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THE SINGULARLY PERTURBED SPLINE COLLOCATION METHOD FOR BOUNDARY VALUE PROBLEMS WITH MIXED BOUNDARY CONDITIONS

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ABSTRACT

The spline collocations method given in [7] for solving boundary value problems without a singular perturbation is adapted for problems with singular perturbation. The exponential features of the exact solution are transferred to spline coefficients by „artifical viscousity". In this way a uniformly convergent method for solving problem: $\varepsilon y'' + p(x)y' + q(x)y = f(x), y'(0) - \alpha y(0) = \alpha_0$, $y(1) = \alpha_0$, $\alpha \geq 0$, $p(x) > 0$, $q(x) \equiv 0$ is achieved. The numerical results indicate a uniform convergence when $q(x) \neq 0$.

The following problem is considered:

$$(1)\qquad \begin{cases} Ly = \varepsilon y'' + p(x)y' + q(x)y = f(x),\ 0 < x < 1, \\ y'(0) - \alpha y(0) = \alpha_0,\ y(1) = \alpha_1, \text{ where} \\ p(x) \geq p > 0,\ q(x) \leq 0;\ \alpha_1,\alpha_2,\alpha,\ \varepsilon \in R;\ \alpha \geq 0; \\ \varepsilon \in (0,1]. \end{cases}$$

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The approximate solution to problem (1) is sought in the form of a cubic spline

$$S(x) = \sum_{i=-1}^{n+1} \beta_i B_i(x)$$

on the mesh $x_i = ih$, $i = \overline{-3(1)n+1}$, $n = 1/n$. $B_i(x)$ are basic splines determined in [7]. The coefficients β_i are determined from the conditions:

$$(3)\quad \begin{cases} L_h S(x_i) = \sigma(x_i)S''(x_i) + p(x_i)S'(x_i) + \\ \qquad\qquad + q(x_i)S(x_i) = f(x_i), \quad i = 0(1)n, \\ S'(0) - \alpha S(0) = \alpha_0 \\ S(1) = \alpha_1 \end{cases}$$

where $\sigma(x_i) = \{hp(x_i)/2\}cth\{hp(x_i)/2\varepsilon\}$.
By replacing (2) into (3) and using the characteristics of basic splines as in [7] we obtain the following system of linear equations:

$$(4)\quad \begin{cases} \beta_{i-1}K_i + \beta_i L_i + \beta_{i+1}M_i = f(x_i), \quad i = 0(1)n, \\ \beta_{-1}(h+3\alpha) + 4\beta_0 h + \beta_1(h-3\alpha) = 6\alpha_0 h \\ \beta_{n-1} \qquad + 4\beta_n \quad + \beta_{n+1} \qquad = 6\alpha_1 \end{cases}$$

where

$$K_i = \frac{1}{h^2}\left(\sigma(x_i) - \frac{p(x_i)h}{2} + \frac{q(x_i)h^2}{6}\right),$$

$$M_i = \frac{1}{h^2}\left(\sigma(x_i) + \frac{p(x_i)h}{2} + \frac{q(x_i)h^2}{6}\right),$$

$$L_i = - M_i - K_i + q(x_i).$$

Excluding the unknown β_{-1} and β_{n+1} from the above equations for $i = 0$ and $i = n$, we obtain the system:

$$(5)\quad \begin{cases} \beta_0\overline{L}_0 + \beta_1\overline{M}_0 = d_0 \\ \beta_{i-1}K_i + \beta_i L_i + \beta_{i+1}M_i = f(x_i),\ i = 1(1)\overline{n-1}. \\ \beta_{n-1}\overline{K}_n + \beta_n\overline{L}_n = d_n, \end{cases}$$

where

$$\overline{L}_0 = L_0 - 4hK_0/(h+3\alpha),\ \overline{M}_0 = M_0 - (h-3\alpha)K_0/(h+3\alpha),$$

$$d_0 = f(x_0) - 6\alpha_0 h/(h+3\alpha),\ \overline{K}_n = K_n - M_n,$$

$$\overline{L}_n = L_n - 4M_n,\ \ d_n = f(x_n) - 6\alpha_1 M_n.$$

System (6) can be written in the form $A\beta = d$, where A is a matrix of the type $(n+1) \times (n+1)$ with elements a_{ij} $(i,j = 0(1)n)$, β and d are vectors with elements β_i, and d_i; $d_i = f(x_i)$, $i = 1(1)n$.
The matrix A is diagonally dominant and irreducible. System (4) has a unique solution which we can obtain by the methods given in [4].

