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FIXED POINT THEOREMS FOR MULTIVALUED MAPPINGS IN PROBABILISTIC METRIC SPACES WITH A CONVEX STRUCTURE

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ABSTRACT

In this paper we shall introduce the notion of a probabilistic metric space with a convex structure and prove fixed point theorems for multivalued mappings in such spaces.

1. INTRODUCTION

In [16] K.Menger introduced the notion of a probabilistic metric space and there are many papers from the theory of probabilistic metric spaces(for bibliographies see the books [4] , [15] and [27]).Since 1972 ,when V.Sehgal and A.Bharucha-Reid published the paper [29] ,there is an increasing interest in the fixed point theory in probabilistic metric spaces and this theory is now an important part of the stochastic analysis [1] .

Fixed point theorems for singlevalued and multivalued mappings in probabilistic metric spaces are proved in [3],[5]-[14], [19] -[24],[31][32] and [37] .Some very interesting results from the theory of probabilistic metric spaces are obtained by a group of Romanian mathematicians from the University of Timisoara.

Japan mathematician W.Takahashi introduced in[35] the notion of a metric space with a convex structure . This class of metric spaces includes normed linear spaces and metric spaces of

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hyperbolic type (see the books[33] and [34]). Iterative processes on metric spaces of hyperbolic type are investigated by W.A.Kirk and K.Goebel ([33], [34]). Some fixed point theorems in such spaces are proved in [18], [25], [33], [34], [35] and [36].

In this paper we shall generalize this notion to the class of probabilistic metric spaces, give a nontrivial example of such a space and prove fixed point theorems for multivalued mappings of nonexpansive type, which are defined on such spaces.

2. PRELIMINARIES

In this section we shall give necessary definitions and notations. By Δ we shall denote the set of all distribution functions F such that $F(0) = 0$ (F is a nondecreasing, leftcontinuous mapping from R into $[0,1]$ so that $\sup_{x\in R} F(x) = 1$).

The ordered pair (S,F) is a <u>probabilistic metric space</u> if S is a nonempty set and $F:S\times S\to\Delta$ ($F(p,q)$ for $p,q\in S$ is denoted by $F_{p,q}$) so that the following conditions are satisfied:

1. $F_{u,v}(x) = 1$, for every $x>0 \Rightarrow u = v$, and F is symmetric.

2. $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1 \Rightarrow F_{u,w}(x+y) = 1$.
   for every $(u,v,w) \in S\times S\times S$ and $(x,y) \in R^+ \times R^+$.

A <u>Menger space</u> is a triple (S,F,t) where (S,F) is a probabilistic metric space and t is a T-norm [27] so that:

$$F_{u,v}(x+y) \geq t(F_{u,v}(x), F_{v,w}(y))$$

for every $u,v,w\in S$ and every $x,y\in R^+$.

The (ε,λ)-topology is introduced by the family of neighbourhoods given by:

$$\mathcal{U} = \{U_v(\varepsilon,\lambda)\}_{(v,\varepsilon,\lambda)\in S\times R^+\times(0,1)}$$

where $U_v(\varepsilon,\lambda)$ is defined in the following way:

$$U_v(\varepsilon,\lambda) = \{u \mid u\in S, F_{u,v}(\varepsilon) > 1-\lambda\}.$$

Let A be a nonempty subset of S where (S,F) is a probabilistic metric space. The function $D_A(\cdot)$, defined on R^+ by:

$$D_A(x) = \sup_{t<x} \inf_{p,q\in A} F_{p,q}(t), \quad x\in R^+$$

is called the <u>probabilistic diameter of A</u> and the set A is <u>probabilistic bounded</u> if and only if [2]

$$\sup_{x\in R} D_A(x) = 1 \quad .$$

The notion of a random normed space is introduced by Sherstnev in [30] . A random normed space (S,F,t) is an ordered triple where S is a real or complex vector space, t is a T-norm which is stronger then T-norm $T_m: T_m(x,y) = \max\{x+y-1,0\}$ and the mapping $F: S \to \Delta$ satisfies the following conditions :

