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A THEOREM ON COINCIDENCE POINTS FOR MULTIVALUED MAPPINGS IN CONVEX METRIC SPACES

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Abstract In this paper a theorem on coincidence points for the family $\{A_i\}_{i\in N}$ of multivalued mappings and singlevalued mappings S and T in convex metric spaces is proved. The obtained theorem contains, as special cases, the theorems from [1], [2] and [5].

1. Introduction

An extension of the contraction principle in convex metric spaces is obtained in [1].

THEOREM *A Let* (M,d) *be a complete convex metric space,* K *a nonempty closed subset of* M, $A: K \to CB(M)$ *(the family of all bounded, closed and nonempty subsets of* M*) such that* $A(\partial K) \subseteq K$ *and there exists* $q \in (0,1)$ *so that*

(1) $H(Ax,Ay) \le qd(x,y)$, *for every* $x,y\in K$.

Then there exists $x \in K$ *such that* $x\in Ax$.

Let us recall that (M,d) is a convex metric space if for any $x,y\in M$, $x\neq y$ there exists an element $z\in M$ such that $x\neq y\neq z$ and

$$d(x,z) + d(z,y) = d(x,y).$$

By H the Hausdorff metric is denoted. A generalization of Theorem A is proved in [4], where condition (1) is replaced by condition (2):

(2) $H(Ax,Ay)\le\alpha d(x,y) + \beta[d(x,Ax) + d(y,Ay)] + \gamma[d(x,Ay) + d(y,Ax)]$,

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for every $x,y \in K$ where $\alpha,\beta,\gamma\geq 0$, $\dfrac{(\alpha+\beta+\gamma)\ (1+\beta+\gamma)}{(1-\beta-\gamma)^2} < 1$.

A further generalization of Theorem A is given in [5].

DEFINITION *1. Let* K *be a nonempty subset of a metric space* (M,d) *and* S,T:K → CB(M). *Then* (S,T) *is said to be a generalized contraction pair on* K *if there exist* $\alpha,\beta,\gamma \geq 0$ *with* $\alpha + 2\beta + 2\gamma < 1$ *such that for any* $x,y\in K$

(3) $H(Sx,Ty) \leq \alpha d(x,y) + \beta[d(x,Sx) + d(y,Ty)] + \gamma[d(x,Ty) + d(y,Sx)]$.

THEOREM B [5] *Let* (M,d) *be a complete convex metric space,* K *a nonempty and closed subset of* M, (S,T) *be a generalized contraction pair on* K *so that*

$$S(\partial K) \cup T(\partial K) \subseteq K \text{ and } \frac{(\alpha+\beta+\gamma)(1+\beta+\gamma)}{(1-\beta-\gamma)^2} < 1.$$

Then there exists $z\in K$ *such that* $z\in Sz$ *and* $z\in Tz$.

In [2] inequality (3) is replaced by inequality (4):

(4) $H(Sx,Ty) \leq \alpha d(fx,fy) + \beta[d(fx,Sx) + d(fy,Ty)] + \gamma[d(fx,Ty) + d(fy,Sx)]$, for every $x,y \in K$, where $f:K \to M$, and under some additional conditions it was proved the existence of an element $z \in K$ such that $fz \in Sz$ and $fz \in Tz$.

We shall introduced the following definition.

DEFINITION 2 *Let* K *be a nonempty subset of a mertic space* (M,d), *for every* $i \in \mathbb{N}$, $A_i:K \to CB(M)$ *and* S,T: K → M. *The family* $\{A_i\}_{i\in\mathbb{N}}$ *is said to be a generalized* (S,T) *contraction family if there exist* $\alpha,\beta,\gamma \geq 0$ *such that* $\dfrac{(\alpha+\beta+\gamma)(1+\beta+\gamma)}{(1-\beta-\gamma)^2} < 1$ *and for every* $i,j \in \mathbb{N}$ $(i\neq j)$:

(5) $H(A_ix,A_jy) \leq \alpha d(Sx,Ty) + \beta[d(Sx,A_ix) + d(Ty,A_jy)] + \gamma[d(Sx,A_jy) + d(Ty,A_ix)]$, for every $x,y \in K$.

