ZBORNIK RADOVA
Prirodno-matematičkog fakulteta
Univerziteta u Novom Sadu
Serija za matematiku, 15,2 (1985)

REVIEW OF RESEARCH
Faculty of Science
University of Novi Sad
Mathematics Series, 15, 2 (1985)

FIXED POINT THEOREMS IN RANDOM PARANORMED SPACES

Olga Hadžić

Institute of Mathematics, University of Novi Sad, Dr Ilije Djuričića 4, 21000 Novi Sad, Yugoslavia

ABSTRACT

In this paper we shall introduce the notion of a random paranormed space. The admissibility of a class of subsets in random paranormed spaces is proved and fixed point theorems are obtained.

1. INTRODUCTION

K. Menger introduced in [26] the notion of a probabilistic metric space. Some fixed point theorems in probabilistic metric spaces are proved in [4], [5], [10], [12], [28], [29], [30], [32].

The notion of a random normed space was introduced by Šerstnev in [31] and some fixed point theorems in such spaces are proved in [2], [8], [11].

Every random normed space is a probabilistic metric space and under some additional conditions it is also a topological vector space. There are some very important non-locally

AMS Mathematics Subject Classification (1980): 60H25.

Key words and phrases: Probabilistic metric spaces, random paranormed spaces, fixed point theorems.

convex topological vector spaces like the space S(0,1) (all the equivalence classes of real Lebesgues measurable functions defined on the interval (0,1)) in which the topology can be introduced by a paranorm. Hence, it will be of interest to introduce the notion of a random paranormed space and to obtain some fixed point theorems in such spaces. Some fixed point theorems in paranormed spaces are obtained in [14], [15], [17], [33].

2. PRELIMINARIES

First, we shall give some definitions. Let $R = (-\infty,\infty)$ $\mathcal{D}$ be the set of distribution functions ($F \in \mathcal{D}$ if $F : R \to [0,1]$ is left continuous, $\inf F = 0$, $\sup F = 1$, F is monotone nondecreasing) and

$$H(t) = \begin{cases} 1, & t > 0 \\ 0, & t \le 0. \end{cases}$$

Definition 1. [26] *A Menger space is a triple* $(E,\mathcal{F},t)$ *where* E *is a nonempty set,* t *is a* T*-norm and* $\mathcal{F} : E \times E \to \mathcal{D}$ *so that the following conditions are satisfied:*

1. $F_{x,y}(u) = H(u)$, *for every* $u > 0$ *if and only if* $x = y$.
2. $F_{x,y}(0) = 0$, *for every* $(x,y) \in E \times E$.
3. $F_{x,y} = F_{y,x}$, *for every* $(x,y) \in E \times E$.
4. $F_{x,y}(u_1 + u_2) \ge t(F_{x,z}(u_1),F_{z,y}(u_2))$ *for every* $x,y,z \in E$ *and every* $u_1,u_2 \ge 0$.

The (ε,λ)-topology is introduced by the family of neighbourhoods $\mathcal{V} = \{V_u(\varepsilon,\lambda) \mid (u,\varepsilon,\lambda) \in E \times R^+ \times (0,1)\}$, where $V_u(\varepsilon,\lambda) = \{v \mid F_{u,v}(\varepsilon) > 1-\lambda\}$.

This topology is metrizable if $\sup_{a<1} t(a,a) = 1$. A well known example of a Menger space $(E,\mathcal{F},t_m)$ ($t_m(a,b) = \max\{a+b-1,0\}$) is the following. Let (M,d) be a separable metric space and (Ω,A,P) a probability measure space. By E we shall denote the space of all the equivalence classes of measurable mappings from Ω into M. For every $X,Y \in E$ and $\varepsilon > 0$ let:

$$F_{X,Y}(\varepsilon) = P\{\omega | \omega \in \Omega, d(X(\omega),Y(\omega)) < \varepsilon\}.$$

It is known that the triple $(E,\mathcal{F},t_m)$ is a Menger space. The convergences in the (ε,λ) topology and in the probability are identical. A further example of a Menger space is the following [32]. Let $\mathcal{D}^+ = \{F | F \in \mathcal{D}, F(0) = 0\}$.