In order to prove the uniform convergence in the case $q(x) \equiv 0$, we shall derive the corresponding difference scheme. On each interval $[x_j,x_{j+1}]$, the spline $S(x)$ has the form:

$$(6)\quad S(x) = v_j(x) = v_j^{(0)} + (x - x_j)v_j^{(1)} + \frac{(x - x_j)^2 v_j^{(2)}}{2!} + \frac{(x - x_j)^3 v_j^{(3)}}{3!},$$

$$x \in [x_j,x_{j+1}].$$

Constants $v_j^{(k)}$, $k = 0,1,2,3$, satisfy the following system:

$$(7)\quad \sigma_j v_j^{(2)} + p_j v_j^{(1)} \doteq f_j, \quad j = 0(1)\overline{n-1}$$

$$\sigma_n(v_{n-1}^{(2)} + hv_n^{(3)}) + p_n(v_{n-1}^{(1)} + hv_{n-1}^{(2)} + h^2v_{n-1}^{(3)}/2) = f_n.$$

$$v_j^{(0)} = v_{j-1}^{(0)} + v_{j-1}^{(1)}h + h^2 v_{j-1}^{(2)}/2 + h^3 v_{j-1}^{(3)}/6, \quad j = 1(1)\overline{n-1}.$$

$$v_j^{(1)} = v_{j-1}^{(1)} + h v_{j-1}^{(2)} + h^2 v_{j-1}^{(3)}/2, \quad j = 1(1)\overline{n-1}$$

(8) $$v_j^{(2)} = v_{j-1}^{(2)} + h v_{j-1}^{(3)}, \; j = 1(1)\overline{n-1}$$

(9) $$v_0^{(1)} - \alpha v_0^{(0)} = \alpha_0, \; v_n^{(0)} = \alpha_1, \text{ where}$$

$$\sigma_j = \sigma(x_j), \; p_j = p(x_j), \; f_j = f(x_j).$$

By the elimination of $v_j^{(1)}$, $v_j^{(2)}$ and $v_j^{(3)}$, we obtain the scheme (see [3]):

(10) $$R_h v_j = Q_h f_j, \; j = 0(1)\overline{n-1}, \text{ where}$$

$$v_j = v_j^{(0)}$$

$$R_h v_j = r_j^- v_{j-1} + r_j^c v_j + r_j^+ v_{j+1}, \; j = 1(1)\overline{n-2}$$

$$Q_h f_j = q_j^- f_{j-1} + q_j^c f_j + q_j^+ f_{i+1}, \; j = 1(1)\overline{n-2}$$

$$R_h v_0 = -(1/\gamma_1 + \alpha)v_0 + (1/\gamma_1)v_1$$

$$Q_h f_0 = \alpha_0 + [h^2 f_0(\omega_1 + 1)/(6\sigma_0(\omega_1 + 1)) + $$
$$+ h^2 f_1 \omega_1/(6\sigma_1(\omega_1 + 1))]/\gamma_1$$

$$R_h v_{n-1} = r_{n-1}^- v_{n-2} + r_{n-1}^c v_{n-1}$$

$$Q_h f_{n-1} = q_{n-1}^- f_{n-2} + q_{n-1}^c f_{n-1} + q_{n-1}^+ f_n - r_{n-1}^+ \alpha_1$$

$$r_j^- = 3(\omega_{j-1} - 1)/(hA_{j-1}), \; r_j^+ = 3(\omega_{j+1}+1)/(hA_j)$$

$$r_j^c = - r_j^+ - r_j^-$$

$$q_j^- = 1/(p_{j-1}A_{j-1}), \; q_j^+ = 1/(p_{j+1}A_j)$$

$$q_j^c = [(2\omega_{j-1} - 1)/A_{j-1} + (2\omega_{j+1} + 1)/A_j]/(\omega_j p_j)$$

