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A THEOREM ON PARTIAL MIDDLING FOR FUNCTIONAL-DIFFERENTIAL EQUATIONS OF THE NEUTRAL TYPE

T. Janiak[*] *and E. Łuczak-Kumorek*[**]

[*] *Institute of Mathematics, Pedagogical University, Zielona Góra, Poland*

[**] *Institute of Mathematics and Physics, Technical University, Zielona Góra, Poland*

ABSTRACT

In this paper a generalization of Plotnikov's result from [5] is obtained for functional-differential equations of the form: $\dot{x}(t) \in F(t,x_t,\dot{x}_t)$, where F is a multifunction with values which are nonempty compact convex subsets of n-dimensional space R^n.

1. INTRODUCTION

In classical system of functional-differential equations it is possible to middle both complete and partial equations. Complete middling was presented by Bogolubov ([1]).

In this paper, we use a partial middling method in the case of generalized functional-differential equations of the form

$$\dot{x}(t) \in F(t,x_t,\dot{x}_t)$$

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where F is a multifunction with values that are nonempty compact convex subsets of n-dimensional Euclidean space R^n. The application of this method leads to a reduced form of the initial equations system and is useful in the case when the means of certain functions do not exist.

The results of this paper generalize the results of W. Płotnikov ([5]), where the generalized system $\dot{x}(t) \in F(t,x)$ was investigated.

Let C_α and $\mathcal{L}_\alpha$, $\alpha \geq 0$ denote the Banach spaces of all continuous and Lebesgue integrable functions, respectively, of $[-r,\alpha]$ into R^n with the norms

$$\|x\|_\alpha = \sup_{-r\leq t\leq \alpha} |x(t)| \quad \text{and} \quad |y|_\alpha = \int_{-r}^{\alpha} |y(t)|dt$$

for $x \in C_\alpha$ and $y \in \mathcal{L}_\alpha$ respectively, where $|\cdot|$ denotes the Euclidean norm. For a given function $u : [-r,T] \to R^n$ and fixed $t \in [0,T]$, we denote $u_t(s) = u(t+s)$ for $s \in [-r,0]$, $r \geq 0, T > 0$. Finally, let us denote by $(\text{Comp } R^n, H)$ and $(\text{Conv } R^n, H)$ the metric spaces of all nonempty compact and nonempty compact and convex, respectively, subsets of n-dimensional Euclidean space R^n with the Hausdorff metric H.

2. THE THEOREM ON PARTIAL MIDDLING

Let $F^i : [0,\infty) \times C_0 \times \mathcal{L}_0 \to \text{Conv } R^n$ $(i = 1,2)$ satisfy the following conditions:

(a) $F^i(\cdot,u,v) : [0,\infty) \to \text{Conv } R^n$ is measurable for fixed $(u,v) \in C_0 \times \mathcal{L}_0$,

(b) there exists a $M > 0$ such that $H(F^i(t,u,v),\{0\}) \leq M$ for $(t,u,v) \in [0,\infty) \times C_0 \times \mathcal{L}_0$,

(c) $F^i(t,\cdot,\cdot) : C_0 \times \mathcal{L}_0 \to \text{Conv } R^n$ satisfies for fixed $t \in [0,\infty)$ the Lipschitz condition of the form $H(F^i(t,u,v),F^i(t,\bar{u},\bar{v})) \leq k[\|u-\bar{u}\|_0 + |v-\bar{v}|_0]$ where $k > 0$, $u,\bar{u} \in C_0$ and $v,\bar{v} \in \mathcal{L}_0$,

(d) there exists a limit

$$\lim_{L\to\infty} \frac{1}{L} H\left(\int_0^L F^1(t,u,v)dt, \int_0^L F^2(t,u,v)dt\right) = 0$$

uniformly with respect to $(u,v) \in C_0 \times \mathcal{L}_0$.

In this part we shall study differential inclusions of the form

(1) $\dot{x}^1(t) \in \varepsilon F^1(t,x_t^1,\dot{x}_t^1)$ for a.e. $t \geq 0$

and

(2) $\dot{x}^2(t) \in \varepsilon F^2(t,x_t^2,\dot{x}_t^2)$ for a.e. $t \geq 0$

where $\varepsilon > 0$ is a small parameter. We shall consider (1) and (2) together with the initial conditions

(3) $x^1(t) = x^2(t) = \varphi(t)$ for $t \in [-r,0]$

where $\varphi : [-r,0] \to R^n$ is a given absolutely continuous function.