If $A:K \to CB(M)$ we say that A is H continuous if A is continuous as a mapping of (K,d) into (CB(M),H)

In this paper we shall prove a theorem on coincidence points for the family $\{A_i\}_{i\in\mathbb{N}}$, S and T if the family $\{A_i\}_{i\in\mathbb{N}}$ is a generalized (S,T) contraction.

THEOREM *Let* (M,d) *be a complete, convex metric space,* K *a nonempty closed subset of* M, S *and* T *continuous mappings from* K *into* M, $\{A_i\}_{i\in\mathbb{N}}$ *a family of mappings from* K *into* CB(M), *which is a generalized* (S,T) *contraction family, so that the following conditions are satisfied:*

1. *For every* $m\in N$ *and every* $y\in K$:

$$Ty\in K \Rightarrow T(A_m y \cap K) \subseteq A_m Ty$$
$$Sy\in K \Rightarrow S(A_m y \cap K) \subseteq A_m Sy.$$

2. $\partial K\subseteq SK \cap TK$, $A_m K \cap K\subseteq SK \cap TK$, *for every* $m\in N$ *and the following implications hold:*

$$Tx\in\partial K \Rightarrow A_m x \subseteq K, \text{ for every } m\in N,$$
$$Sx\in\partial K \Rightarrow A_m x \subseteq K, \text{ for every } m\in N.$$

Then there exists $z\in K$ *such that one of the following families of inequalities is satisfied:*

$$d(Sz, A_m z)\le \frac{\alpha+\beta+\gamma}{1-\beta-\gamma}d(Tz,Sz), \text{ for every } m\in N$$

$$d(Tz, A_m z)\le \frac{\alpha+\beta+\gamma}{1-\beta-\gamma}d(Tz,Sz), \text{ for every } m\in N.$$

If $T,S: M \to M$, A_1 *is H continuous, for every* $1\in N$ *and for every* $y\in K$ *and* $m\in N$:

$$Ty\in K \Rightarrow T(A_m y) \subseteq A_m Ty;\ Sy\in K \Rightarrow S(A_m y) \subseteq A_m Sy$$

then $\{Tz, Sz\} \cap A_m z \neq \emptyset$, *for every* $m\in N$.

Proof: Let $x\in\partial K$. Since $\partial K \subseteq SK \cap TK$ it follows that there exists $p_0\in K$ such that $Tp_0=x$. Further, from $Tp_0\in\partial K$ and the implication $Tu\in\partial K \Rightarrow A_1 u\subseteq K$ we conclude that $A_1 p_0\subseteq K$. From $A_1 K\cap K\subseteq SK\cap TK$ we have that $A_1 p_0\subseteq SK$ and hence there exists $p_1\in K$ such that $Sp_1 = p_1'\in A_1 p_0$.

Let $p_2'\in A_2 p_1$ so that

$$d(p_1', p_2') \le H(A_1 p_0, A_2 p_1) + \frac{1-\beta-\gamma}{1+\beta+\gamma}\cdot k,\ k = \frac{(\alpha+\beta+\gamma)(1+\beta+\gamma)}{(1-\beta-\gamma)^2}.$$

If $p_2'\in K$ from $p_2'\in A_2 K\cap K\subseteq TK$ it follows that there exists $p_2\in K$ so that $Tp_2=p_2'$. If $p_2'\notin K$ then there exists $p_2\in K$ so that $Tp_2\in\partial K$ and

$$d(Sp_1, Tp_2) + d(Tp_2, p_2') = d(Sp_1, p_2').$$

Let $p_3'\in A_3 p_2$ so that

$$d(p_2', p_3') \le H(A_2 p_1,\ A_3 p_2) + \frac{1-\beta-\gamma}{1+\beta+\gamma}\cdot k^2.$$

If $p_3'\in K$ from $p_3'\in A_3 K\cap K \subseteq SK$ it follows that there exists $p_3\in K$ so that $Sp_3=p_3'$. If $p_3'\notin K$ then there exists $p_3\in K$ so that $Sp_3\in\partial K$ and

$$d(Tp_2, Sp_3)+d(Sp_3, p_3') = d(Tp_2, p_3').$$

Continuing in this way we obtain that there exist two sequences $\{p_n\}_{n\in N}$ and $\{p'_n\}_{n\in N}$ such that:

1. For every $n\in N$, $p'_n \in A_n p_{n-1}$

2. For every $n\in N$ the following implications hold:

$$p'_{2n}\in K \Rightarrow p'_{2n} = Tp_{2n};$$

$$p'_{2n+1}\in K \Rightarrow p'_{2n+1} = Sp_{2n+1};$$

$$p'_{2n}\notin K \Rightarrow Tp_{2n}\in \partial K \text{ and}$$

$$d(Sp_{2n-1}, Tp_{2n}) + d(Tp_{2n}, p'_{2n}) = d(Sp_{2n-1}, p'_{2n}) ;$$

$$p'_{2n+1}\notin K \Rightarrow Sp_{2n+1}\in \partial K \text{ and}$$

$$d(Tp_{2n}, Sp_{2n+1}) + d(Sp_{2n+1}, p'_{2n+1}) = d(Tp_{2n}, p'_{2n+1}) .$$

3. For every $n\in N$

$$d(p'_n, p'_{n+1}) \le H(A_n p_{n-1}, A_{n+1} p_n) + k^n \frac{1-\beta-\gamma}{1+\beta+\gamma} .$$

We shall prove that there exists $z\in K$ so that $z = \lim_{n\to\infty} Tp_{2n} = \lim_{n\to\infty} Sp_{2n+1}$.

Let P_0, Q_0, P_1, Q_1 be define in the following way:

$P_0 = \left\{p_{2n};\ n\in N \text{ and } p'_{2n} = Tp_{2n}\right\}$,

$Q_0 = \left\{p_{2n+1};\ n\in N \text{ and } p'_{2n+1} = Sp_{2n+1}\right\}$,

$P_1 = \left\{p_{2n};\ n\in N \text{ and } p'_{2n} \ne Tp_{2n}\right\}$,

$Q_1 = \left\{p_{2n+1};\ n\in N \text{ and } p'_{2n+1} \ne Sp_{2n+1}\right\}$.

It is easy to prove that the following implications hold:

$$p_{2n} \in P_1 \Rightarrow p_{2n+1} \in Q_0 \text{ and } p_{2n-1} \in Q_0 ;$$

$$p_{2n-1} \in Q_1 \Rightarrow p_{2n} \in P_0 \text{ and } p_{2n-2} \in P_0 .$$

Hence, we have the following possibilities:

$$(p_{2n},p_{2n+1}) \in P_0\times Q_0 \ ; \ (p_{2n},\ p_{2n+1}) \in P_0\times Q_1 \ ;$$

$$(p_{2n},p_{2n+1}) \in P_1\times Q_0 \ .$$

a) If $(p_{2n},p_{2n+1}) \in P_0\times Q_0$ then from (5) we have

$$d(Tp_{2n},Sp_{2n+1}) = d(p'_{2n},p'_{2n+1}) \le H(A_{2n}p_{2n-1},\ A_{2n+1}p_{2n}) + k^{2n}\frac{1-\beta-\gamma}{1+\beta+\gamma} \le$$

$$\le \alpha d(Sp_{2n-1},\ Tp_{2n}) + \beta[d(Sp_{2n-1},\ A_{2n}p_{2n-1}) + d(Tp_{2n},A_{2n+1}p_{2n})] +$$

$$+ \gamma[d(Sp_{2n-1},A_{2n+1}p_{2n}) + d(Tp_{2n},\ A_{2n}p_{2n-1})] + k^{2n}\frac{1-\beta-\gamma}{1+\beta+\gamma} \le \alpha d(Sp_{2n-1},Tp_{2n})+$$

$$+\beta[d(Sp_{2n-1},\ Tp_{2n}) + d(Tp_{2n},Sp_{2n+1})] + \gamma[d(Sp_{2n-1},Sp_{2n+1}) + d(Tp_{2n},Tp_{2n})]+$$

$$k^{2n}\frac{1-\beta-\gamma}{1+\beta+\gamma} \le \alpha d(Sp_{2n-1},Tp_{2n}) + (\beta+\gamma)d(Sp_{2n-1},Tp_{2n}) + (\beta+\gamma)d(Tp_{2n},Sp_{2n+1}) +$$