$$\sigma_j = \varepsilon\rho_j\omega_j, \quad \omega_j = cth(p(x_j)/2\varepsilon)),$$

$$A_j = 3\omega_j\omega_{j+1} + 2\omega_j - 2\omega_{j+1} - 1.$$

$$\gamma_{j+1} = hA_j/(3\omega_j(\omega_{j+1} + 1))$$

At the same time, we have that

$$\bar{R}_h v_j^{(1)} = \bar{Q}_h f_j, \; j = 1(1)n. \tag{11}$$

$$\bar{R}_h = a_j v_j^{(1)} - b_j v_{j-1}^{(1)}, \; \bar{Q}_h f_j = \frac{h}{2}\left(\frac{f_{j-1}}{\sigma_{j-1}} + \frac{f_j}{\sigma_j}\right)$$

$$a_j = 1 + \frac{hp_j}{2\sigma_j}, \qquad b_j = 1 - \frac{hp_{j-1}}{2\sigma_{j-1}}.$$

Before proving the uniform convergence, we shall show some important properties of operators R_h and $\bar{R}_h$. The local truncation errors of these operators are defined in the usual way, i.e.,

$$\tau_j(y) = R_h(y(x_j) - v_j) = R_h y(x_j) - Q_h(Ly(x_j))$$

$$\bar{\tau}_j(y') = \bar{R}_h(y'(x_j) - v_j^{(1)}) = R_h y'(x_j) - \bar{Q}_h(Ly(x_j)).$$

If $y(x) = exp(-px/\varepsilon)$ and $p(x) = p = const.$, then $\tau_j = 0$, $j = 1(1)\overline{n-1}$, $\bar{\tau}_j = 0$, $j = 1(1)n$. This is the consequence of the fact that in this case

$$r_j^-/r_j^+ = exp(-ph/\varepsilon), \; a_j/b_j = exp(-ph/\varepsilon).$$

The scheme of this type for the Dirichlet's boundary conditions has been derived in [5]. It has the first order of the uniform convergence. The same has been proved in [6] for scheme (10). We shall outline that proof and show that the same holds for the first derivative (Theorem 1). The convergence between grid points we consider in Theorem 2. The following lemmas are used in the comparison function proof to bound the truncation error (see [1]).

Lemma 1. ([6]) *Let* $\{v_j\}$ *be a set of values at the grid points* x_j *satisfying* $R_h v_j \geq 0$, $j = 0(1)\overline{n-1}$. *Then* $v_j \leq 0$, $j = 0(1)\overline{n-1}$.

Lemma 2. ([6]) *Let* $p(x), q(x) \in C^3[a,b]$. *Then the solution of (1) can be written in the form*

$$y(x) = u(x) + w(x),$$

where

$$u(x) = \delta_1 \varepsilon \exp(-p(0)x/\varepsilon),$$

$$|w^{(i)}(x)| \leq M(1 + \varepsilon^{-i+2}\exp(-2\delta_0 x/\varepsilon)), (i = 1,2,3,4),$$

δ_0, δ_1 *and* M *are constants independent of* ε.

Lemma 3. ([6]) *There are constants* c_1 *and* c_2 *independent of* h *and* ε, *such that for* $h \leq c_1$, $0 < \delta \leq c_2$, $j = 1(1)\overline{n-1}$,

a) $R_h \varphi_j \geq M\frac{h^2}{\varepsilon^2}$ *for* $h \leq \varepsilon$,

b) $R_h \varphi_j \geq M$ *for* $\varepsilon \leq h$,

c) $R_h \psi_j \geq M\mu^j(\delta)/h$ *for* $\varepsilon \leq h$,

d) $R_h \varphi_j \geq M\frac{h^2}{\varepsilon^2}\frac{1}{\varepsilon}\mu^j(\delta)$ *for* $h \leq \varepsilon$,

e) $R_h \varphi_0 \geq M$

f) $R_h \psi_0 \geq M\varepsilon^{-1} \exp(-t_0\delta)$ *for* $h \leq \varepsilon$,

g) $R_h \psi_0 \geq Mh^{-1} \exp(-t_0\delta)$ *for* $\varepsilon \leq h$,

$t_j = x_j/\varepsilon$, $\varphi_j = -2 + x_j$, $\psi_j = -exp(-\delta t_j)$,

$\mu(\delta) = exp(-\delta h/\varepsilon)$.