(a) $F_p = H \Leftrightarrow p = \theta$, where θ is the neutral element of S and the mapping H is defined by :

$$H(x) = \begin{cases} 0 & ,x \leq 0 \\ 1 & ,x > 0 \end{cases} \quad .$$

(b) For every $p \in S$, every $x \in R^+$ and every $\lambda \in K \setminus \{\theta\}$ (K is the scalar field of S):

$$F_{\lambda p}(x) = F_p\left(\frac{x}{|\lambda|}\right)$$

(c) For every $p,q \in S$ and every $x,y \in R^+$:

$$F_{p-q}(x+y) \geq t(F_p(x), F_q(y)) \quad .$$

Every random normed space is a Menger space, where $F: S \times S \to \Delta$ is defined by $F(p,q) = F_{p-q}$, for every $p,q \in S$.

The notion of a metric space with a convex structure is introduced in [35] by Takahashi and we shall generalize this notion on a Menger space .

Definition 1. *Let* (S,F,t) *be a Menger space . A mapping* $W: S \times S \times [0,1] \to S$ *is said to be a convex structure if for every* $(x,y) \in S \times S$:

$$W(x,y,0) = y \;, W(x,y,1) = x$$

and for every $\lambda \in (0,1)$:

$$F_{u,W(x,y,\lambda)}(2\varepsilon) \geq t\left(F_{u,x}\left(\frac{\varepsilon}{\lambda}\right), F_{u,y}\left(\frac{\varepsilon}{1-\lambda}\right)\right)$$

for every $\varepsilon \in R^+$ *and every* $(u,x,y) \in S \times S \times S$.

Let us prove that every metric space (S,d) with a convex structure in the sense of Takahashi is a Menger space with a convex structure .

Let (S,d) be a metric space with a convex structure W which means that $W: S \times S \times [0,1] \to S$ so that for every $(x,y,\lambda) \in S \times S \times [0,1]$

(1) $$d(u,W(x,y,\lambda)\leqslant\lambda d(u,x)+(1-\lambda)d(u,y)\ ,\text{for every } u\in S\ .$$

It is known that (S,F,min) is a Menger space ,where :

$$F_{u,v}(x)=\begin{cases}0,d(u,v)\geqslant x\\ 1,d(u,v)<x\end{cases}\quad .$$

Let us prove that :

(2) $$F_{u,W(x,y,\lambda)}(2\varepsilon)\geqslant\min\{F_{u,x}(\tfrac{\varepsilon}{\lambda}),F_{u,y}(\tfrac{\varepsilon}{1-\lambda})\}$$

for every $(u,x,y)\in S\times S\times S$ and every $(\varepsilon,\lambda)\in R^{+}\times(0,1)$. Suppose that $F_{u,x}(\frac{\varepsilon}{\lambda})=1$ and $F_{u,y}(\frac{\varepsilon}{1-\lambda})=1$. Then $d(u,x)<\frac{\varepsilon}{\lambda}$ and $d(u,y)<\frac{\varepsilon}{1-\lambda}$ and (1) implies that :

$$d(u,W(x,y,\lambda))<\lambda\frac{\varepsilon}{\lambda}+(1-\lambda)\frac{\varepsilon}{1-\lambda}=2\varepsilon$$

Hence,we have that :

$$F_{u,W(x,y,\lambda)}(2\varepsilon)=1=\min\{F_{u,x}(\tfrac{\varepsilon}{\lambda}),F_{u,y}(\tfrac{\varepsilon}{1-\lambda})\}\quad .$$

If $\min\{F_{u,x}(\frac{\varepsilon}{\lambda}),F_{u,y}(\frac{\varepsilon}{1-\lambda})\}=0$ then from $F_{u,W(x,y,\lambda)}(2\varepsilon)\geqslant 0$ it follows that (2) is satisfied.

