_1,\dots,x_n)$, then for all terms p_i, $i\in\{1,\dots,n\}$ infer

$$\sigma(p_1,\dots,p_n) \approx \tau(p_1,\dots,p_n).$$

□

The following lemmas are about some syntactic properties of sets of identities, therefore the proofs are "technical".

Lemma 1. *Let* K *be a variety in* $\mathcal{L}$ *and* (A,R) *a finite presentation in* K. *Then, for every* $t\in\mathrm{Sub}(R)$, *we have*

$R' \vdash t \approx C_t$.

Proof. By induction on the length of t.

(i) If $|t| = 1$, then $t \approx C_t \in R'$ so that

$R' \vdash t \approx C_t$.

(ii) Let $t = f(t_1,\dots,t_n)$. Then, since $t \in Sub(R)$, it follows that $t_1,\dots,t_n \in Sub(R)$ and by the induction hypothesis

$R' \vdash t_i \approx C_{t_i}, \quad i \in \{1,2,\dots,n\}$.

By the definition of R', $f(C_{t_1},\dots,C_{t_n}) \approx C_t \in R'$. From the rules of equational logic it follows that

$R' \vdash f(t_1,\dots,t_n) \approx C_t$ i.e.

$R' \vdash t \approx C_t$.

□

Lemma 2. *Let* $\hat{R}$ *be the set of identities, in the language* $\mathcal{L}UA$, *which appears from the set* R' *in such a way that in every identity in* R' *every symbol* C_t $(t \in Sub(R))$ *is replaced by* t. *Then, for all the identities* $p \approx q$ *in* $\mathcal{L}UA$, *and any set* Σ *of identities in* $\mathcal{L}$, *we have*

$$\Sigma \cup \hat{R} \vdash p \approx q \quad \text{iff} \quad \Sigma \cup R \vdash p \approx q.$$

Proof. Let $e \in R'$. Prove that identity $\hat{e}$, which appears from e when we replace all the symbols C_t by t, is in R or it is a trivial identity of the form $\tau \approx \tau$. If $e \in R'$ we have four cases.

1) e is $t \approx C_t$, $|t| = 1$. Then, $\hat{e}$ is the trivial identity $t \approx t$.

2) e is $f(C_{t_1},\dots,C_{t_n}) \approx C_t$, where $t = f(t_1,\dots,t_n)$. But, then again, we have $t \approx t$.

3) e is $C_p \approx C_q$, where $|p| = 1$ and $p \approx q \in R$. Then $\hat{e}$ is $p \approx q$, which belongs to R.

4) e is $f(C_{t_1},\dots,C_{t_n}) \approx C_q$, where $p \approx q \in R$ and $p = f(t_1,\dots,t_n)$. Then $\hat{e}$ is $f(t_1,\dots,t_n) \approx q$, which belongs to R.

Therefore

$$R \vdash \hat{R}.$$

Conversely, $R \subseteq \hat{R}$ since if $p \approx q \in R$ and $p = f(t_1,\dots,t_n)$, then $f(C_{t_1},\dots,C_{t_n}) \approx C_q \in R'$, so that $f(t_1,\dots,t_n) \approx q \in \hat{R}$ i.e. $p \approx q \in \hat{R}$. Therefore, $\hat{R} \vdash R$ and the proof is ended.

□

Example 2. Let L be the variety of all the lattices and (A,R) be the same as in Example 1. Then,

$$\hat{R} = \{a \approx a,\ b \approx b,\ d \approx d,\ a \wedge b \approx a \wedge b,\ (a \vee d) \wedge b \approx (a \vee d) \wedge b,$$

$$a \vee d \approx a \vee d,\ d \wedge a \approx d \wedge a,\ a \wedge b \approx d,\ (a \vee d) \wedge b \approx d \wedge b\}$$

□

4. MAIN RESULT

From the set R' we obtained (by the replacement of some symbols) the set $\hat{R}$, which is of the same "deductive power" as the set R. On the other hand, our aim is to obtain from the set R' such a set of identities R* which can be the positive diagram of some partial algebra. First of all, from the set R' we have to take out the identities of the form $t \approx C_t$, where $|t| = 1$. Further on, in the set R' there are identities of the form $f(C_{t_1},\ldots,C_{t_n}) \approx C_p$ and $f(C_{t_1},\ldots,C_{t_n}) \approx C_q$, where $C_p \neq C_q$, and therefore are not supposed to be in the positive diagram of some partial algebra.

