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REGULAR PERMUTATIONS OF PARASTROPHY INVARIANT n-QUASIGROUPS

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ABSTRACT

An n-quasigroup (Q,f) is called a G-n-quasigroup iff $f = f^{\sigma}$ for all $\sigma \in G$, where G is a subgroup of the symmetric group of degree n+1 and f^{σ} is defined by

$$f^{\sigma}(x_{\sigma 1},\dots,x_{\sigma n}) = x_{\sigma(n+1)} \Leftrightarrow f(x_1,\dots,x_n) = x_{n+1}.$$

In the paper regular permutations (Definitions 1, 2 and 3) of several classes of such n-quasigroups are considered and some of their properties described.

1. INTRODUCTION

In the theory of binary quasigroups there exists a well known close relation between nuclei and groups of regular permutations. In the case of n-ary quasigroups the situation is similar, although there exist several different generalizations of the notions of nuclei and regular permutations (see [1], [3], [4]). In this paper we shall consider some classes of

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regular permutations which were defined and considered in [2], [4], [5], [6], [8]. In [2], [5] and [8] regular permutations of totally symmetric (TS) n-quasigroups were studied. But TS--n-quasigroups, as well as cyclic n-quasigroups ([9]) and alternating symmetric (AS) n-quasigroups ([10]), are only special cases of a class of parastrophy invariant n-quasigroups - they are all G-n-quasigroups, where G is a transitive permutation group ([7]). In the paper we shall consider regular permutations of some classes of parastrophy invariant n-quasigroups, in particular G-n-quasigroups, where G is transitive. Since TS, cyclic and AS n-quasigroups are special cases of G-n--quasigroups, where G is transitive, some of the obtained results generalize the corresponding theorems from [5], [8], [9] and [10].

2. NOTATIONS AND DEFINITIONS

We shall give some basic definitions and notations. Other notions from the theory of n-quasigroups can be found in [2].

The sequence $x_m, x_{m+1}, \dots, x_n$ we shall denote by $\{x_i\}_{i=m}^n$ or by x_m^n. If $m > n$, then x_m^n will be considered empty. The sequence $x, x, \dots, x$ (n times) will be denoted by $\overset{n}{x}$. If $n \le 0$, then $\overset{n}{x}$ will be considered empty.

An n-ary groupoid (n-groupoid) (Q,f) is called an n--quasigroup iff the equation $f(a_1^{i-1}, x, a_{i+1}^n) = b$ has a unique solution x for every $a_1^n, b \in Q$ and every $i \in N_n = \{1,\dots,n\}$. An n-quasigroup (Q,f) is an n-loop iff there exists $e \in Q$ such that $f(\overset{i-1}{e}, x, \overset{n-i}{e}) = x$ for all $x \in Q$ and all $i \in N_n$, and e is called a unit of that n-loop.

An n-quasigroup (Q,f) is called idempotent iff $f(\overset{n}{x}) = x$ for all $x \in Q$.

An n-quasigroup (Q,f) is isotopic to an n-quasigroup (Q,g) iff there exists a sequence $T = (\alpha_1^{n+1})$ of permutations of Q such that the following identity

$$g(x_1^n) = \alpha_{n+1}^{-1} f(\{\alpha_i x_i\}_{i=1}^n)$$

holds. T is called an isotopism, g is an isotope of f, and by $f^T = g$ we denote that f is isotopic to g by T. T^{-1} is defined by $T^{-1} = (\{\alpha_i^{-1}\}_{i=1}^{n+1})$. If T is an isotopism of (Q,f) to itself, that is $f^T = f$, then T is called an autotopism of f. The set of all autotopisms of an n-quasigroup (Q,f) under the compositions of autotopisms is a group which we denote by A(f). The automorphism group of (Q,f) we denote by Aut(f).

By S_n we denote the symmetric group of degree n.