Furthermore from (1) we obtain ,for $\lambda=0$ and $u=y$ that $W(x,y,0)=y$ and for $\lambda=1$ and $u=x$ that $W(x,y,1)=x$.

Every random normed space is a Menger space with the convex structure defined by $W(x,y,\lambda)=\lambda x+(1-\lambda)y$,for every $(x,y,\lambda)\in S\times S\times[0,1]$. This follows from the inequality:

$$F_{u,W(x,y,\lambda)}(2\varepsilon)=F_{u-\lambda x-(1-\lambda)y}(2\varepsilon)=F_{\lambda(u-x)+(1-\lambda)(u-y)}(2\varepsilon)$$
$$\geqslant t(F_{u-x}(\tfrac{\varepsilon}{\lambda}),F_{u-y}(\tfrac{\varepsilon}{1-\lambda}))\ .$$

EXAMPLE Let us give a nontrivial example of a probabilistic metric space with a convex structure . Suppose that (M,d) is a separable metric space with a convex structure W so that for every $\lambda\in[0,1]$ the mapping $(x,y)\mapsto W(x,y,\lambda)$ is continuous and (Ω,A,P) is a probability space .

Let S be the space of all measurable mappings from Ω into M(i.e.the space of all equivalence classes) . It is well known [4] that the triple (S,F,T_m) is a Menger space , if for every $X\in S,Y\in S$,$\varepsilon>0,u\in[0,1]$ and $v\in[0,1]$:

$$F_{X,Y}(\varepsilon) = P\ \{\omega \mid d(X(\omega),Y(\omega))<\varepsilon\}$$

$$T_m(u,v) = \max\{u+v-1,0\}\ .$$

Let $\overline{W}:S\times S\times[0,1]\to S$ be defined by the relation:

$$\overline{W}(X,Y,\lambda)(\omega) = W(X(\omega),Y(\omega),\lambda)\ ,\text{for every}\ \ \omega\in\Omega$$

and for every $X\in S$,$Y\in S$,$\lambda\in[0,1]$.

Since X and Y are measurable mappings and W is,for every fixed $\lambda\in[0,1]$,a continuous mapping it follows that for every $X\in S$ and $Y\in S$,$\overline{W}(X,Y,\lambda)\in S$.

Now ,prove that for every $U\in S,X\in S,Y\in S$ and $\lambda\in(0,1)$:

$$(*)\quad F_{U,\overline{W}(X,Y,\lambda)}(2\varepsilon)\geq T_m(F_{U,X}(\tfrac{\varepsilon}{\lambda}),F_{U,Y}(\tfrac{\varepsilon}{1-\lambda}))\quad ,\ \text{for every}\quad \varepsilon>0.$$

From the definition of the mapping F it follows that:

$$F_{U,\overline{W}(X,Y,\lambda)}(2\varepsilon) = P\ \{\omega \mid d(U(\omega),W(X(\omega),Y(\omega),\lambda))<2\varepsilon\ \}\ .$$

Further,from (1) it follows that for every $\omega\in\Omega$:

$$d(U(\omega),W(X(\omega),Y(\omega),\lambda))\leq \lambda d(U(\omega),X(\omega)) + (1-\lambda)d(U(\omega),Y(\omega)).$$

This inequality implies that :

$$\{\omega \mid d(U(\omega),W(X(\omega),Y(\omega),\lambda))< 2\varepsilon\}\supset\{\omega \mid d(U(\omega),X(\omega))<\tfrac{\varepsilon}{\lambda}\ \}$$

$\cap\ \{\omega \mid d(U(\omega),Y(\omega))<\frac{\varepsilon}{1-\lambda}\ \}$ and so we obtain that :