$$k^{2n}\frac{1-\beta-\gamma}{1+\beta+\gamma} .$$

This implies that

$$d(Tp_{2n},Sp_{2n+1}) \le \frac{\alpha+\beta+\gamma}{1-\beta-\gamma} d(Sp_{2n-1},Tp_{2n}) + k^{2n}\frac{1}{1+\beta+\gamma} .$$

b) If $(p_{2n},p_{2n+1}) \in P_0\times Q_1$ then $d(Tp_{2n},Sp_{2n+1}) \le d(Tp_{2n},p'_{2n+1}) =$

$$= d(p'_{2n},p'_{2n+1}) \le H(A_{2n}p_{2n-1},A_{2n+1}p_{2n}) + k^{2n}\frac{1-\beta-\gamma}{1+\beta+\gamma}$$

which implies that

$$d(Tp_{2n},Sp_{2n+1}) \le \frac{\alpha+\beta+\gamma}{1-\beta-\gamma} d(Sp_{2n-1},Tp_{2n}) + k^{2n}\frac{1}{1+\beta+\gamma} .$$

c) If $(p_{2n},p_{2n+1}) \in P_1\times Q_0$ we shall prove that

$$d(Tp_{2n},Sp_{2n+1}) \le \frac{(1+\beta+\gamma)(\alpha+\beta+\gamma)}{(1-\beta-\gamma)^2} d(Sp_{2n-1},Tp_{2n-2}) + k^{2n-1}\frac{1}{1+\beta+\gamma}+k^{2n}\frac{1}{1+\beta+\gamma}$$

We have

$$d(p'_{2n},p'_{2n+1}) \le H(A_{2n}p_{2n-1},A_{2n+1}p_{2n}) + \frac{1-\beta-\gamma}{1+\beta+\gamma}k^{2n} \le \alpha d(Sp_{2n-1},Tp_{2n}) +$$

$$\beta[d(Sp_{2n-1},A_{2n}p_{2n-1}) + d(Tp_{2n},A_{2n+1}p_{2n})] + \gamma[d(Sp_{2n-1},A_{2n+1}p_{2n}) + d(Tp_{2n},$$

$$A_{2n}p_{2n-1})] + \frac{1-\beta-\gamma}{1+\beta+\gamma}k^{2n} = \alpha d(Sp_{2n-1},Tp_{2n}) + \beta[d(Sp_{2n-1},p'_{2n}) + d(Tp_{2n},p'_{2n+1})]$$

$$+ \gamma[d(Sp_{2n-1},p'_{2n+1}) + d(Tp_{2n},p'_{2n})] + \frac{1-\beta-\gamma}{1+\beta+\gamma}k^{2n} = \alpha d(Sp_{2n-1},Tp_{2n}) +$$

$$\beta[d(Sp_{2n-1},p'_{2n}) + d(Tp_{2n},Sp_{2n+1})] + \gamma[d(Sp_{2n-1},Sp_{2n+1}) + d(Tp_{2n},p'_{2n})]$$

$$+ \frac{1-\beta-\gamma}{1+\beta+\gamma}k^{2n} \le \alpha d(Sp_{2n-1},Tp_{2n}) + \beta[d(Sp_{2n-1},p'_{2n}) + d(Tp_{2n},Sp_{2n+1})] +$$

$$\gamma[d(Sp_{2n-1},Tp_{2n}) + d(Tp_{2n},Sp_{2n+1}) + d(Tp_{2n},p'_{2n})] + \frac{1-\beta-\gamma}{1+\beta+\gamma}k^{2n} = (\alpha+\gamma)$$

$$d(Sp_{2n-1},Tp_{2n}) + \beta d(Sp_{2n-1},p'_{2n}) + (\beta+\gamma)d(Tp_{2n},Sp_{2n+1}) + \gamma d(Tp_{2n},p'_{2n}) +$$

$$\frac{1-\beta-\gamma}{1+\beta+\gamma}k^{2n}.$$

From this we obtain that:

$$d(Tp_{2n},Sp_{2n+1}) \le d(Tp_{2n},p'_{2n}) + d(p'_{2n},p'_{2n+1}) \le (1+\gamma)\ d(Tp_{2n},p'_{2n}) +$$

$$+ (\alpha+\gamma)d(Sp_{2n-1},Tp_{2n}) + \beta d(Sp_{2n-1},p'_{2n}) + (\beta+\gamma)d(Tp_{2n},Sp_{2n+1})] + \frac{1-\beta-\gamma}{1+\beta+\gamma}k^{2n}$$

and since $\alpha<1$ and $d(Sp_{2n-1},Tp_{2n})+d(Tp_{2n},p'_{2n}) = d(Sp_{2n-1},p'_{2n})$ we have that

$$d(Tp_{2n},Sp_{2n+1}) \le \frac{1+\beta+\gamma}{1-\beta-\gamma}(Sp_{2n-1},p'_{2n}) + k^{2n}\frac{1}{1+\beta+\gamma}.$$