If (Q,f) is an n-quasigroup and $\sigma \in S_{n+1}$, then the n-quasigroup f^σ defined by

$$f^\sigma(\{x_{\sigma i}\}_{i=1}^{n}) = x_{\sigma(n+1)} \Leftrightarrow f(x_1^n) = x_{n+1}$$

is called a σ-parastrophe (or simply parastrophe) of f. If $\sigma,\tau \in S_{n+1}$, then $(f^\sigma)^\tau = f^{\sigma\tau}$. If $T = (\alpha_1^{n+1})$ is an isotopism of f, then $(f^T)^\sigma = (f^\sigma)^{T^\sigma}$, where $T^\sigma = (\{\alpha_{\sigma i}\}_{i=1}^{n+1})$.

If (Q,f) is an n-quasigroup and $\sigma \in S_{n+1}$ such that $f = f^\sigma$, then σ is called an autoparastrophism of f. The set of all autoparastrophisms of f is a subgroup of S_{n+1}. If (Q,f) is an n-quasigroup and G is a subgroup of S_{n+1} such that $f = f^\sigma$ for every $\sigma \in G$, then (Q,f) is called a G-n-quasigroup ([7]). We also say that (Q,f) is a G-permutable n-quasigroup. G-n--quasigroup are called parastrophy invariant n-quasigroups. Of course, if H is a subgroup of G, then every G-permutable n--quasigroup is also H-permutable.

Let (Q,f) be a G-n-quasigroup. If $G = S_{n+1}$, then (Q,f) is called totally symmetric, if G is alternating subgroup of S_{n+1}, then (Q,f) is called alternating symmetric and if G is generated by the cycle (1 2 ... n+1), then (Q,f) is called cyclic.

If Q is a nonempty set, by ε we denote the identity mapping of Q.

3. REGULAR PERMUTATIONS

As we have noted before regular permutations of binary quasigroups can be generalized to n-ary case in several ways. Here we shall consider regular permutations of n-quasigroups as defined in [8], [2], [4], [5].

Definition 1. ([2], [8]) *Let* (Q,f) *be an* n-*quasigroup,* $i \in N_n$. *A permutation* α *of* Q *is said to be* i-*inverse regular for* f *iff* $(\overset{i-1}{\varepsilon},\alpha,\overset{n-i}{\varepsilon},\alpha^{-1}) \in A(f)$. *A permutation of* Q *which is* i-*inverse regular for* f *for all* $i \in N_n$ *is called inverse regular for* f. *The set of all inverse regular permutations for* f *will be denoted by* V.

Definition 2. ([4], [5]) *Let* (Q,f) *be an* n-*quasigroup,* $i \in N_n$. *A permutation* α *of* Q *is* i-*outer regular for* f *iff* $(\overset{j-1}{\varepsilon},\alpha,\overset{n-j}{\varepsilon},\alpha) \in A(f)$ *for all* $j \in N_n \setminus \{i\}$. *The set of all* i-*outer regular permutations for* f *will be denoted by* Λ_i.

Definition 3. ([4], [5]) *Let* (Q,f) *be an* n-*quasigroup,* $i \in N_n$. *A permutation* α *of* Q *is* i-*inner regular for* f *iff there exist permutations* β_j^*, $j \in N_n \setminus \{i\}$, *such that* $(\overset{i-1}{\varepsilon},\alpha,\overset{j-i-1}{\varepsilon},\beta_j^{*-1},\overset{n-j+1}{\varepsilon}) \in A(f)$ *for all* $j \in N_n \setminus \{i\}$. *The permutation* β_j^* *is said to be* j-*conjugate to* α. *The set of all* i-*inner regular permutations for* f *will be denoted by* Φ_i, *the set of all* j-*conjugate permutations to all* i-*inner regular permutations by* Φ_{ij}^*.

Each of the sets V, Λ_i, Φ_i, Φ_{ij}^* under the composition of mappings is a group.

Proposition 1. *Let* (Q,f) *be an* n-*quasigroup. Then every inverse regular permutation for* f *is* i-*inner regular permutation for* f *for all* $i \in N_n$, *i.e.* $V \subseteq \Phi_i$.