$P[\{\omega \mid d(U(\omega),W(X(\omega),Y(\omega),\lambda))<2\varepsilon\}]\geq P[\{\omega \mid\ d(U(\omega),X(\omega))<\frac{\varepsilon}{\lambda}\ \}$ $\cap\{\omega \mid d(U(\omega),Y(\omega))<\frac{\varepsilon}{1-\lambda}\}]$. Since for every $A,B\in\mathcal{A}$:$P(A\cap B)=P(A)+P(B)-P(A\cup B)$ we obtain that:

$$P[\{\omega \mid\ d(U(\omega),X(\omega))<\tfrac{\varepsilon}{\lambda}\}\cap\{\omega \mid d(U(\omega),Y(\omega))<\tfrac{\varepsilon}{1-\lambda}\ \}]$$

$$= P[\{\omega \mid\ d(U(\omega),X(\omega))<\tfrac{\varepsilon}{\lambda}\}]\ +\ P[\{\omega \mid\ d(U(\omega),\ Y(\omega))<\tfrac{\varepsilon}{1-\lambda}\}]$$

$$-P[\{\omega \mid\ d(U(\omega),X(\omega))<\tfrac{\varepsilon}{\lambda}\}\ \cup\ \{\omega \mid d(U(\omega),X(\omega))< \tfrac{\varepsilon}{1-\lambda}\}\ \ ]$$

$\geq F_{U,X}(\frac{\varepsilon}{\lambda})+F_{U,Y}(\frac{\varepsilon}{1-\lambda})\ -\ 1$,which completes the proof of (*) .

In a Menger space with a convex structure the notion of a star-convex subset can be introduced similarly as in the case of a normed space .

Definition 2. *Let* (S,F,t) *be a Menger space with a convex structure* $W:S\times S\times[0,1]\to S$ *and* M *a subset of* S . *The set* M *is said to be star-convex if there exists* $x_o\in M$ *such that* :

$$W(x,x_o,\lambda)\in M \text{ ,for every } (x,\lambda)\in S\times[0,1] \quad .$$

Then x_o *is a star point of the set* M .

In this paper we shall suppose that (S,F,t) is a Menger space with a convex structure $W:S\times S\times[0,1]\to S$ so that the following condition is satisfied :

$$(3) \qquad F_{W(x,z,\lambda),W(y,z,\lambda)}(\lambda\varepsilon)\geq F_{x,y}(\varepsilon)$$

for every $(\varepsilon,\lambda)\in R^+\times(0,1)$ and every $(x,y,z)\in S\times S\times S$.

A similar condition for metric spaces with a convex structure is introduced in [25]. If (S,F,t) is a random normed space condition (3) is satisfied since :

$$F_{\lambda x+(1-\lambda)z-\lambda y-(1-\lambda)z}(\lambda\varepsilon) = F_{x-y}(\varepsilon) \quad \text{,for every } (x,y,z)\in$$

$S\times S\times S$ and every $(\varepsilon,\lambda)\in R^+\times(0,1)$.

If M is a nonempty subset of S ,by 2^M we shall denote the family of all nonempty subsets of S and by 2^M_c the family of all nonempty,closed subsets of M .

Let $T:M\to 2^S$ $(M\subset S)$. The mapping T is <u>demicompact</u> if for every two sequences $\{x_n\}_{n\in N}$ and $\{y_n\}_{n\in N}$ such that $x_n\in M(n\in N)$, $y_n\in Tx_n(n\in N)$ and that :

$$\lim_{n\to\infty} F_{x_n,y_n}(\varepsilon) = 1 \quad \text{,for every} \quad \varepsilon>0$$

there exists a convergent subsequence $\{x_{n_k}\}_{k\in N}$ .This definition is introduced in [8] .

If a singlevalued mapping is demicompact in the sense of a normed space it is also demicompact in the above sense.In the case of normed spaces it is well known that a densifying mapping is an example of a demicompact mapping .