In paper [4] the following theorem has been proved.

Theorem 1. *Let* $\delta : [0,T] \to R$ *be a nonnegative Lebesgue integrable function and* $\varphi \in C_0$ *be absolutely continuous. Suppose* $F : [0,T] \times C_0 \times \mathcal{L}_0 \to \text{Comp } R^n$ *satisfies (a) (b) and (c) of the form*

$$H(F(t,u,v), F(t,\bar{u},\bar{v})) \leq k(t)[\|u - \bar{u}\|_0 + |v - \bar{v}|_0]$$

where $k : [0,T] \to R^+$ *is a Lebesgue integrable function,* $u,\bar{u} \in C_0$ *and* $v,\bar{v} \in \mathcal{L}_0$. *Furthermore let* $y : [-r,T] \to R^n$ *be an absolutely continuous mapping such that*

(e) $y(t) = \varphi(t)$ *for* $t \in [-r,0]$,

(f) $d(\dot{y}(t), F(t,y_t,\dot{y}_t)) \leq \delta(t)$ *for a.e.* $t \in [0,T]$.

Then there is a solution $x(\cdot)$ *of an initial-value problem*

$$\begin{cases} \dot{x}(t) \in F(t,x_t,\dot{x}_t) & \textit{for a.e. } t \in [0,T] \\ x(t) = \varphi(t) & \textit{for } t \in [-r,0] \end{cases}$$

such that

$$|x(t) - y(t)| \le \xi(t) \textit{ for } t \in [0,T]$$

and

$$|\dot{x}(t) - \dot{y}(t)| \le \delta(t) + 2k(t)\xi(t) \textit{ for a.e. } t\in[0,T]$$

where

$$\xi(t) = \int_0^t \delta(s)\, e^{2[m(t)-m(s)]}ds \textit{ and } m(t) = \int_0^t k(r)dr.$$

Now we can prove the main result of this paper.

Theorem 2. *Suppose* $F^i : [0,\infty) \times Co \times \mathcal{L}_o \to Conv\, R^n$, $(i = 1,2)$ *satisfy the conditions (a) - (d). Then, for each* $\eta > 0$ *and* $T > 0$ *there exists a* $\varepsilon^o(\eta,T) > 0$ *such that for every* $\varepsilon\in(0,\varepsilon^o]$ *the following conditions are satisfied:*

(i) *for each solution* $x^1(\cdot)$ *of (1) there exists a solution* $x^2(\cdot)$ *of (2) such that*

(4) $$|x^1(t) - x^2(t)| \le \eta \quad \textit{for} \quad t \in [-r,T/\varepsilon]$$

(ii) *for each solution* $x^2(\cdot)$ *of (2) there exists a solution* $x^1(\cdot)$ *of (1) such that (4) holds.*

Proof. Let $x^1(\cdot)$ be a solution of (1) on $[-r,\infty)$. To prove this theorem we shall consider the solution $x^2(\cdot)$ of the inclusion (2) in such a way that for $t \in [-r,0]$, $x^1(t) = x^2(t) = \varphi(t) = const.$, hence $|x^1(t) - x^2(t)| = 0 < \eta$ and that $t \in [0,T/\varepsilon]$ inequality (4) was satisfied too.

To do this divide the interval $[0,T/\varepsilon]$ on m-subintervals $[t_i,t_{i+1}]$, $i = 0,1,2,\dots,m-1$, and write a solution $x^1(\cdot)$ in the form:

$$(5)\qquad \begin{cases} x^1(t) = \varphi(t) = \text{const. for } t \in [-r,0], \\ x^1(t) = x^1(t_i) + \varepsilon\int_{t_i}^{t} v^1(\tau)d\tau \text{ for } t \in [t_i,t_{i+1}] \\ \text{where } t_i = iT/\varepsilon m,\ v^1(t) \in F^1(t,x_t^1,\dot{x}_t^1). \end{cases}$$