Proof. If $\alpha \in V$, then for all $i \in N_n$, $T_i = (\overset{i-1}{\varepsilon},\alpha,\overset{n-i}{\varepsilon},\alpha^{-1}) \in A(f)$ and $T^{-1} \in A(f)$. Thus, for a fixed $i \in N_n$ and every $j \in N_n \setminus \{i\}$ $T_iT_j^{-1} = (\overset{j-1}{\varepsilon},\alpha^{-1},\overset{i-j-1}{\varepsilon},\alpha,\overset{n-i}{\varepsilon}) \in A(f)$, hence $\alpha \in \Phi_i$.

Proposition 2. *Let* (Q,f) *be an* n*-quasigroup. If* α *is an* i*-inner regular permutation for* f*, then every* j*-conjugate permutation to* α *is* j*-inner regular permutation for* f*, i. e.*

$$\Phi^{*}_{ij} \subseteq \Phi_j .$$

Proof. Let $\alpha \in \Phi_i$, and β^{*}_j be j-conjugate to α, $j \in N_n \setminus \{i\}$. Then for all $j \in N_n \setminus \{i\}$, $T_{ij} = ({}^{i-1}_{\varepsilon}, \alpha, {}^{j-i-1}_{\varepsilon}, \beta^{*-1}_j, {}^{n-j+1}_{\varepsilon}) \in A(f)$ and $T^{-1}_{ij} \in A(f)$. Hence for a fixed $j \in N_n \setminus \{i\}$ and all $k \in N_n \setminus \{i,j\}$.

$$T^{-1}_{ij}\, T_{ik} = ({}^{j-1}_{\varepsilon}, \beta^{*}_j, {}^{k-j-1}_{\varepsilon}, \beta^{*-1}_j, {}^{n-k+1}_{\varepsilon}) \in A(f),$$

and since $T^{-1}_{ij} \in A(f)$, it follows that $\beta^{*}_j \in \Phi_j$.

Proposition 3. *If* $T = (\alpha_1^{n+1})$ *is an autotopism of a* G*-*n*-quasigroup* (Q,f) *and* $\sigma \in G$*, then* $T^{\sigma} = (\alpha_{\sigma 1}^{\sigma(n+1)})$ *is also an autotopsm of* f*.*

Proof. Since $f^{T} = f$ and $f^{\sigma} = f$, it follows that $f = (f^{T})^{\sigma} = (f^{\sigma})^{T^{\sigma}} = f^{T^{\sigma}}$, i.e. T is an autotopism of f.

Proposition 4. *If for some* $i,j \in N_n$, $i \neq j$, $({}^{i-1}_{\varepsilon}, \alpha, {}^{j-i-1}_{\varepsilon}, \beta, {}^{n-j+1}_{\varepsilon})$ *is an autotopism of a* G*-*n*-quasigroup* (Q,f)*, where* G *is transitive, then* $\beta = \alpha^{-1}$.

Proof. Since $({}^{i-1}_{\varepsilon}, \alpha, {}^{j-i-1}_{\varepsilon}, \beta, {}^{n-j+1}_{\varepsilon}) \in A(f)$, the following identity

$$(1) \qquad f(x_1^{i-1}, \alpha x_i, x_{i+1}^{n}) = f(x_1^{j-1}, \beta^{-1} x_j, x_{j+1}^{n})$$

holds. Putting in (1) $x_1 = \ldots = x_n = x$, by the transitivity of G we get

$$f({}^{i-1}x, \alpha x, {}^{n-i}x) = f({}^{j-1}x, \beta^{-1}x, {}^{n-j}x) = f({}^{i-1}x, \beta^{-1}x, {}^{n-i}x),$$

which implies $\alpha x = \beta^{-1}x$, i.e. $\beta = \alpha^{-1}$.

Corollary 1. *Let* (Q,f) *be a* G-n-*quasigroup, where* G *is transitive.*

(i) *If* $\alpha \in \Lambda_i$, *then* $\alpha^2 = \varepsilon$, *i.e.* Λ_i *is a boolean group.*

(ii) *If* $\alpha \in \Phi_i$ *and* β_j^* *is* j-*conjugate to* α, $j \in N_n \setminus \{i\}$, *then* $\alpha = \beta_j^*$.