3. FIXED POINT THEOREMS

The following fixed point theorem is a generalization of the well known Nadler's fixed point theorem [17] and it is proved in [8] .

Theorem *Let* (S,F,t) *be a complete Menger space with a continuous* T-*norm* t *,* M *a nonempty closed subset of* S, $T:M\to 2^M_c$ *so that the following conditions are satisfied:*

(i) For every $u,v\in M$, *every* $x\in Tu$ *and every* $\delta>0$ *there exists* $y\in Tv$ *such that:*

$$F_{x,y}(\varepsilon)\geq F_{u-v}\left(\frac{\varepsilon-\delta}{q}\right), \text{ for every } \varepsilon>0 \text{ where } q\in(0,1).$$

(ii) T *is demicompact or the family* $\{t_n(u)\}_{n\in N}$ *is equicontinuous at the point* $u=1$ *, where*

$$t_n(u) = \underbrace{t(t(\ldots t(t(u,u),u)\ldots))}_{n\text{-times}} .$$

Then there exists at least one element $x\in M$ *such that* $x\in Tx$.

If (S,d) is a metric space and $(S,F,\min)$ the induced Menger space the condition (i) is satisfied if:

$$D(Tu,Tv)\leq q\, d(u,v) \quad (u,v\in M \,, q\in(0,1))$$

and D is the Hausdorff metric $(T:M\to CB(M))$.

This was proved in [8] and since for T-norm $t=\min$ the family $\{t_n(u)\}_{n\in N}$ is equicontinuous at the point $u=1$ it follows that the Nadler fixed point theorem is a corollary of the above fixed point theorem .

Let us give an example of a T-norm $t\neq\min$ such that the family $\{t_n(u)\}_{n\in N}$ is equicontinuous at the point $u=1$.

Let $\bar{t}$ be a continuous t-norm and for every $m\in N\cup\{0\}$:

$$I_m = [1-2^{-m}, 1-2^{-m-1}] .$$

We shall define T-norm t in the following way:

$$t(x,y) = \begin{cases} 1-2^{-m}+2^{-m-1}\bar{t}(2^{m+1}(x-1+2^{-m}), 2^{m+1}(y-1+2^{-m})) \\ \text{for } (x,y)\in I_m\times I_m \\ \\ \min\{x,y\} \quad , \text{for } (x,y)\notin \bigcup_{m\in N\cup\{0\}} I_m\times I_m . \end{cases}$$

It it easy to see that the family $\{t_n(u)\}_{n\in N}$ is equi - continuous at the point $u\neq 1$.

Using the above theorem we shall prove a fixed point theorem for multivalued mappings which are of nonexpansive type .

Theorem 1. *Let* (S,F,t) *be a complete Menger space with a convex structure* W *and continuous* T-*norm* t, M *a nonempty, closed star-convex subset of* S, $T:M\to 2^M_c$ *so that the set* $T(M)$ *is probabilistically precompact (in the sense of* [2] *) and the following condition is satisfied: For every* $u,v\in M$, *every* $x\in Tu$ *and every* $\delta>0$ *there exists* $y\in Tv$ *such that* :

(4) $$F_{x,y}(\varepsilon)\geq F_{u,v}(\varepsilon-\delta)\ \text{, for every}\quad \varepsilon>0.$$

Then there exists at least one element $x\in M$ *such that* $x\in Tx$.

Proof: Let x_o be the star-point of the set M and $\{k_n\}_{n\in N}$ a sequence of real numbers from $(0,1)$ such that $\lim\limits_{n\to\infty} k_n = 1$.
For every $n\in N$ and $x\in M$ let $T_n x = \bigcup\limits_{z\in Tx} W(z,x_o,k_n)$. We shall prove that for every $n\in N$ there exists $x_n\in M$ such that $x_n\in T_n x_n$.