Let us consider a function $y^1(\cdot)$ defined by

$$(6)\qquad \begin{cases} y^1(t) = \varphi(t) = \text{const. for } t \in [-r,0], \\ y^1(t) = y^1(t_i) + \varepsilon\int_{t_i}^{t} z_{i+1}^1(\tau)d\tau \text{ for } t \in [t_i,t_{i+1}] \end{cases}$$

where $z_{i+1}^1(\cdot)$, $i = 0,1,\dots,m-1$, are measurable functions such that

$$z_{i+1}^1(t) \in F^1(t,y_{t_i}^1,\dot{y}_{t_i}^1)$$

and

$$|v^1(t) - z_{i+1}^1(t)| = d(v^1(t),F^1(t,y_{t_i}^1,\dot{y}_{t_i}^1)) = \min_{z(t)\in F^1(t,y_{t_i}^1,\dot{y}_{t_i}^1)} |v_1(t) - z(t)|.$$

Such measurable mappings z_{i+1}^1 exist because set-valued function $F^1(\cdot,y_{t_i},\dot{y}_{t_i}^1)$ is measurable and has compact and convex values ([3]).

By virtue of (5), for every $t \in [t_i,t_{i+1}]$ we have

$$(7)\qquad \begin{aligned} &|x^1(t) - y^1(t_i)| \le \\ &\le |x^1(t) - x^1(t_i)| + |x^1(t_i) - y^1(t_i)| = \\ &= |\varepsilon\int_{t_i}^{t} v^1(\tau)d\tau| + |x^1(t_i) - y^1(t_i)| \le \\ &\le \varepsilon M(t - t_i) + \delta_i, \end{aligned}$$

where $\delta_i = |x^1(t_i) - y^1(t_i)|$, $i = 0,1,\dots,m-1$.

Furthermore for $t \in [t_i, t_{i+1}]$

$$(8) \qquad |v^1(t) - z^1_{i+1}(t)| \le$$

$$\le H(F^1(t, x^1_t, \dot{x}^1_t), F^1(t, y^1_{t_i}, \dot{y}^1_{t_i})) \le$$

$$\le k[\|x^1_t - y^1_{t_i}\|_0 + |\dot{x}^1_{t_i} - \dot{y}^1_{t_i}|_0].$$

But

$$\|x^1_t - y^1_{t_i}\|_0 \le \|x^1_t - x^1_{t_i}\|_0 + \|x^1_{t_i} - y^1_{t_i}\|_0 =$$

$$= \sup_{-r \le s \le 0} |x^1(t + s) - x^1(t_i + s)| +$$

$$+ \sup_{-r \le s \le 0} |x^1(t_i + s) - y^1(t_i + s)|.$$

By the absolutely continuous function $x^1(\cdot)$ it follows that for $t \in [t_i, t_{i+1}]$, $x^1(\cdot)$ is uniformly continuous. Therefore, for every given above $\varepsilon > 0$ there exists $\delta(\varepsilon) > 0$ such that $|x^1(t + s) - x^1(t_i + s)| < \varepsilon$ for each $t \in [t_i, t_{i+1}]$ whenever $|t - t_i| \le (T/\varepsilon m) < \delta(\varepsilon)$.

Hence

$$\sup_{-r \le s \le 0} |x^1(t + s) - x^1(t_i + s)| < \varepsilon .$$

Furthermore

$$\sup_{-r \le s \le 0} |x^1(t_i + s) - y^1(t_i + s)| =$$

$$= \sup_{t_i - r \le \tau \le t_i} |x^1(\tau) - y^1(\tau)|.$$

On the ground of (5) and (6), we have

$$|x^1(\tau) - y^1(\tau)| \le |x^1(t_i) - y^1(t_i)| +$$

$$+ \varepsilon \int_\tau^{t_i} |v^1(s) - z^1_{i+1}(s)| ds \le$$

$$\leq \delta_i + 2\varepsilon M(t_i - \tau) \leq \delta_i + 2\varepsilon Mr.$$

Hence

$$\sup_{-r\leq s\leq 0} |x^1(t_i + s) - y^1(t_i + s)| \leq \delta_i + 2\varepsilon Mr.$$

