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CARATHEODORY SELECTIONS, RANDOM FIXED-POINT THEOREMS AND EXISTENCE OF EQUILIBRIA

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ABSTRACT

In this paper a Caratheodory-type selection theorem is proved. As an application, a random fixed point theorem is obtained. A result on the existence of equilibria in abstract economies with a measure space of agents and with an infinite-dimensional strategy space is also included.

1. INTRODUCTION

In a series of papers, Kim-Prikry-Yannelis [2, 3, 4] have proved a Caratheodory-type selection theorem and used it to prove general theorems on the existence of random fixed--points and the existence of equilibria in abstract economies with a measure space of agents and with an infinite-dimensional strategy space.

The object of this paper is to prove similar results. The paper is organized as follows. Section 2 contains definitions and preliminary material. In section 3, a Caratheodory-type selection theorem is proved. In section 4, a random fixed-point theorem is proved. In section 5, a theorem on the

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existence of equilibrium in abstract economies is proved by using the Caratheodory-type selection theorem established in section 3.

2. PRELIMINARIES

Let X,Y be two topological spaces. The *graph* of the correspondence $\phi : X \to 2^Y$ is denoted by $G_\phi = \{(x,y) \in X \times \times Y : y \in \phi(x)\}$. The correspondence $\phi : X \to 2^Y$ is said to have a *closed graph* if the set G_ϕ is closed in $X \times Y$. A correspondence $\phi: X \to 2^Y$ is said to have *open lower sections* if for each $y \in Y$ the set $\phi^{-1}(y) = \{x \in X : y \in \phi(x)\}$ is open in X. An *open cover* of a topological space X is a collection $U = \{u_a : a \in A\}$ of open subsets of X whose union is X, i.e. $\bigcup_{a \in A} u_a = X$. An open cover $U = \{u_a : a \in A\}$ is *locally finite* if every $x \in X$ has a neighborhood intersecting only finitely many $u \in U$. Let (T,τ,μ) be a complete finite measure space, i.e. μ is a realvalued, non-negative, countably additive measure defined in a complete σ-field τ of subsets of T such that $\mu(T) < \infty$. The correspondence $\phi: T \to 2^X$ is a said to have a *measurable graph* if $G_\phi \in \tau \times B(X)$, where B(X) denotes Borel σ-algebra on X and $\times$ denotes σ-product field.

Let now X be a Banach space. $L_1(\mu,X)$ denotes the space of equivalence classes of X-valued Bochner integrable functions $f : T \to X$ normed by

$$\|f\| = \int_T \|f(t)\| d\mu(t).$$

A correspondence $\phi : T \to 2^X$ is said to be *integrably bounded* if there exists a map $g \in L_1(\mu)$ such that for almost all $t \in T$, $\sup\{\|x\| : x \in \phi(t)\} \leq g(t)$. A Banach space X has the *Radon-Nikodym property* with respect to (T,τ,μ) if for each μ-continuous vector measure $G : \tau \to X$ of bounded variation there exists $g \in L_1(\mu,X)$ such that $G(E) = \int_E g d\mu$ for all $E \in \tau$.

Let X be a topological space and Y be a linear topological space. Let $\phi : X \to 2^Y$ be a nonempty valued correspondence. A function $f : X \to Y$ is said to be a *continuous se-*

lection from ϕ if $f(x) \in \phi(x)$ for all $x \in X$, and f is continuous. Let T be an arbitrary measure space. Let $\psi : T \to 2^Y$ be a nonempty valued correspondence. A function $f : T \to Y$ is said to be a *measurable selection* from ψ if $f(t) \in \psi(t)$ for all $t \in T$, and f is measurable. Let Z be a topological space and $\phi : T \times Z \to 2^Y$ be a nonempty valued correspondence. A function $f : T \times Z \to Y$ is said to be a *Caratheodory-type selection* from ϕ if $f(t,z) \in \phi(t,z)$ for all $(t,z) \in T \times Z$ and $f(\cdot,z)$ is measurable for all $z \in Z$ and $f(t,\cdot)$ is continuous for all $t \in T$.

3. SELECTION THEOREM

The following selection theorem is due to Kim-Prikry-Yannelis [2].