Since x_o is a star-point of the set M it follows that $\bigcup\limits_{z\in Tx} W(z,x_o,k_n) \subset M$ and so $T_n x\subset M$ for every $n\in N$ and every $x\in M$.
From (3) it follows that the mapping W is continuous in respect to the first variable . Since Tx is closed it follows that Tx is compact (as a subset of TM) and so the set $W(Tx,x_o,k_n)$ is closed for every $n\in N$. This implies that the set $T_n x$ is closed for every $n\in N$ and every $x\in M$.

We shall prove that for every $u,v\in M$, every $x\in T_n u$ and every $\delta>0$ there exists $y\in T_n v$ such that :

$$F_{x,y}(\varepsilon)\geq F_{u,v}\left(\frac{\varepsilon-\delta}{k_n}\right)\quad \text{, for every}\quad \varepsilon>0 .$$

Let $u,v\in M$, $\delta>0$ and $x\in T_n u$. Then there exists $z\in Tu$ such that $x = W(z,x_o,k_n)$. From (4) it follows that there exists $y'\in Tv$ such that :

$$F_{z,y'}(\varepsilon')\geq F_{u,v}\left(\varepsilon'-\frac{\delta}{k_n}\right)\quad \text{, for every}\quad \varepsilon'>0 .$$

Let $y = W(y',x_o,k_n)\in T_n v$. Then we have that :

$$F_{x,y}(\varepsilon) = F_{W(z,x_o,k_n),W(y',x_o,k_n)}\left(k_n\frac{\varepsilon}{k_n}\right)\geq F_{z,y'}\left(\frac{\varepsilon}{k_n}\right)$$
$$\geq F_{u,v}\left(\frac{\varepsilon-\delta}{k_n}\right) .$$

The set $T(M)$ is probabilistically precompact . This means that for every $\varepsilon>0$ and every $\lambda\in(0,1)$ there exists a finite cover of $T(M)$, $\{A_i\}_{i\in I}$ (I is finite) such that $D_{A_i}(\varepsilon)>1-\lambda$, $i\in I$ where D is the probabilistic diameter ,which is defined by $D_A(x) = \sup_{t<x} \inf_{p,q\in A} F_{p,q}(t)$. From this it it obvious that TM is a probabilistically bounded subset of $S(\sup_x D_{TM}(x)= 1)$ and in [2] it is proved that TM is precompact in respect to the metric ρ which metrizises the uniformity of S generated by the (ε,λ)-topology . Hence ,the set $\overline{T(M)}$ is compact . From the continuity of the mapping W in respect to the first variable it follows that the set $T_n(M) = W(T(M),x_o,k_n)$ $(n\in N)$ is relatively compact . Let us prove that T_m is a demicompact mapping .

Suppose that $\{x_n\}_{n\in N}$ and $\{y_n\}_{n\in N}$ are two sequences from M such that $y_n\in T_m x_n$ and $\lim_{n\to\infty} F_{x_n,y_n}(\varepsilon) = 1$,for every $\varepsilon>0$. Then there exists a convergent subsequence $\{y_{n_k}\}$ of the sequence $\{y_n\}_{n\in N}$ and suppose that $\lim_{k\to\infty} y_{n_k} = z$. Then from the inequality $F_{x_{n_k},z}(\varepsilon)\geq t(F_{x_{n_k},y_{n_k}}(\frac{\varepsilon}{2}),F_{y_{n_k},z}(\frac{\varepsilon}{2}))$ it follows that $\lim_{k\to\infty} x_{n_k} = z$.

This means that the mapping T_m is demicompact .Hence, all the conditions of the Theorem are satisfied and for every $n\in N$ there exists $x_n\in M$ such that $x_n\in T_n x_n$.