Then

(9) $$\| x^1_t - y^1_{t_i} \|_o \leq \varepsilon + \delta_i + 2\varepsilon Mr.$$

By the definition of the norm $|\cdot|_o$ and the definitions of (5) and (6), we obtain

$$|\dot{x}^1_t - \dot{y}^1_{t_i}|_o = \int_{-r}^{0} |\dot{x}^1(t + s) - \dot{y}^1(t_i + s)|\,ds =$$

$$= \begin{cases} 0 \text{ for every } (t + s),(t_i + s) \in [-r,0], \\ \varepsilon \int_{-r}^{0} |v^1(t + s) - z^1_{i+1}(t_i + s)|\,ds \leq 2\varepsilon Mr \\ \qquad \text{for every } (t+s),(t_i+s) \in [0,T/\varepsilon], \\ \int_{-r}^{0} |\varepsilon v^1(t + s) - \dot{\varphi}(t_i + s)|\,ds \leq \varepsilon Mr \text{ for every} \\ \qquad (t + s) \in [0,T/\varepsilon] \text{ and } (t_i + s) \in [-r,0]. \end{cases}$$

Hence

(10) $$|x^1_t - y^1_{t_i}|_o \leq 2\varepsilon Mr .$$

Therefore inequality (8) for $t \in [t_i,t_{i+1}]$ can be written down in the form

(11) $$|v^1(t)-z^1_{i+1}(t)| \leq k(\varepsilon + \delta_i + 4\varepsilon Mr) .$$

By virtue of (5), (6) and (11), it follows

$$\delta_i = |x^1(t_i) - y^1(t_i)| \leq |x^1(t_{i-1}) - y^1(t_{i-1})| +$$

$$+ \varepsilon \int_{t_{i-1}}^{t_i} |v^1(\tau) - z^1(\tau)| d\tau \le$$

$$\le \delta_{i-1} + k\varepsilon(t_i - t_{i-1})(\varepsilon + \delta_{i-1} + 4\varepsilon Mr) =$$

$$= \delta_{i-1} + \frac{kT}{m}(\varepsilon + \delta_{i-1} + 4\varepsilon Mr) =$$

$$= \delta_{i-1}(1 + \frac{kT}{m}) + \frac{\varepsilon kT}{m}(1 + 4Mr) =$$

$$= \delta_{i-1}(1 + \frac{a}{m}) + \frac{b}{m},$$

where $a = kT$ and $b = \varepsilon kT(1 + 4Mr)$.

Hence

$$\delta_i \le (1 + \frac{a}{m})[(1 + \frac{a}{m})\delta_{i-2} + \frac{b}{m}] + \frac{b}{m}$$

$$\le (1 + \frac{a}{m})^i \delta_0 + (1 + \frac{a}{m})^{i-1} \frac{b}{m} + \ldots + \frac{b}{m} =$$

$$= \frac{b}{m}(1 + (1 + \frac{a}{m}) + \ldots + (1 + \frac{a}{m})^{i-1}) = \tag{12}$$

$$= \frac{b}{a}[(1 + \frac{a}{m})^i - 1] \le \frac{b}{a}(e^a - 1) =$$

$$= \varepsilon(1 + 4Mr)(e^{kT} - 1),$$

where $i = 0,1,\ldots,m-1$.

For $t \in [t_i, t_{i+1}]$ we have

$$|x^1(t) - x^1(t_i)| = |x^1(t_i) + \varepsilon \int_{t_i}^{t} v^1(\tau) d\tau - x^1(t_i)| \le \tag{13}$$

$$\le \varepsilon M(t - t_i) \le \frac{MT}{m}$$

and

$$|y^1(t) - y^1(t_i)| \le \frac{MT}{m}. \tag{14}$$

On the ground of (12), (13) and (14), we obtain

$$|x^1(t) - y^1(t)| \le |x^1(t) - x^1(t_i)| + \tag{15}$$

$$+ |x^1(t_i) - y^1(t_i)| + |y^1(t_i) - y^1(t)|$$

$$\leq \frac{2MT}{m} + \varepsilon(1 + 4Mr)(e^{kT} - 1).$$

Now we shall consider the function

$$(16) \quad \begin{cases} y^2(t) = \varphi(t) = \text{const. for } t \in [-r,0], \\ y^2(t) = y^2(t_i) + \varepsilon\int_{t_i}^{t} z^2_{i+1}(\tau)d\tau \text{ for } t \in [t_i,t_{i+1}], \\ \qquad\qquad i = 0,1,\dots,m-1, \end{cases}$$

where $z^2_{i+1}(\cdot)$, $i = 0,1,\dots,m-1$, are measurable functions, such that $z^2_{i+1}(t) \in F^2(t,y^1_{t_i},\dot{y}^1_{t_i})$ and $|z^1_{i+1}(t) - z^2_{i+1}(t)| =$ $= d(z^1_{i+1}(t),F^2(t,y^1_{t_i},\dot{y}^1_{t_i})) = \min\limits_{z(t)\in F^2(t,y^1_{t_i},\dot{y}^1_{t_i})} |z^1_{i+1}(t) - z(t)|$.