Theorem 1 *Let* (T,τ,μ) *be a complete measure space, Y be a complete, metrizable and separable Hausdorff linear topological space and Z be a complete metrizable and separable topological space. Let* $X : T \to 2^Y$ *be a correspondence with measurable graph, and* $\phi : T \times Z \to 2^Y$ *be a convex valued correspondence (possibly empty) with measurable graph, such that:*

(i) *for each* $t \in T$, $\phi(t,x) \subseteq X(t)$ *for all* $x \in Z$.

(ii) *for each* $t, \phi(t,\cdot)$ *has open lower sections in* Z, *i.e. for each* $t \in T$, *and each* $y \in Y$, $\phi_t^{-1}(y) = \{x \in Z : y \in \phi(t,x)\}$ *is open in* Z.

(iii) *for each* $(t,x) \in T \times Z$, *such that* $\phi(t,x) \neq \emptyset$, $\phi(t,x)$ *has a nonempty interior in* $X(t)$.

Let $U = \{(t,x) \in T \times Z : \phi(t,x) \neq \emptyset\}$ *and for each* $x \in Z$, $U_x = \{t \in T : (t,x) \in U\}$ *and for each* $t \in T$, $U^t = \{x \in Z : (t,x) \in U\}$. *Then for each* $x \in Z$, U_x *is a measurable set in* T *and there exists a Caratheodory-type selection from* $\phi|_U$, *i.e., there exists a function* $f : U \to Y$ *such that* $f(t,x) \in \phi(t,x)$ *for all* $(t,x) \in U$ *and for each* $x \in Z$, $f(\cdot,x)$ *is measurable on* U_x *and for each* $t \in T$, $f(t,\cdot)$ *is continuous on* U^t. *Moreover,* $f(\cdot,\cdot)$ *is jointly measurable.*

We now prove the following selection theorem.

Theorem 2 *Let* T,Y *and* Z *be as in Theorem 1. Suppose that* $\phi : T \times Z \to 2^Y$ *is a convex-valued correspondence (possibly empty) such that the following conditions hold:*

(i)′ *if* S *is a countable dense subset of* Y, *then for each* $t \in T$, *and* $x \in U^t = \{z \in Z : (t,z) \in U\}$, *there exists* $y \in S$ *such that* $x \in \mathrm{int}\,\phi_t^{-1}(y)$ *where* $\phi_t^{-1}(y) = \{z \in Z : y \in \phi(t,z)\}$

(ii)′ *for each* $t \in T$, *and* $y \in Y$ *the correspondence* $B : T \to 2^Z$ *given by* $B(t) = \mathrm{int}\{z \in Z : y \in \phi(t,z)\}$ *has a measurable graph.*

Then the conclusion of Theorem 1 holds, i.e. ϕ *has a Caratheodory-type selection* f, *that is jointly measurable.*

Remark The Theorem 2 is different from the Theorem 1 in the following ways:

1. The conditions (i) and (iii) of Theorem 1 are altogether removed in Theorem 2.
2. The condition (i)′ of Theorem 2 is weaker than the conditions (ii) and (iii) of Theorem 1. To see this suppose that conditions (ii) and (iii) of Theorem 1 hold. Let S be a countable dense subset of Y and $t \in T$ with $x \in U^t$.

Now $x \in U^t$ implies that $\phi(t,x) \neq \phi$. Hence, the condition (iii) implies that there exists $y \in \phi(t,x) \cap S$, since S is dense. Now $y \in \phi(t,x)$ implies $x \in \phi_t^{-1}(y)$. Condition (ii) implies that $\phi_t^{-1}(y)$ is open. Hence, $x \in \mathrm{int}\,\phi_t^{-1}(y) = \phi_t^{-1}(y)$ and condition (i)′ of theorem holds. Thus conditions (ii) and (iii) of Theorem 1 imply condition (i)′ of Theorem 2.

3. The measurability of $\phi : T \times Z \to 2^Y$ in Theorem 1 has been replaced by the condition (ii)′. We do not know if the two measurability assumptions are related. It seems that they are not comparable.

Proof The general line of argument is the same as in Kim-Prikry-Yannelis [2]. As in [2], U_x is a measurable set. Since Y is separable there exists a countable dense subset $\{y_1,y_2,\dots,\}$ of Y. For each n = 1,2,..., define a function $f_n : T \to Y$ by $f_n(t) = y_n$ for all $t \in T$. Each f_n is clearly measurable.