It is easy to see that $p'_{2n-1}=Sp_{2n-1}$, since $p_{2n}\in P_1$ implies that $p_{2n-1}\in Q_0$ and so:

$$d(Tp_{2n},Sp_{2n+1}) \le \frac{1+\beta+\gamma}{1-\beta-\gamma}d(p'_{2n-1},p'_{2n}) + k^{2n}\frac{1}{1+\beta+\gamma}.$$

Similarly as in case b) we can prove that $d(p'_{2n-1},p'_{2n}) \le \frac{\alpha+\beta+\gamma}{1-\beta-\gamma}\cdot$

$$\cdot\ d(Sp_{2n-1},Tp_{2n}) + k^{2n}\frac{1}{1+\beta+\gamma}$$

which implies that

$$d(Tp_{2n},Sp_{2n+1}) \le \frac{1+\beta+\gamma}{1-\beta-\gamma}\left[\frac{\alpha+\beta+\gamma}{1-\beta-\gamma}d(Sp_{2n-1},Tp_{2n-2}) + k^{2n-1}\frac{1}{1+\beta+\gamma}\right] + k^{2n}\frac{1}{1+\beta+\gamma} =$$

$$= \frac{(1+\beta+\gamma)(\alpha+\beta+\gamma)}{(1-\beta-\gamma)^2}d(Sp_{2n-1},Tp_{2n-2}) + k^{2n-1}\frac{1}{1-\beta-\gamma} + k^{2n}\frac{1}{1+\beta+\gamma}.$$

Similar inequality can be obtained for $d(Sp_{2n-1},Tp_{2n})$ and as in the Itoh paper it follows that there exists $z\in K$ so that

$$\lim_{n\to\infty} Sp_{2n-1} = \lim_{n\to\infty} Tp_{2n} = z.$$

Since $p_{2n}\in P_1$ implies that $p_{2n+1}\in Q_0$ and $p_{2n-1}\in Q_1$ implies that $p_{2n}\in P_0$ we conclude that there exists at least one sequence $\{Tp_{2n_k}\}_{k\in\mathbb{N}}$ or $\{Sp_{2n_k-1}\}_{k\in\mathbb{N}}$ such that: $Tp_{2n_k} \in A_{2n_k}p_{2n_k-1}$, for every $k\in\mathbb{N}$ or $Sp_{2n_k-1} \in$ $\in A_{2n_k-1}p_{2n_k-2}$ for every $k\in\mathbb{N}$.

Suppose that there exists a sequence $\{n_k\}_{k\in\mathbb{N}}$ such that $Tp_{2n_k} \in$ $A_{2n_k}p_{2n_k-1}$ for every $k\in\mathbb{N}$.

Since from condition 1 it follows that $STp_{2n_k} \in A_{2n_k}Sp_{2n_k-1}$, $k\in\mathbb{N}$ we have that $d(STp_{2n_k},A_mz) \le H(A_{2n_k}Sp_{2n_k-1}, A_mz)$ for every $k\in\mathbb{N}$ and every $m\in\mathbb{N}$.

Further, for $m\ne 2n_k$:

$$H(A_{2n_k}Sp_{2n_k-1}, A_mz) \le \alpha d(SSp_{2n_k-1},Tz) + \beta[d(SSp_{2n_k-1},A_{2n_k}Sp_{2n_k-1}) +$$
$$+ d(Tz,A_mz)] + \gamma[d(SSp_{2n_k-1},A_mz) + d(Tz, A_{2n_k}Sp_{2n_k-1})]$$

and since $STp_{2n_k}\in A_{2n_k}Sp_{2n_k-1}$ we have that

$$H(A_{2n_k}Sp_{2n_k-1},A_mz) \le \alpha d(SSp_{2n_k-1},Tz) + \beta[d(SSp_{2n_k-1},STp_{2n_k}) + d(Tz,A_mz)]+$$
$$+\gamma[d(SSp_{2n_k-1},A_mz) + d(Tz,ST_{2n_k})].$$