Let us notice that by virtue of condition (d) for each $\eta_1 > 0$ and $T > 0$ there exists a $\varepsilon^0(\eta_1,T) > 0$ such that for every $\varepsilon \leq \varepsilon^0$, we have inequalities

$$(17) \quad H\left(\frac{\varepsilon m}{iT}\int_0^{iT/\varepsilon m} F^1(t,y^1_{t_i},\dot{y}^1_{t_i})dt, \frac{\varepsilon m}{iT}\int_0^{iT/\varepsilon m} F^2(t,y^1_{t_i},\dot{y}^1_{t_i})dt\right) \leq$$

$$\leq \frac{\eta_1}{2i}$$

and

$$(18) \quad H\left(\frac{\varepsilon m}{(i+1)T}\int_0^{\frac{(i+1)T}{\varepsilon m}} F^1(t,y^1_{t_i},\dot{y}^1_{t_i})dt, \frac{\varepsilon m}{(i+1)T}\int_0^{\frac{(i+1)T}{\varepsilon m}} F^2(t,y^1_{t_i},\dot{y}^1_{t_i})dt\right)$$

$$\leq \frac{\eta_1}{2(i+1)}$$

where $i = 1,2,\dots,m-1$.

By virtue of (17), (18) and the Hausdorff metric condition (see Lemma 3, ([2])), we have

$$H\left(\int_{\frac{iT}{\varepsilon m}}^{\frac{(i+1)T}{\varepsilon m}} F^1(t,y^1_{t_i},\dot{y}^1_{t_i})dt, \int_{\frac{iT}{\varepsilon m}}^{\frac{(i+1)T}{\varepsilon m}} F^2(t,y^1_{t_i},\dot{y}^1_{t_i})dt\right) \leq$$

$$\le H\left(\int_0^{\frac{iT}{\varepsilon m}} F^1(t,y^1_{t_i},\dot y^1_{t_i})dt,\ \int_0^{\frac{iT}{\varepsilon m}} F^2(t,y^1_{t_i},\dot y^1_{t_i})dt\right) +$$

$$+ H\left(\int_0^{\frac{(i+1)T}{\varepsilon m}} F^1(t,y^1_{t_i},\dot y^1_{t_i})dt,\ \int_0^{\frac{(i+1)T}{\varepsilon m}} F^2(t,y^1_{t_i},\dot y^1_{t_i})dt\right) \le$$

$$\le \frac{\eta_1 T}{\varepsilon m}$$

Then

$$\left|\int_{\frac{iT}{\varepsilon m}}^{\frac{(i+1)T}{\varepsilon m}} (z^1_{i+1}(\tau) - z^2_{i+1}(\tau))d\tau\right| \le \frac{\eta_1 T}{\varepsilon m}$$

and

$$|y^1(t_{i+1}) - y^2(t_{i+1})| \le |y^1(t_i) - y^2(t_i)| +$$

(19) $$+ \left|\varepsilon\int_{t_i}^{t_{i+1}}(z^1_{i+1}(\tau) - z^2_{i+1}(\tau))d\tau\right| \le$$

$$\le |y^1(t_i) - y^2(t_i)| + \frac{\eta_1 T}{m} \le \dots \le$$

$$\le \frac{m\eta_1 T}{m} = \eta_1 T, \quad \text{where } i = 0,1,\dots,m-1.$$

Using the inequality (14), (19) and the fact that for $t \in [t_i,t_{i+1}]$, $|y^2(t) - y^2(t_i)| \le \frac{MT}{m}$ we have

$$|y^1(t) - y^2(t)| \le |y^1(t) - y^1(t_i)| +$$

(20) $$+ |y^1(t_i) - y^2(t_i)| + |y^2(t_i) - y^2(t)|$$

$$\le \frac{2MT}{m} + \eta_1 T .$$

By assumption (c) it follows that

$$H(F^2(t,y_t^2,\dot{y}_t^2),F^2(t,y_{t_i}^1,\dot{y}_{t_i}^1)) \le$$
$$k[\|y_t^2 - y_{t_i}^1\|_0 + |\dot{y}_t^2 - \dot{y}_{t_i}^1|_0].$$