Now for each $t \in T$ and $x \in U^t$, assumption (i)′ implies that there exists $f_n(t)$ for some n, such that $x \in \mathit{int}\,\phi_t^{-1}(f_n(t)) = B_n(t)$. This implies that the open sets $\{B_n(t) : n = 1,2,\dots\}$ form an open cover of the set U^t, and $B_n(t) \subseteq \{z \in Z : f_n(t) \in \phi(t,z)\}$. Assumption (ii)′ implies that $B_n(\cdot)$ has a measurable graph.

For each m = 1,2,... define the operator $(\ )_m$ by $(W)_m = \{w \in W : \mathit{dist}(w, Z\setminus W) \geq \frac{1}{2^m}\}$. For n = 1,2,... and $t \in T$, let $C_n(t) = B_n(t)\setminus \bigcup_{k=1}^{n-1} (B_k(t))_n$.
Then $C_n(t)$ is open in Z and it can be easily checked that $\{C_n(t) : n = 1,2,\dots\}$ is a locally finite open cover of the set $\{x \in Z : (t,x) \in U\} = U^t$.

Since $B_n(\cdot)$ has a measurable graph, it follows that $C_n(\cdot)$ has a measurable graph by Lemmas 4.6 and 4.8 of Kim-Prikry-Yannelis [2].

Define, for n = 1,2,...,
$$g_n(t,x) = \frac{\mathit{dist}(x, Z\setminus C_n(t))}{\sum_{k=1}^{\infty} \mathit{dist}(x, Z\setminus C_k(t))}.$$

This is a partition of unity subordinated to the open cover $\{C_n(t) : n = 1,2,\dots\}$.
Define $f : U \to Y$ by $f(t,x) = \sum_{n=1}^{\infty} g_n(t,x) f_n(t)$. Since $\{C_n(t) : n = 1,2,\dots\}$ is locally finite, each x has a neighborhood N_x which intersects only finitely many $C_n(t)$. Hence, $f(t,\cdot)$ is a finite sum of continuous functions on N_x and it is therefore continuous. Furthermore, for any n such that $g_n(t,x) > 0$, $x \in C_n(t) \subseteq B_n(t) \subseteq \{z \in Z : f_n(t) \in \phi(t,z)\}$, i.e., $f_n(t) \in \phi(t,x)$. So $f(t,x)$ is a convex combination of elements $f_n(t)$ from the convex set $\phi(t,x)$. Consequently, $f(t,x) \in \phi(t,x)$ for all $(t,x) \in U$. Since $C_n(\cdot)$ has a measurable graph, $\mathit{dist}(x, Z \setminus C_n(\cdot))$ is a measurable function by Lemmas 4.6 and 4.7 of

Kim-Prikry-Yannelis [2]. Therefore for each n and x, $g_n(\cdot,x)$ is a measurable function. Since for each n, $f_n(\cdot)$ is a measurable function, it follows that $f(\cdot,x)$ is measurable for each x, i.e., $f(t,x)$ is a Caratheodory-type selection from $\phi|_U$.

The joint measurability of f follows from Lemma 4.12 of Kim-Prikry-Yannelis [4].

4. A RANDOM FIXED-POINT THEOREM

Let T be any measure space and X be a nonempty subset of any linear topological space. Let ϕ be a correspondence from $T \times X$ into 2^X. The correspondence ϕ is said to have a *random fixed point* if there exists a measurable function $x : T \to X$ such that $x(t) \in \phi(t,x(t))$ for almost all $t \in T$.

Below we prove a random fixed-point theorem.

Theorem 3. *Let* (T,τ,μ) *be a complete finite measure space, and* K *be a nonempty compact convex subset of a separable, complete and metrizable locally convex linear topological space* E. *Let* $\phi : T \times K \to 2^K$ *be a non-empty convex valued map such that*

(i) *if* S *is a countable dense subset of* E, *then for each* $t \in T$, $x \in U^t = \{z \in K : \phi(t,z) \neq \phi\}$ *there exists* $y \in S$ *such that* $x \in int\ \phi_t^{-1}(y)$ *where* $\phi_t^{-1}(y) = \{z \in K : y \in \phi(t,z)\}$.