Proposition 5. *Let* (Q,f) *be an* n-*quasigroup. If at least one of the following conditions holds*

(i) Q *is finite,*

(ii) (Q,f) *is* G-*permutable, where* G *is transitive,*

then for all $i,j \in N_n$

$$\Phi_i = \Phi_{ij}^* = \Phi_j .$$

Proof. (i) Let Q be finite. Since the group Φ_{ij}^* is antiisomorphic to Φ_i, these groups are isomorphic. Hence by Proposition 2 $\Phi_i \simeq \Phi_{ij} \subseteq \Phi_j$. Since also $\Phi_j \simeq \Phi_{ji}^* \subseteq \Phi_i$, it follows that $\Phi_i = \Phi_j$, which implies $\Phi_i = \Phi_{ij}^* = \Phi_j$ for all $i,j \in N_n$.

(ii) Let (Q,f) be G-permutable, where G is transitive. If $\alpha \in \Phi_i$, i.e. there exist β_m^*, $m \in N_n \setminus \{i\}$, such that $(\overset{i-1}{\varepsilon}, \overset{m-i-1}{\varepsilon}, \beta_m^{*-1}, \overset{n-m+1}{\varepsilon}) \in A(f)$, then for any $k \in N_n$ by the transitivity of G we obtain that $(\overset{k-1}{\varepsilon}, \alpha, \overset{j-k-1}{\varepsilon}, \beta_m^{*-1}, \overset{n-j+1}{\varepsilon}) \in A(f)$ for all $j \in N_n \setminus \{k\}$. Hence $\Phi_i \subseteq \Phi_j$ for all $i,j \in N_n$, which gives $\Phi_i = \Phi_j$, for all $i,j \in N_n$.

If $\alpha \in \Phi_i$ and β_j^* is j-conjugate to α, $j \in N_n \setminus \{i\}$, then by Corollary 1 $\alpha = \beta_j^*$, $j \in N_n \setminus \{i\}$, that is, $\alpha \in \Phi_{ij}^*$ for all $j \in N_n \setminus \{i\}$. We have obtained that $\Phi_i \subseteq \Phi_{ij}^*$ for all $i,j \in N_n$. By Proposition 2 it follows $\Phi_i = \Phi_{ij}^* = \Phi_j$ for all $i,j \in N_n$.

Theorem 1. *Let* (Q,f) *be a* G-n-*quasigroup, where* G *is transitive. Then*

$$\Lambda_i = \Lambda_j \subseteq V = \Phi_i = \Phi^*_{ij}$$

for all $i,j \in N_n$.

Proof. If $\alpha \in \Lambda_i$, then by Proposition 4 $(\overset{j-1}{\varepsilon},\alpha,\overset{n-j}{\varepsilon},\alpha^{-1}) \in A(f)$ for all $j \in N_n \setminus \{i\}$. The transitivity of G implies that $(\overset{i-1}{\varepsilon},\alpha,\overset{k-i-1}{\varepsilon},\alpha^{-1},\overset{n-k+1}{\varepsilon}) \in A(f)$ for some $k \in N_n \setminus \{i\}$. But $(\overset{k-1}{\varepsilon},\alpha,\overset{n-k}{\varepsilon},\alpha^{-1})(\overset{i-1}{\varepsilon},\alpha,\overset{k-i-1}{\varepsilon},\alpha^{-1},\overset{n-k+1}{\varepsilon}) = (\overset{i-1}{\varepsilon},\alpha,\overset{n-i}{\varepsilon},\alpha^{-1}) \in A(f)$, hence $\alpha \in \Lambda_j$, for all $j \in N_n$. Consequently, $\Lambda_i = \Lambda_j$ for all $i,j \in N_n$, and $\Lambda_i \subseteq V$.