Corollary 1. *If* $k_1(h,\varepsilon) \ge 0$ *and* $k_2(h,\varepsilon) \ge 0$ *are such functions that*

$$R_h(k_1\varphi_j + k_2\psi_j) \ge R_h(\pm z_j) = \pm\tau_j(y), \quad j = 0(1)\overline{n-1}$$

then

$$|z_j| \le k_1|\varphi_j| + k_2|\psi_j|, \quad z_j^{(k)} = y^{(k)}(x_j) - v_j^{(k)}, \quad k=0,1,2,3.$$

Throughout the paper, M (or δ) denotes the possible different constants independent of ε and h.

Theorem 1. *Let* $\{v_j\}$ *and* $\{v_j^{(1)}\}$ *be the approximation to* $y(x_j)$ *and* $y'(x_j)$, $j = 0(1)n$, *obtained by using (10) and (11), respectively. Let* $q(x) \equiv 0$, $p(x) \in C^3[0,1]$, $p(x) \ge \ge p > 0$, $\alpha \ge 0$.

Then

$$|y(x_j) - v_j| + |y^{(1)}(x_j) - v_j^{(1)}| \le Mh^2/(\varepsilon + h). \tag{12}$$

Proof. In the main part of the proof we shall estimate the truncation error for u(x) and w(x) separately.

$$\tau_j(y) = \tau_j(u) + \tau_j(w), \quad z_j^{(k)} = z_j^{(k)}(u) + z_j^{(k)}(w),$$

$$\overline{\tau}_j(y^{(1)}) = \overline{\tau}_j(u^{(1)}) + \overline{\tau}_j(w^{(1)})$$

In [6] it has been shown that

$$|\tau_j(y)| \le \begin{cases} M\dfrac{h^4}{\varepsilon^2(h+\varepsilon)} + \dfrac{h^2}{\varepsilon}exp(-\delta t_j), & h \le \varepsilon, \ j = 1(1)\overline{n-1} \\ M(h + \varepsilon exp(-\delta t_{j-1}), & \varepsilon \le h \end{cases}$$

$$|\tau_0(w)|\le\begin{cases} M(\frac{h^2}{\varepsilon+h}+\frac{h^3}{\varepsilon^2}exp(-\delta t_0)), & h\le\varepsilon.\\ M(\frac{h^2}{\varepsilon+1}+h\ exp(-\delta t_0)), & \varepsilon\le h,\end{cases}$$

$$|\tau_0(u)|\le\begin{cases} Mh^2exp(-\delta t_0)/\varepsilon^2, & h\le\varepsilon,\\ M\ exp(-\delta t_0), & \varepsilon\le h,\end{cases}$$

and by Corollary 1 and Lemma 3 we have that the contribution to the error from $\tau_j(y)$ satisfies (12). As in [3], we have

$$\overline{\tau}_j(y') = \overline{\tau}_j(y', p_j, p_{j-1}) = R_2(x_{j-1}, x_j, y') -$$

$$- hR_1(x_{j-1}, x_j, y'')/2 + h(\eta_{j-1}/\sigma_{j-1} + \eta_j/\sigma_j)/2,$$

where

$$R_i(x_{j-1},x_j,g) = g^{(i+1)}(\zeta)\frac{(x_j-x_{j-1})^{i+1}}{(i+1)!} =$$

$$= \frac{1}{i!}\int_{x_{j-1}}^{x_j}(x_j-s)^i g^{(i+1)}(s)ds$$

$x_{j-1}\le\zeta\le x_j$, $\eta_j = y''(x_j)(\sigma_j-\varepsilon)$.