We shall formulate two rules:

(α) If a set of identities I contains an identity of the form $p \approx q$, where $|p| = |q| = 1$, then we take out this identity from the set I and in all the other identities we replace the symbol q by p.

(β) If a set of identities I contains some identities of the form $t \approx t_1$, $t \approx t_2$, where $t_1 \neq t_2$, then from I we take out the identity $t \approx t_2$ and in all the other identities we replace the symbol t_2 by t_1.

□

Let I be a set of identities. Denote by $\alpha(I)$ the set of identities which appears from I, if the rule α is applied, and by $\beta(I)$ the set of identities which appears from I, when the rule β is applied.

We say that the set of identities I is α-pure if $\alpha(I)=I$. Analogously, I is β-pure if $\beta(I)=I$. Obviously, if I is a finite set of identities, then there are natural numbers m,n such that the set $\alpha^n(I)$ is α-pure and the set $\beta^m(I)$ is β-pure.

Definition 5. *Let* (A,R) *be a finite presentation in a variety* **K**. *Let* n *be a natural number such that* $\alpha^n(R')$ *is* α*-pure and* m *be a natural number such that* $\beta^m(\alpha^n(R'))$ *is* β*-pure. Then, let*

$$R^* = \beta^m(\alpha^n(R'))$$

and A* *be the set of all these symbols from* $A' \cup const(\mathcal{L})$ *which appear in the identities of* R*.

□

It is not difficult to see that $R^* \subseteq Eq(\mathcal{L} \cup A^*)$ and that R* is the positive diagram of a partial algebra (in the language $\mathcal{L}$) with a carrier A*. Denote this algebra by $\mathcal{A}^*$.

We can assume, not losing generality, that $A \subseteq A^*$ i.e. that in application of the rule α we have $q \notin A$ and in the application of rule β, we have $t_2 \notin A$.

Example 3. Let L be the variety of all the lattices and (A,R) the same as in Example 1. Then,

$$\alpha^3(R') = \{a\wedge b \approx C_{a\wedge b},\ C_{a\vee d}\wedge b \approx C_{(a\vee d)\wedge b},\ a\vee d \approx C_{a\vee d},$$

$$d\wedge a \approx C_{d\wedge a},\ a\wedge b \approx d,\ C_{a\vee d}\wedge b \approx C_{d\wedge a}\},$$

$$R^* = \beta^2(\alpha^3(R')) = \{a\vee d \approx C_{a\vee d},\ d\wedge a \approx C_{d\wedge a},\ a\wedge b \approx d,$$

$$C_{a\vee d}\wedge b \approx C_{d\wedge a}\},$$

$$A^* = \{a,\ b,\ d,\ C_{a\vee d},\ C_{d\wedge a}\},$$

$$\mathcal{A}^* = (A^*, \wedge, \vee), \text{ where } \Delta(\mathcal{A}^*) = R^*$$

□

Lemma 3.

(i) *Let* $\Sigma(c,d)$ *be a set of identities in* $\mathcal{L}$ *which contain the symbols* c *and* d *as constant symbols and let* $\varphi(c)$ *be an identity not containing* d. *Then,*

$$\Sigma(c,d) \cup \{c \approx d\} \vdash \varphi(c) \quad \text{iff} \quad \Sigma(c,c) \vdash \varphi(c).$$

(ii) In point (i), we can put, instead of the constants c *and* d, *any closed terms* t_1 *and* t_2.

Proof.

(←). Let $\Sigma(c,c)\vdash\varphi(c)$. Then, since

$\Sigma(c,d)\cup\{c\approx d\}\vdash\Sigma(c,c)$,

we have

$\Sigma(c,d)\cup\{c\approx d\}\vdash\varphi(c)$.

(→). Prove the following:

If $\Sigma(c,d)\cup\{c\approx d\}\vdash\psi(c,d)$ and if

(*) $\psi_1(c,d),\ \psi_2(c,d),\dots,\psi_n(c,d) = \psi(c,d)$

is a proof-sequence for $\psi(c,d)$, then

(**) $\psi_1(c,c),\ \psi_2(c,c),\dots,\psi_n(c,c) = \psi(c,c)$

is a proof-sequence for $\psi(c,c)$ from $\Sigma(c,c)$.