Propositions 1 and 5 imply that $V \subseteq \Phi_i = \Phi^*_{ij}$ for all $i,j \in N_n$. If $\alpha \in \Phi_i$, then $T_j = (\overset{i-1}{\varepsilon},\alpha,\overset{j-i-1}{\varepsilon},\beta^{*-1}_j,\overset{n-j+1}{\varepsilon}) \in A(f)$ for all $j \in N_n \setminus \{i\}$, but by Corollary 1 $\beta^*_j = \alpha$, for all $j \in N_n \setminus \{i\}$. Also, since G is transitive there is $\sigma \in G$, $\sigma(n+1) = i$, such that $(T^\sigma_j)^{-1} = (\overset{j-1}{\varepsilon},\alpha,\overset{n-j}{\varepsilon},\alpha^{-1}) \in A(f)$, for all $j \in N_n$, i.e. $\alpha \in V$, which completes the proof.

Since in G-n-quasigroups, where G is transitive, $\Lambda_i = \Lambda_j$, $\Phi_i = \Phi^*_{ij} = \Phi_j$, for all $i,j \in N_n$, when dealing with such n-quasigroups we shall omit indexes and write Λ instead of Λ_i and Φ instead of Φ_i and Φ^*_{ij}.

Theorem 2. *Let* (Q,f) *be an idempotent* G-n-*quasigroup, where* G *is transitive. Then*

(i) *If* $\alpha \in \Phi$, *then* $\alpha^{n+1} = \varepsilon$.

(ii) *If* n *is even,* Λ *consists of the identity mapping only.*

(iii) Φ *is a normal subgroup of the automorphism group* Aut(f).

Proof. (i) If $\alpha \in \Phi$, then by Proposition 4 and Theorem 1 $(\overset{i-1}{\varepsilon},\alpha,\overset{j-i-1}{\varepsilon},\alpha^{-1},\overset{n-j+1}{\varepsilon}) \in A(f)$ for all $i,j \in N_n$.

Hence

$$\prod_{j=2}^{n+1} (\alpha, \overset{j-2}{\varepsilon}, \alpha^{-1}, \overset{n-j+1}{\varepsilon}) = (\alpha^n, \alpha^{-1}, \ldots, \alpha^{-1}) \in A(f),$$

that is, $f(\alpha^n x_1, \{\alpha^{-1}x_i\}_{i=2}^{n}) = \alpha^{-1} f(x_1^n)$. Putting in the preceding equality $x_1 = \ldots = x_n = x$, it follows $f(\alpha^n x, \alpha^{-1}x, \ldots, \alpha^{-1}x) = \alpha^{-1}x$, i.e. $f(\alpha^{n+1}x, \overset{n-1}{x}) = x$, which implies $\alpha^{n+1} = \varepsilon$.

(ii) Since $\Lambda \subseteq \Phi$, from $\alpha \in \Lambda$ it follows that $\alpha^{n+1} = \varepsilon$, and by Corollary 1 $\alpha^2 = \varepsilon$. Hence if n is even $\alpha = \varepsilon$.

(iii) First we prove that $\Phi \subseteq \mathrm{Aut}(f)$. If $\alpha \in \Phi$, we have proved that $(\alpha^n, \alpha^{-1}, \ldots, \alpha^{-1}) \in A(f)$ and $\alpha^n = \alpha^{-1}$, hence $\alpha \in \mathrm{Aut}(f)$. Also, if $\varphi \in \mathrm{Aut}(f)$, i.e. $T = (\overset{n+1}{\varphi}) \in A(f)$, and $\alpha \in \Phi$, then $T^{-1}(\overset{i-1}{\varepsilon}, \alpha, \overset{j-i-1}{\varepsilon}, \alpha^{-1}, \overset{n-j+1}{\varepsilon})\, T = (\overset{i-1}{\varepsilon}, \varphi^{-1}\alpha\varphi, \overset{j-i-1}{\varepsilon}, \varphi^{-1}\alpha^{-1}\varphi, \overset{n-j+1}{\varepsilon}) \in A(f)$ for all $i,j \in N_n$. Consequently, $\varphi^{-1}\Phi\varphi \subseteq \Phi$.

Proposition 6. *Let* (Q,f) *be a* G-n-*loop, where* G *is transitive. If* $\alpha \in \Phi$, *then* $\alpha^2 = \varepsilon$.