(ii) *for each* $t \in T$ *and* $y \in Y$ *the map* $B : T \to 2^K$ *given by* $B(t) = int\{z \in K : y \in \phi(t,z)\}$ *has a measurable graph.*

Then ϕ *has a random fixed-point.*

Proof. All the asumptions of Theorem 2 are satisfied for ϕ. Hence, Theorem 2 implies that there exists a jointly measurable Caratheodory-type selection f of ϕ, i.e., $f(t,x) \in \phi(t,x)$ for all $(t,x) \in T \times K$.

Define $F : T \to K$ by $F(t) = \{x \in K : x - f(t,x) = 0\}$.

$F(t) \neq \phi$ by Tychonoff's fixed point theorem. F has a measurable graph since f is jointly measurable. Hence, Aumann's measurable selection theorem implies that there exists a measurable function $x^* : T \to K$ such that $x^*(t) \in F(t)$ for almost all $t \in T$. Consequently, $x^*(t) = f(t,x^*) \in \phi(t,x^*)$ for almost all $t \in T$, and the theorem is proved.

Remark The theorem proved above is similar to Theorem 3.3 of Kim-Prikry-Yannelis [4]. It should be noted that unlike Kim-Prikry-Yannelis we do not assume that ϕ has closed values or that E is a separable Banach space.

5. EQUILIBRIUM EXISTENCE THEOREM

Let (T,τ,μ) be a finite, positive, complete measure space. Let Y be a separable Banach space whose dual possesses the Radon-Nikodym property. For any correspondence $X : T \to 2^Y$, $L_1(\mu,X)$ will denote the subset of $L_1(\mu,X)$ consisting of those $x \in L_1(\mu,X)$ which satisfy $x(t) \in X(t)$ for almost all t in T.

An *abstract economy* Γ is a quadruple $[(T,\tau,\mu),X,P,A]$, where

(1) (T,τ,μ) is a measure space of agents;

(2) $X : T \to 2^Y$ is a strategy correspondence;

(3) $P : T \times L_1(\mu,X) \to 2^Y$ is a preference correspondence such that $P(t,x) \subseteq X(t)$ for all $(t,x) \in T \times L_1(\mu,X)$;

(4) $A : T \times L_1(\mu,X) \to 2^Y$ is a correspondence such that $A(t,x) \subseteq X(t)$ for all $(t,x) \in T \times L_1(\mu,X)$.

Notice that since P is a mapping from $T \times L_1(\mu,X)$ to 2^Y, we have allowed for interdependent preferences. The interpretation of these preference correspondences is that $y \in P(t,x)$ means that agent t strictly prefers y to x(t) if the given strategies of other agents are fixed. Notice that preferences need not be transitive or complete and therefore need not be representable by utility functions. However, it will be assumed that $x(t) \notin \text{con}\, P(t,x)$, the convex hull of $P(t,x)$, for all $x \in L_1(\mu,X)$ and for almost all t in T, which implies that

$x(t) \notin P(t,x)$ for all $x \in L_1(\mu,X)$ and almost all t in T, i.e., $P(t,\cdot)$ is *irreflexive* for almost all t in T.

An *equilibrium* for Γ is an $x^* \in L_1(\mu,X)$ such that for almost all t in T the following conditions are satisfied:

(i) $x^*(t) \in c\ell A(t,x^*)$, and

(ii) $P(t,x^*) \cap c\ell A(t,x^*) = \phi$.

The following equilibrium existence theorem is due to Kim-Prikry-Yannelis [2].