This implies that

$$\lim_{k\to\infty} d(STp_{2n_k}, A_m z) = d(Sz, A_m z) \le \alpha d(Sz,Tz) + \beta[d(Sz,Sz) + d(Sz,Tz) +$$

$$+ d(Sz, A_m z)] + \gamma[d(Sz, A_m z) + d(Tz,Sz)]$$

and so

$$d(Sz, A_m z) \le \frac{\alpha+\beta+\gamma}{1-\beta-\gamma} d(Tz,Sz), \quad m\in\mathbb{N}.$$

We shall prove that from the assumption:

$$T:M \to M \text{ and } Ty\in K \Rightarrow T(A_m y) \subseteq A_m Ty, \quad m\in\mathbb{N}$$

it follows that $Tz\in Az$, for every $m\in\mathbb{N}$.

First, we shall prove that

(6) $$\lim_{k\to\infty} d(Tp_{2n_k}, A_m p_{2n_k}) = 0.$$

Since $Tp_{2n_k} \in A_{2n_k} p_{2n_k-1}$, $k\in\mathbb{N}$ we have that

$$d(A_m p_{2n_k} Tp_{2n_k}) \le H(A_m p_{2n_k}, A_{2n_k} p_{2n_k-1})$$

since $Tp_{2n_k} \in A_{2n_k} p_{2n_k-1}$. Further,

$$H(A_m p_{2n_k}, A_{2n_k} p_{2n_k-1}) \le \alpha d(Sp_{2n_k-1}, Tp_{2n_k}) + \beta[d(Sp_{2n_k-1}, A_{2n_k} p_{2n_k-1}) +$$

$$d(Tp_{2n_k}, A_m p_{2n_k})] + \gamma[d(Sp_{2n_k-1}, A_m p_{2n_k}) + d(Tp_{2n_k}, A_{2n_k} p_{2n_k-1})]$$

and since $Tp_{2nk} \in A_{2n_k} p_{2n_k-1}$ we have that

$$d(A_m p_{2n_k}, Tp_{2n_k}) \le \alpha d(Sp_{2n_k-1}, Tp_{2n_k}) + \beta[d(Sp_{2n_k-1}, Tp_{2n_k}) + d(Tp_{2n_k}, A_m p_{2n_k})]$$

$$+\gamma[d(Sp_{2n_k-1}, A_m p_{2n_k}) + d(Tp_{2n_k}, Tp_{2n_k})] \le \alpha d(Sp_{2n_k-1}, Tp_{2n_k}) + \beta[d(Sp_{2n_k-1}, Tp_{2n_k})$$

$$+ d(Tp_{2n_k}, A_m p_{2n_k})] + \gamma[d(Sp_{2n_k-1}, Tp_{2n_k}) + d(Tp_{2n_k}, A_m p_{2n_k})]$$

and so

$$d(A_m p_{2n_k}, Tp_{2n_k}) \le \frac{\alpha+\beta+\gamma}{1-\beta-\gamma} d(Sp_{2n_k-1}, Tp_{2n_k}).$$

From this we obtain that

$$\lim_{k\to\infty} d(Tp_{2n_k}, A_m p_{2n_k}) \le \frac{\alpha+\beta+\gamma}{1-\beta-\gamma} d(z,z) = 0$$

and (6) is proved. For every $k \in \mathbb{N}$ there exists $z_k \in A_m p_{2n_k}$ such that

$$d(Tp_{2n_k}, z_k) < d(Tp_{2n_k}, A_m p_{2n_k}) + \frac{1}{k}$$

and from (5) we obtain that $\lim_{k\to\infty} d(Tp_{2n_k}, z_k) = 0$ which implies that $\lim_{k\to\infty} z_k = z$, since $\lim_{k\to\infty} Tp_{2n_k} = z$. Using the implication: $Ty \in K \Rightarrow T(A_m y) \subseteq A_m Ty$ from $z_k \in A_m p_{2n_k}$ we obtain that $Tz_k \in A_m Tp_{2n_k}$. Since A_m is H continuous, A_m is closed and from the continuity of T we obtain that $Tz \in A_m z$, for every $m \in \mathbb{N}$.