Proof. If $\alpha \in \Phi$, then $(\overset{i-1}{\varepsilon}, \alpha, \overset{j-i-1}{\varepsilon}, \alpha^{-1}, \overset{n-j+1}{\varepsilon}) \in A(f)$ for all $i,j \in N_n$. Hence for all $x_1^n \in Q$

$$f(x_1^{i-1}, \alpha x_i, x_{i+1}^n) = f(x_1^{j-1}, \alpha x_j, x_{j+1}^n).$$

Putting in the preceding equality $x_k = e$, $k \neq i$, where e is a unit of f, we get

$$\alpha x_i = f(\overset{i-1}{e}, x_i, \overset{j-i-1}{e}, \alpha e, \overset{n-j}{e}).$$

Since G is transitive, there is $\sigma \in G$ such that $\sigma(n+1) = i$. Applying σ to the last equality, we obtain

$$x_i = f(\overset{p-1}{e}, \alpha x_i, \overset{p-q-1}{e}, \alpha e, \overset{n-q}{e}),$$

where $\sigma p = n+1$, $\sigma q = j$. But $(\overset{p-1}{\varepsilon}, \alpha, \overset{p-q-1}{\varepsilon}, \alpha^{-1}, \overset{n-q+1}{\varepsilon}) \in A(f)$, hence

$$x_i = f(\overset{p-1}{e}, \alpha^2 x_i, \overset{n-p}{e}) = \alpha^2 x_i,$$

i.e. $\alpha^2 = \varepsilon$.

Corollary 2. *If* n *is even, the group* Φ *of all inner regular permutations of an idempotent* G-n-*loop, where* G *is transitive, consists of the identity mapping only.*

Theorem 3. *Let* (Q,f) *be a* G-n-*quasigroup, where* G *is transitive and let* $\alpha \in \Lambda$, $\alpha \neq \varepsilon$. *Then*

(i) α *is an automorphism of* (Q,f) *iff* n *is odd.*

(ii) *If* n *is even and* (Q,g) *is isotopic to* (Q,f), $f^T = g$, *where* $T = (\overset{n}{\varepsilon}, \alpha)$, *then* g *is isomorphic to* f.

Proof. Since $\alpha \in \Lambda$, by Proposition 4 and Theorem 1 $(\overset{i-1}{\varepsilon}, \alpha, \overset{j-i-1}{\varepsilon}, \alpha, \overset{n-j+1}{\varepsilon}) \in A(f)$ for all $i,j \in N_n$.

(i) Hence

$$\prod_{i=1}^{\left[\frac{n-1}{2}\right]} (\overset{2i}{\varepsilon}, \overset{2}{\alpha}, \overset{n-2i-1}{\varepsilon}) = \begin{cases} (\overset{n}{\alpha}, \varepsilon) \in A(f) & \text{if n is even,} \\ (\overset{n+1}{\alpha}) \in A(f), & \text{if n is odd.} \end{cases}$$

Since two autotopisms differing in only one component must be equal, it follows that if α is an automorphism of f and n is even, then $\alpha = \varepsilon$, which is a contradiction.

(ii) If n is even, we have proved that $S = (\overset{n}{\alpha}, \varepsilon) \in A(f)$. Therefore $g = (f^S)^T = f^{ST}$, where $ST = (\overset{n+1}{\alpha})$.

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REZIME

REGULARNE PERMUTACIJE PARASTROFNO INVARIJANTNIH n-KVAZIGRUPA

n-Kvazigrupa (Q,f) se naziva G-n-kvazigrupa ako i samo ako je $f = f^{\sigma}$ za svako $\sigma \in G$, gde je G podgrupa simetrične grupe stepena $n+1$ a f^{σ} je definisano sa

$$f^{\sigma}(x_{\sigma 1}, \ldots, x_{\sigma n}) = x_{\sigma(n+1)} \Leftrightarrow f(x_1, \ldots, x_n) = x_{n+1}.$$

U ovom radu su posmatrane regularne permutacije (definicije 1, 2 i 3) nekih klasa G-n-kvazigrupa i ispitane neke njihove osobine.

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