Theorem 4 *Let* $\Gamma = [(T,\tau,\mu),X,P,A]$ *be an abstract economy satisfying the assumptions* (A.1)-(A.4) *where:*

(A.1) (T,τ,μ) *is a finite, positive, complete, separable measure space.*

(A.2) $X : T \to 2^Y$ *is an integrably bounded correspondence with measurable graph such that for all* $t \in T$, $X(t)$ *is a nonempty, convex and weakly compact subset of* Y, *where* Y *is a separable Banach space whose dual possesses the Radon-Nikodym property.*

(A.3) $A : T \times L_1(\mu,X) \to 2^Y$ *is a correspondence such that:*

(a) $\{(t,x,y) \in T \times L_1(\mu,X) \times Y : y \in A(t,x)\} \in \tau \times B_w(L_1(\mu,X)) \times B(Y)$ *where* $B_w(L_1(\mu,X))$ *is the Borel* σ*-algebra for the weak topology on* $L_1(\mu,X)$ *and* $B(Y)$ *is the Borel* σ*-algebra for the norm topology on* Y;

(b) *it has weakly open lower sections, i.e., for each* $t \in T$ *and for each* $y \in Y$, *the set* $A^{-1}(t,y) = \{x \in L_1(\mu,X) : y \in A(t,x)\}$ *is weakly open in* $L_1(\mu,X)$;

(c) *for all* $(t,x) \in T \times L_1(\mu,X)$, $A(t,x)$ *is convex and has a nonempty norm interior in* $X(t)$;

(d) *for each* $t \in T$, *the correspondence* $\bar{A}(t,\cdot) : L_1(\mu,X) \to 2^Y$, *defined by* $\bar{A}(t,x) = c\ell A(t,x)$

for all $(t,x) \in T \times L_1(\mu,X)$, *is u.s.c. in the sense that the set* $\{x \in L_1(\mu,X) : \bar{A}(t,x) \subseteq V\}$ *is weakly open in* $L_1(\mu,X)$ *for every norm open subset* V *of* Y.

(A.4) $P : T \times L_1(\mu,X) \to 2^Y$ *is a correspondence such that*

(a) $\{(t,x,y) \in T \times L_1(\mu,X) \times Y : y \in \mathrm{con}P(t,x)\} \in \tau \times B_w(L_1(\mu,X)) \times B(Y)$

(b) *it has weakly open lower sections, i.e., for each* $t \in T$ *and each* $y \in Y$, $P^{-1}(t,y) = \{x \in L_1(\mu,X) : y \in P(t,x)\}$ *is weakly open in* $L_1(\mu,X)$;

(c) *for all* $(t,x) \in T \times L_1(\mu,X)$, $P(t,x)$ *is norm open* $X(t)$;

(d) $x(t) \notin \mathrm{con}\, P(t,x)$ for all $x \in L_1(\mu,X)$ *and for almost all* t *in* T.

Then Γ *has an equilibrium.*

We now use Theorem 2 to weaken some of the topological assumptions Theorem 4.

Theorem 5 *Assume that all the conditions of Theorem 3, except (A.3b) and (A.4b), hold. Suppose that in addition, the following conditions hold:*

(i) *if* S *is a countable dense subset of* Y, *then for each* $t \in T$, $x \in L_1(\mu,X)$ *such that* $A(t,x) \cap \mathrm{con}P(t,x) \neq \phi$, *there exists* $y \in S$ *such that* $x \in \mathrm{int}(A_t^{-1}(y) \cap \mathrm{con}P_t^{-1}(y))$

(ii) *for each* $t \in T$, *and* $y \in Y$, *the correspondence* $B : T \to 2^{L_1(\mu,X)}$ *given by* $B(t) = \mathrm{int}\{z \in L_1(\mu,X) : y \in A(t,z) \cap \mathrm{con}P(t,z)\}$ *has a measurable graph.*

Then the economy Γ *has an equilibrium.*

Proof Although the proof is similar to that in [3], we include it for the sake of completeness. Define $\psi : T \times L_1(\mu,X) \to 2^Y$ by $\psi(t,x) = con\ P(t,x)$ for $(t,x) \in T \times L_1(\mu,X)$ and $\phi : T \times L_1(\mu,X) \to 2^Y$ by $\phi(t,x) = A(t,x) \in \psi(t,x)$.