Similarly, as in the proof of the inequality (9) and (10), using the definitions of the norm $\|\cdot\|_0$ and $|\cdot|_0$ and the absolutely continuous function $y^2(\cdot)$ and making use of the inequality (20) we obtain

$$\|y_t^2 - y_{t_i}^1\|_0 \le \varepsilon + \frac{2MT}{m} + \eta_1 T \text{ and } |y_t^2 - y_{t_i}^1|_0 \le 2\varepsilon Mr.$$

Hence

$$H(F^2(t,y_t^2,\dot{y}_t^2),F^2(t,y_{t_i}^1,\dot{y}_{t_i}^1)) \le$$
$$\le k(\varepsilon + \frac{2MT}{m} + \eta_1 T + 2\varepsilon Mr).$$

By virtue of (16), we have

$$d(\dot{y}^2(t),\varepsilon F^2(t,y_t^2,\dot{y}_t^2)) \le d(\dot{y}^2(t),\varepsilon F^2(t,y_{t_i}^1,\dot{y}_{t_i}^1)) +$$
$$+ H(\varepsilon F^2(t,y_{t_i}^1,\dot{y}_{t_i}^1),\varepsilon F^2(t,y_t^2,\dot{y}_t^2)) \le$$
$$\le \varepsilon k(\varepsilon + \frac{2MT}{m} + \eta_1 T + 2\varepsilon Mr).$$

Now, on the ground of Filippov's type theorem (see Theorem 1) there exists the solution $x^2(\cdot)$ of (2) that for $t \in [0,T/\varepsilon]$

$$|y^2(t) - x^2(t)| \le \varepsilon k(\varepsilon + \frac{2MT}{m} + \eta_1 T + 2\varepsilon Mr)\int_0^t e^{2\varepsilon k(t-s)}ds$$
$$\le (\frac{\varepsilon}{2} + \frac{MT}{m} + \frac{\eta_1 T}{2} + \varepsilon Mr)(e^{2kT} - 1).$$

Hence, and by inequality (15) and (20), it follows

$$|x^1(t) - x^2(t)| \le |x^1(t) - y^1(t)| + |y^1(t) - y^2(t)| +$$
$$+ |y^2(t) - x^2(t)| \le \frac{2MT}{m} + (\varepsilon + 4\varepsilon Mr)(e^{kT} - 1) + \frac{2MT}{m} +$$

$$+ \eta_1 T + \left(\frac{\varepsilon}{2} + \frac{MT}{m} + \frac{\eta_1 T}{2} + \varepsilon Mr\right)(e^{2kT} - 1) \le$$

$$\le \frac{MT}{m}(3 + e^{2kT}) + \frac{\eta_1 T}{2}(1 + e^{2kT}) + \frac{\varepsilon e^{2kT}}{2}(10Mr + 3).$$

Therefore, choosing

$$m > \frac{3MT(3 + e^{2kT})}{\eta}, \qquad \eta_1 = \frac{2\eta}{3T(1 + e^{2kT})}$$

and

$$\varepsilon < \frac{2\eta}{3e^{2kT}(10Mr + 3)},$$

we get the inequality

$$|x^1(t) - x^2(t)| \le \eta \quad \text{for } t \in [0, T/\varepsilon].$$

Adopting now the procedure presented above we get condition (ii). In this way the proof is completed for $t \in$ $\in [-r, T/\varepsilon]$.

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REZIME

TEOREMA O DELIMIČNOM USREDNJENJU ZA FUNKCIONALNO-DIFERENCIJALNE JEDNAČINE NEUTRALNOG TIPA

U ovom radu dokazana je jedna generalizacija rezultata Plotnikova iz [5] za funkcionalno-diferencijalne jednačine oblika $\dot{x}(t) \in F(t,x_t,\dot{x}_t)$ gde je F višeznačno preslikavanje sa vrednostima koje su neprazni kompaktni konveksni podskupovi n-dimenzionalnog prostora R^n.

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