Since $(\sigma_j-\varepsilon)\le Mh^2/(\varepsilon+h)$, we have

(13) $$|\overline{\tau}_j(\omega',p_j,p_{j-1})|\le Mh^2/(\varepsilon+h)+h^3exp(-\delta t_j)/\varepsilon^2,\ h\le\varepsilon.$$

$$\overline{\tau}_j(u',p_j,p_{j-1}) = \overline{\tau}_j(u',p_j,p_{j-1}) - \overline{\tau}_j(u',p(0),p(0))$$

because

$$\overline{\tau}_j(u',p(0),p(0)) = 0$$

$$|\overline{\tau}_j(u',p_j,p_{j-1}) - \overline{\tau}_j(u',p(0),p(0))|\le$$

$$\le h|(\eta_{j-1}/\sigma_{j-1} - \eta_0/\sigma_0 + \eta_j/\sigma_j - \eta_0/\sigma_0)|/2.$$

After the Taylor development we obtain

(14) $$|\bar{\tau}_j(u',p_j,p_{j-1})| \leq M\,\frac{h}{\varepsilon}\left(\frac{x_{j-1}h^2u(x_{j-1})}{\sigma_0\sigma_{j-1}} + \frac{x_j h^2 u(x_j)}{\sigma_0\sigma_j}\right) \leq M\,\frac{h^3}{\varepsilon^2}exp(-\delta t_j),\ h \leq \varepsilon,\ j = 1(1)n.$$

For $j = 1$, the first term is equal to zero.
Since

(15) $z_0^{(1)} - \alpha z_0^{(0)} = 0$, we have that $|z_0^{(1)}| \leq Mh^2/(\varepsilon+h)$

and from (13), (14) and

(16) $$a_j z_j^{(1)} = b_j z_{j-1}^{(1)} + \bar{\tau}_j(y',p_j,p_{j-1}),\ j = 1(1)n$$

we have that (11) holds for $h \leq \varepsilon$.

Let $\varepsilon \leq h$. Then we consider $z_j^{(1)}(u)$ and $z_j^{(1)}(w)$. From

(17) $$z_j^{(2)} = (\eta_j - p_j z_j^{(1)})/\sigma_j\ ,\ \text{and}$$

$$z_j^{(1)} = z_{j-1}^{(1)} + hz_{j-1}^{(2)} + R_1(x_{j-1},x_j,z')$$

we have

(18) $$z_j^{(1)} = z_{j-1}^{(1)}(1 - hp_{j-1}/\sigma_{j-1}) + h\eta_{j-1}/\sigma_{j-1} + \pm R_1(x_{j-1},x_j,z').$$

Using the integral form of the remainder term, we have

$$|R_1(x_{j-1},x_j,w')| \leq Mh(h + exp(-\delta t_{j-1})),\ j = 1(1)n.$$

From (17), (15) and $|v_j^{(3)}(w)| \leq Mh^{-1}$, we obtain

(19) $$|z_j^{(1)}(w)| \leq Mh^2/(\varepsilon+h).$$

For $z_j^{(1)}(u)$ we take form (16). Since

$$\sigma_0 - \sigma_j = \omega_j(hp(0) - hp(x_i))/2 + hp(0)(\omega_0 - \omega_j)/2$$

$$|\sigma_0 - \sigma_j| \le Mhx_i,$$

we have

(20) $$|\bar{\tau}_j(u', p_j, p_{j-1})| \le Mh\,exp(-\delta t_{j-1}), \quad j = 1(1)n.$$

Then from (15), (16), (19) and (20) there follows (11).

Theorem 2. *Let the conditions of Theorem 1 be satisfied. Then*

(21) $$|y(x) - S(x)| \le Mh^2/(\varepsilon+h),$$

M *is a constant independent of* h *and* ε.

Proof. The function $z(x) = y(x) - S(x)$ has the form

(22) $$z(x) = z_j + (x - x_j)z_j^{(1)} + \frac{(x-x_j)^2}{2!} z_j^{(2)} + \frac{(x-x_j)^3}{3!} z_j^{(3)} + R_3(x_{j-1}, x_j, y),$$

on each subinterval $x \in [x_j, x_{j+1}]$.