Remark 1. We shall prove that the following inequality is satisfied

$$(7) \qquad d(z, A_m z) \le \frac{\alpha+\beta+\gamma}{1-\beta-\gamma} d(Tz, z), \quad m \in \mathbb{N}.$$

We have for every $m \in \mathbb{N}$ and $k \in \mathbb{N}$ such that $2n_k \ne m$ that

$$d(Tp_{2n_k}, A_m z) \le H(A_{2n_k} p_{2n_k-1}, A_m z) \le \alpha d(Sp_{2n_k-1}, Tz) + \beta[d(Sp_{2n_k-1}, A_{2n_k} p_{2n_k-1}) + d(Tz, A_m z)] + \gamma[d(Sp_{2n_k-1}, A_m z) + d(Tz, A_{2n_k} p_{2n_k-1})] \le \alpha d(Sp_{2n_k-1}, Tz) + \beta[d(Sp_{2n_k-1}, Tp_{2n_k}) + d(Tz, A_m z)] + \gamma[d(Sp_{2n_k-1}, A_m z) + d(Tz, Tp_{2n_k})].$$

From this we have

$$\lim_{k\to\infty} d(Tp_{2n_k}, A_m z) = d(z, A_m z) \le \alpha d(z, Tz) + \beta[d(Tz,z) + d(z, A_m z)] + \gamma d(z, A_m z) + \gamma d(Tz, z)$$

which implies:

$$d(z, A_m z)(1-\beta-\gamma) \le (\alpha+\beta+\gamma)\, d(Tz, z).$$

Hence, (7) is proved.

Remark 2. Suppose that A_m is a singlevalued mapping for every $m \in \mathbb{N}$ and prove that $T,S:M \to M$ implies that $z = Sz = Tz = A_m z$, for every $m \in \mathbb{N}$.

For every $m \in \mathbb{N}$ and $k \in \mathbb{N}$ such that $2n_k \neq m$ we have

$$d(Tp_{2n_k}, A_{2n_k}Sp_{2n_k-1}) \le d(Tp_{2n_k}, A_m p_{2n_k}) +$$

$$d(A_m p_{2n_k}, A_{2n_k}Sp_{2n_k-1}) \le d(Tp_{2n_k}, A_m p_{2n_k}) + \alpha d(Tp_{2n_k}, SSp_{2n_k-1}) +$$

$$+\beta[d(Tp_{2n_k}, A_m p_{2n_k}) + d(SSp_{2n_k-1}, A_{2n_k}Sp_{2n_k-1})] +$$

$$\gamma[d(Tp_{2n_k}, A_{2n_k}Sp_{2n_k-1}) + d(SSp_{2n_k-1}, A_m p_{2n_k})].$$

If $k \to \infty$ we obtain that

$$d(z,Sz) \le \alpha d(z,Sz) + \beta d(Sz,Sz) + 2\gamma d(Sz,z).$$

Since $\alpha + 2\beta + 2\gamma < 1$ we conclude that $d(z,Sz) = 0$.

We have proved that $T:M \to M$ implies that $Tz = A_m z$, for every $m \in \mathbb{N}$.

Using the inequality

$$d(Sz,Tz) = d(Sz,A_m z) \le \frac{\alpha+\beta+\gamma}{1-\beta-\gamma} d(Sz,Tz)$$

for $\frac{\alpha+\beta+\gamma}{1-\beta-\gamma} < 1$ we obtain that $d(Sz,Tz) = 0$ and so

$$z = Sz = Tz = A_m z, \quad \text{for every } m \in \mathbb{N}.$$

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REZIME

TEOREMA O TAČKAMA KOINCIDENCIJE ZA VIŠEZNAČNA PRESLIKAVANJA U KONVEKSNIM METRIČKIM PROSTORIMA

U ovom radu dokazana je teorema o tačkama koincidencije za familiju $\{A_i\}_{i\in N}$ *višeznačnih preslikavanja i jednoznačna preslikavanja* S *i* T *u konveksnim metričkim prostorima. Dobijena teorema sadrži kao specijalne slučajeve, teoreme iz* [1], [2] *i* [5].

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