Since $T_n x_n = \bigcup_{z\in Tx_n} W(z,x_o,k_n)$ it follows that there exists $z_n\in Tx_n$ such that $x_n = W(z_n,x_o,k_n)$. Then we have that :

$$F_{x_n,z_n}(\varepsilon) = F_{z_n,W(z_n,x_o,k_n)}(\varepsilon)\geq t(F_{z_n,z_n}(\frac{\varepsilon}{2k_n}),F_{z_n,x_o}(\frac{\varepsilon}{2(1-k_n)})) =$$

$$= t(1,F_{z_n,x_o}(\frac{\varepsilon}{2(1-k_n)})) = F_{z_n,x_o}(\frac{\varepsilon}{2(1-k_n)}) \text{ ,for every } n\in N .$$

Since TM is probabilistic bounded we have that for every $z\in TM$,$\lim_{n\to\infty} F_{z_n,z}(\frac{\varepsilon}{4(1-k_n)}) = 1$ (for every $\varepsilon>0$) .Using the inequality :

$$F_{z_n,x_o}(\frac{\varepsilon}{2(1-k_n)})\geq t(F_{z_n,z}(\frac{\varepsilon}{4(1-k_n)}),F_{z,x_o}(\frac{\varepsilon}{4(1-k_n)}))$$

and $\lim_{n\to\infty} F_{z,x_o}(\frac{\varepsilon}{4(1-k_n)}) = 1$ (for every $\varepsilon>0$) it follows that $\lim_{n\to\infty} F_{z_n,x_o}(\frac{\varepsilon}{2(1-k_n)}) = 1$ (for every $\varepsilon>0$). Hence we have that:

$$\lim_{n\to\infty} F_{x_n,z_n}(\varepsilon)= 1 \text{ (for every } \varepsilon> 0) . \tag{5}$$

Since $z_n \in Tx_n$ and the set $\overline{T(M)}$ is compact there exists a convergent subsequence $\{z_{n_k}\}_{k\in N}$ and let $z = \lim_{k\to\infty} z_{n_k}$.

Then (5) and the inequality :

$$F_{x_{n_k},z}(\varepsilon) \geq t(F_{x_{n_k},z_{n_k}}(\tfrac{\varepsilon}{2}), F_{z_{n_k},z}(\tfrac{\varepsilon}{2}))$$

implies that $\lim_{k\to\infty} x_{n_k} = z$. Let us prove that $z \in Tz$. We shall prove that $z \in \overline{Tz}$,which implies,since Tz is closed, that $z \in Tz$.

Let $\varepsilon>0$ and $\lambda \in (0,1)$. We shall prove that there exists $b \in Tz$ such that $b \in U_z(\varepsilon,\lambda)$. Let us take in (4) that $\delta = \frac{\varepsilon}{4}$,$u = x_{n_k}$ and $v = z$. Then there exists $b_k \in Tz$ such that :

$$F_{z_{n_k},b_k}(\tfrac{\varepsilon}{2}) \geq F_{x_{n_k},z}(\tfrac{\varepsilon}{4}) \quad .$$

Suppose that $\eta(\lambda)$ be such an element from the interval $(0,1)$ that we have the following implication:

$$x > 1-\eta(\lambda) \Rightarrow t(x,x) > 1-\lambda \ .$$

If $n_o(\varepsilon,\lambda) \in N$ is such that :

$$F_{z,x_{n_k}}(\tfrac{\varepsilon}{4}) > 1-\frac{\eta(\lambda)}{2}, F_{z,z_{n_k}}(\tfrac{\varepsilon}{2}) > 1-\frac{\eta(\lambda)}{2} \text{ ,for every } k > n_o(\varepsilon,\lambda)$$

it follows that :

$$F_{z,b_k}(\varepsilon) \geq t(F_{z,z_{n_k}}(\tfrac{\varepsilon}{2}), F_{z_{n_k},b_k}(\tfrac{\varepsilon}{2})) \geq t(1-\frac{\eta(\lambda)}{2}, 1-\frac{\eta(\lambda)}{2}) > 1-\lambda$$

and so $b_k \in U_z(\varepsilon,\lambda) \cap Tz$. Since Tz is closed ,we conclude that $z \in Tz$.