Assumptions (i) and (ii) imply that all the conditions of Theorem 2 are satisfied. Hence, Theorem 2 implies that there exists a function $f : U \to Y$ such that $f(t,x) \in \phi(t,x)$ for all $(t,x) \in U$, and for each $x \in L_1(\mu,X)$, $f(\cdot,x)$ is measurable on U_x and for each $t \in T$, $f(t,\cdot)$ is continuous on U^t where $L_1(\mu,X)$ is endowed with the weak topology and Y with the norm topology. Moreover, for each $x \in L_1(\mu,X)$, U_x is a measurable set.
Define $\theta : T \times L_1(\mu,X) \to 2^Y$ by

$$\theta(t,x) = \begin{cases} \{f(t,x)\} & \text{if } (t,x) \in U \\ c\ell A(t,x) & \text{if } (t,x) \notin U. \end{cases}$$

By Lemma 4.2 Kim-Prikry-Yannelis [3] for each $x \in L_1(\mu,X)$, the correspondence $c\ell A(\cdot,x) : T \to 2^Y$ has a measurable graph. Therefore, by Lemma 4.3 of Kim-Prikry-Yannelis for each $x \in L_1(\mu,x)$, $\theta(\cdot,x) : T \to 2^Y$ has a measurable graph. Notice that assumption (i) implies that the set U^t is weakly open in $L_1(\mu,X)$. Consequently, by Lemma 4.5 of Kim-Prikry-Yannelis [3], $\theta(t,\cdot) : L_1(\mu,X) \to 2^Y$ is u.s.c. in the sense that the set $\{x \in L_1(\mu,X) : \theta(t,x) \subseteq V\}$ is weakly open in $L_1(\mu,X)$ for every norm open subset V of Y. Moreover, θ is convex and non-empty valued. Define $F : L_1(\mu,X) \to 2^{L_1(\mu,X)}$ by $F(x) = \{y \in L_1(\mu,X) :$ for almost all t in $T, y(t) \in \theta(t,x)\}$. Since for each $x \in L_1(\mu,X)$, $\theta(\cdot,x)$ has a measurable graph, F is nonempty valued by Lemma 4.6 of Kim-Prikry-Yannelis [3]. Since θ is convex valued, so is F. By Lemma 4.9 and Remark 4.2 of Kim-Prikry-Yannelis [3] F is weakly u.s.c. Furthermore, since $X(\cdot)$ is integrably bounded and has a measurable graph, $L_1(\mu,X)$ is non-empty as a consequence of Aumann's measurable selection theorem; also since $X(\cdot)$ is convex valued, $L_1(\mu,X)$ is convex. By

Lemma 4.8. and Remark 4.1 of Kim-Prikry-Yannelis [3] $L_1(\mu,X)$ is weakly compact. Therefore, by the Fan fixed point theorem, there exists $x^* \in L_1(\mu,X)$ such that $x^* \in f(x^*)$, i.e., $x^*(t) \in \theta(t,x^*)$ for almost all t in T. We now show that the fixed point is by construction an equilibrium for Γ. Suppose that for a non-null set of agents S, $(t,x^*) \in U$ for all $t \in S$. Then by the definition of θ $x^*(t) = f(t,x^*) \in \phi(t,x^*) \subseteq conP(t,x^*)$ for all $t \in S$, a contradiction to assumption (A.4)d. Therefore, $(t,x^*) \notin U$ for almost all t in T and so for almost all $t \in T$, $x^*(t) \in c\ell A(t,x^*)$ and $\phi(t,x^*) = A(t,x^*) \cap conP(t,x^*) = \phi$. But, $A(t,x^*) \cap conP(t,x^*) = \phi$ implies that $A(t,x) \cap P(t,x^*) = \phi$. Since by assumption (A.3)c, $P(t,x)$ is norm open in $X(t)$ for all $(t,x) \in T \times L_1(\mu,X)$, the fact that $A(t,x^*) \cap P(t,x^*) = \phi$, implies that $c\ell A(t,x^*) \cap P(t,x^*) = \phi$, i.e., x^* is an equilibrium for Γ. This completes the proof of the main existence theorem.

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REZIME

KARATEODORIJEVA SELEKCIJA, TEOREME O STOHASTIČKOJ NEPOKRETNOJ TAČKI I EGZISTENCIJA EKVILIBRIJUMA

U ovom radu dokazana je teorema selekcije Karateodorijevog tipa. Kao primena dobijena je teorema o stohastičkoj nepokretnoj tački. Rad sadrži i rezultat o postojanju ekvilibrija u abstraktnoj ekonomiji sa merljivim prostorom agenata i beskonačno dimenzionalnim prostorom strategije.

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