Because of

$$z_j^{(3)} = (z_j^{(2)} - z_{j-1}^{(2)} - R_1(x_{j-1}, x_j, y''))/h$$

and

$$|y^{(i)}| \le M(1 + \varepsilon^{-i+1} exp(-\delta t_j)),$$

from (17) we have

$$|z_j^{(i+2)}| \le M \frac{h^{-i}}{\varepsilon+h}, \quad i = 0,1$$

and $j = 0(1)n$ if $h \le \varepsilon$, $j = 1(1)n$ if $\varepsilon \le h$.

Thus from (22), we obtain (21) for $j = 1(1)n$. To complete the proof we must show that the same holds for $\varepsilon \le h$ and $j = 0$.

Indeed,

$$|z(x)| \le |z_0| + |(x - x_0)z'(\zeta)| \le Mh, \quad \zeta \in [x_0,x_1]$$

because

$$|z_i^{(1)}| \le Mh \Rightarrow |v_i^{(k)}| \le Mh^{-k+1}, \quad k = 1,2,3$$

and

$$|v_0^{(1)}(x)| \le M, \quad x \in [x_0,x_1] \quad \text{(see (6), (7), (8))},$$

$$|z'(\zeta)| = |y'(\zeta) - v_0'(\zeta)| \le M$$

Numerical example [2]:

$$\varepsilon u'' + (1 + x^2)u' - (x - \tfrac{1}{2})^2 u =$$

$$= -4(3x^2 - 3x + 1)((x - \tfrac{1}{2})^2 + 2)$$

$$u(0) - u'(0) = 0, \quad u(1) = 0.$$

Our test for the order of uniform convergence and the notation are taken from [2]. Table 1 contains the results for the solution and Table 2 for the first derivative.
The computed order of uniform convergence is 1.20 and the classical one 2.02, (Table 1).
The computed estimate of the order of uniform convergence is 1.22 and the classical one 2.00, (Table 2).

Table 1.

ε\k	0	1	2	3	4	$\bar{p}_\varepsilon$
1/2	1.99	2.00	2.00	1.97	2.12	2.02
1/4	1.95	2.00	2.00	2.00	1.99	1.99
1/8	1.92	1.97	1.99	2.00	2.07	1.99
1/16	1.87	1.87	1.96	1.99	1.96	1.93
1/32	1.89	1.68	1.91	1.98	1.99	1.89
1/64	1.50	1.59	1.74	1.92	1.98	1.75
1/128	1.10	1.53	1.42	1.76	1.93	1.55
1/256	1.03	1.09	1.47	1.40	1.77	1.35
1/512	1.03	1.02	1.09	1.42	1.42	1.20

Table 2.

ε\k	0	1	2	3	4	$\bar{p}_\varepsilon$
1/2	2.00	2.00	2.00	2.00	2.00	2.00
1/4	2.01	2.00	2.00	2.00	2.00	2.00
1/8	2.02	2.00	2.00	2.00	2.00	2.00
1/16	2.02	1.99	2.00	2.00	2.00	2.00
1/32	1.82	1.95	1.99	2.00	2.00	1.95
1/64	1.29	1.83	1.92	1.98	2.00	1.80
1/128	1.00	1.32	1.81	1.93	1.98	1.61
1/256	0.95	1.03	1.33	1.80	1.92	1.41
1/512	0.95	0.98	1.04	1.34	1.80	1.22

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REZIME

SINGULARNO PERTURBIRAN SPLINE KOLOKACIONI METOD ZA RUBNI PROBLEM SA MEŠOVITIM RUBNIM USLOVIMA

Spline kolokacioni metod dat u [7] za rešavanje rubnog problema bez singularne perturbacije je prilagodjen za problem sa singularnom perturbacijom. Eksponencijalne karakteristike tačnog rešenja su prenete na splajn koeficijente pomoću "veštačke viskoznosti". Na taj način dobijen je uniformno konvergentan metod za rešavanje problema:

$$\varepsilon y'' + p(x)y' + q(x)y = f(x), y'(0) - \alpha y(0) = \alpha_0, \; y(1) = \alpha_1,$$

$$\geq 0, \quad p(x) > 0, \; q(x) \equiv 0.$$

Numerički rezultati ukazuju na uniformnu konvergenciju kada je $q(x) \neq 0$.

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