Using Theorem 1 we can prove the following theorem.

Theorem 2. *Let* (S,F,t) *be a complete Menger space with a convex structure* W *and continuous* T-*norm* t *such that the family* $\{t_n(u)\}_{n\in N}$ *is equicontinuous at the point* $u = 1$,M *as in Theorem 1.* T *a demicompact mapping from* M *into the family of all nonempty and compact subsets of* M *such that* $\overline{T(M)}$ *is bounded and the condition* (4) *is satisfied . Then there exists* $x \in M$ *such that* $x \in Tx$.

Proof:As in the proof of Theorem 1 ,let for every $n \in N$ and every $x \in M$:$T_n x = \bigcup_{z\in Tx} W(z,x_o,k_n)$. Since the set Tx is compact ,for every $x \in M$ it follows that $T_n x$ is closed ,for every $n \in N$ and every $x \in M$. From the equicontinuity of the family $\{t_n(u)\}_{n\in N}$ at the point $u = 1$ we obtain that the mapping T_n satisfies all the conditions of Theorem 1 . Hence,there exists,for every $n \in N$, $x_n \in M$ such that

$x_n \in T_n x_n$. Since TM is bounded ,as in Theorem 1 it follows that :

$$\lim_{n\to\infty} F_{x_n,z_n}(\varepsilon) = 1 \text{ ,for every } \varepsilon>0$$

where $z_n \in Tx_n$,for every $n\in N$. Since the mapping T is demicompact we obtain the existence of a convergent sequence $\{x_{n_k}\}_{k\in N}$. The rest of the proof is as in Theorem 1 .

Corollary *Let* (S,d) *be a complete metric space with a convex structure* W *such that for every* $(x,y,z)\in S\times S\times S$ *and every* $\lambda\in[0,1]$:

$$d(W(x,z,\lambda),W(y,z,\lambda)) \leqslant \lambda d(x,y) .$$

Let $\emptyset\neq M\subset S$, T *be a demicompact mapping from* M *into the family of all nonempty and compact subsets of* M,TM *be a bounded subset of the space* S *and the set* M *closed and star-convex* . *If for every* $u\in M$ *and* $v\in M$:

$$D(Tu,Tv) \leqslant d(u,v)$$

there exists at least one element $x\in M$ *such that* $x\in Tx$.

Proof:The triple (S,F,min) is a Menger space where:

$$F_{x,y}(\varepsilon) = \begin{cases} 1 & , d(x,y)<\varepsilon \\ 0 & , d(x,y)\geqslant\varepsilon \end{cases}$$

and the topology induced by the metric d is the (ε,λ)-topology . Further, for t = min the family $\{t_n(u)\}_{n\in N}$ is equicontinuous at the point u = 1 .From the definition of the Hausdorff metric D it follows that for every $\delta>0$,every $u,v\in M$ and every $x\in Tu$ there exists $y\in Tv$ such that :

(6) $$d(x,y) < d(u,v) +\delta .$$

Inequality (6) implies that for $d(u,v)<\varepsilon-\delta$ we obtain that $d(x,y)<\varepsilon$ .Then from the definition of the mapping F it follows that (4) is satisfied. Hence,all the conditions of Theorem 2 are satisfied and so there exists $x\in M$ such that $x\in Tx$.

Remark In the books [33] and [34] furhter information on the existence of a fixed point for nonexpansive mappings defined on some types of metric spaces with a convex structure may be obtained.

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REZIME

TEOREME O NEPOKRETNOJ TAČKI ZA VIŠEZNAČNA PRESLIKAVANJA U VEROVATNOSNIM METRIČKIM PROSTORIMA SA KONVEKSNOM STRUKTUROM

U ovom radu uveden je pojam verovatnosnog metričkog prostora sa konveksnom strukturom i dokazane su teoreme o nepokretnoj tački za višeznačna preslikavanja u ovim prostorima.

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