Clearly, if n=1, then

$\psi(c,d)\in\Sigma(c,d)\cup\{c\approx d\}$

or it is an axiom of equational logic. Then, $\psi(c,c)\in\Sigma(c,c)$ or $\psi(c,c)$ is $c\approx c$ or it is an axiom, so that

$\Sigma(c,c)\vdash\psi(c,c)$.

Suppose the assertion holds for k<n and prove that it holds for n. Then, $\psi_n(c,d)$ is obtained by rules (2)-(5) from some

previous identities in sequence (*). It is not difficult to see then that $\psi_n(c,c)$ is obtained by the *same* rule from the corresponding formulas in sequence (**).

(ii) Analogously to (i).

□

Corollary. *Let* $K = \mathrm{mod}(\Sigma)$ *be a variety in* $\mathcal{L}$, (A,R) *a finite presentation in* K. *Then, for every identity* $p \approx q \in Eq(\mathcal{L} \cup A)$ *with no variables,*

$$\Sigma \cup R \vdash p \approx q \quad \text{iff} \quad \Sigma \cup R^* \vdash p \approx q.$$

Proof. On the one hand,

$\Sigma \cup R' \vdash p \approx q$ iff

iff $\Sigma \cup R' \cup \{t \approx C_t \mid t \in Sub(R)\} \vdash p \approx q$ (Lemma 1.)

iff $\Sigma \cup \hat{R} \vdash p \approx q$ (Lemma 3.)

iff $\Sigma \cup R \vdash p \approx q$. (Lemma 2.)

On the other hand,

$\Sigma \cup R' \vdash p \approx q$

iff $\Sigma \cup R^* \vdash p \approx q$,

because of the construction of R* and Lemma 3. Therefore,

$$\Sigma \cup R \vdash p \approx q \quad \text{iff} \quad \Sigma \cup R^* \vdash p \approx q.$$

□

Proposition 4. *Let* (A,R) *be a finite presentation in a variety* $\mathbf{K} = \mathrm{mod}(\Sigma)$, *in language* $\mathcal{L}$, *and let* A^* *be a* K*-partial algebra. Then, if the problem of the partial algebra* A^* *in* K *is solvable, the word problem for* (A,R) *in* K *is solvable, too.*

Proof. Let $p\approx q \in Eq(\mathcal{L} \cup A)$, with no variables. Then,

$P_{\mathbf{K}}(A,R) \models p\approx q$

$\Sigma \cup R \vdash p\approx q$ (Definition 2.)

iff $\Sigma \cup R^* \vdash p\approx q$ (Corollary)

iff $P_{\mathbf{K}}(A^*,R^*) \models p\approx q$ (Definition 2.)

iff $F\mathbf{K}(A^*) \models p\approx q$. (Proposition 1.)

□

Hence, if the problem of partial K-algebras is solvable, then we can solve the word problem for all those finite presentations (A,R) in K for which the corresponding partial algebra A^* is a K-partial algebra. It is easy to give an example of a variety K and a finite presentation (A,R), so that A^* is *not* K-partial algebra.

Example 4. Let L be the variety of all the lattices, $A = \{a,b,d\}$, $R = \{a\wedge b\approx a,\ b\wedge a\approx d\}$. Then, $R^* = R$ and $A^* = A$. But then A^* is not a partial lattice i.e. it is not a relative subalgebra of any lattice $\mathcal{B}$ (we shall have that a=d).

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REZIME

O JEDNOM PROBLEMU PARCIJALNIH ALGEBRI

Neka je A konačna parcijalna algebra, V varijetet a FV(A) algebra slobodno generisana sa A u V.

Problem parcijalnih V-algebri pita da li postoji univerzalni algoritam koji odlučuje da li za bilo koji identitet $p \approx q \in Eq(\mathcal{L} \cup A)$, bez promenljivih, važi

$$FV(A) \models p \approx q.$$

U radu se ispituje odnos problema reči za V i problema parcijalnih algebri. Pokazano je da rešivost problema reči implicira rešivost problema parcijalnih algebri. Takodje, data je klasa konačnih prezentacija za koju rešivost problema parcijalnih algebri implicira rešivost problema reči.

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