Let E be a real or complex vector space, t is a T--norm stronger then t_m ($t \geq t_m$) and the mapping $\mathcal{F} : E \to \mathcal{D}^+$ satisfies the following conditions:

1. $F_p = H \Leftrightarrow p = \theta$ (θ is the neutral element of E).

2. For every $p \in E$, every $u > 0$ and every $\delta \in K\setminus\{0\}$ (K is the scalar field):

$$F_{\delta p}(u) = F_p(u/|\delta|).$$

3. For every $p,q \in E$ and every $u,v > 0$:

$$F_{p-q}(u+v) \geq t(F_p(u),F_q(v)).$$

Then $(E,\mathcal{F},t)$ is a random normed space ($F_{x-y} = F_{x,y}$).

If t is continuous then E is, in the (ε,λ) topology, a topological vector space.

Every normed space $(E,\|\ \|)$ is a random normed space, where

$$F_x(\varepsilon) = \begin{cases} 1, & \|x\| < \varepsilon \\ 0, & \|x\| \geq \varepsilon. \end{cases}$$

Let E be a vector space and $p : E \to [0,\infty)$ so that the following conditions are satisfied:

(i) $p(x) = 0 \Leftrightarrow x = 0$.

(ii) $p(x) = p(-x)$, for every $x \in E$.

(iii) $p(x+y) \leq p(x) + p(y)$, for every $x,y \in E$.

(iv) If $\lambda_n \to \lambda$ (λ_n,λ are from the scalar field) and $p(x_n - x) \to 0$, ($x_n, x \in E$) then $p(\lambda_n x_n - \lambda x) \to 0$.

Then the pair (E,p) is a paranormed space which is also a topological vector space with the fundamental system of neighbourhoods of zero given by: $\mathcal{V} = \{V_\varepsilon\}_{\varepsilon>0}$, where:

$$V_\varepsilon = \{x \mid x \in E,\ p(x) < \varepsilon\}.$$

The space $S(0,1)$ is a paranormed space with the function p given by:

$$p(\hat{x}) = \int_0^1 \frac{|x(t)|}{1 + |x(t)|}\, dt \quad (\{x(t)\} \in \hat{x}).$$

Now we shall introduce the following definition.

Definition 2. *A random paranormed space is a triple* $(E,\mathcal{F},t)$ *where* E *is a real or complex vector space,* $\mathcal{F} : E \to \mathcal{D}^+$ *and* t *is a T-norm such that* $t \geq t_m$ *and the following conditions are satisfied:*

1. $F_p = H \Leftrightarrow p = 0$.
2. $F_{-x} = F_x$, *for every* $x \in E$.
3. $F_{x+y}(u_1 + u_2) \geq t(F_x(u_1), F_y(u_2))$ *for every* $x,y \in E$ *and every* $u_1,u_2 \geq 0$
4. *If* $\lambda_n \to \lambda$ *and* $F_{x_n - x}(\varepsilon) \to 1 (n \to \infty)$, *for every* $\varepsilon > 0$ *then* $F_{\lambda_n x_n - \lambda x}(\varepsilon) \to 1\ (n \to \infty)$, *for every* $\varepsilon > 0$.

Every paranormed space (E,p) is also a random paranormed space where:

$$F_x(\varepsilon) = \begin{cases} 1, & p(x) < \varepsilon \\ 0, & p(x) \geq \varepsilon. \end{cases}$$

The topology is introduced by the (ε,λ)-topology as in the Menger spaces. It is obvious that a random paranormed space $(E,\mathcal{F},t)$ is also a Menger space which is a topological vector space if t is continuous. Let (X,p) be a separable paranormed space and (Ω,A,P) a probability measure space. By S we shall denote all the equivalence classes of measurable mappings $x : \Omega \to X$. Let $\mathcal{F} : S \to \mathcal{D}^+$ be defined by:

$$F_x(\varepsilon) = P\{\omega | \omega \in \Omega, p(x(\omega)) < \varepsilon\}.$$

Then $(S,\mathcal{F},t_m)$ is a random paranormed space.

Remark. Let $\lambda_n \to \lambda$ and $x_n \to x$ in the (ε,λ)-topology. Then $x_n \to x$ in the probability. Hence there exists a subsequence $\{x_{n_k}\}$ which converges to x almost everywhere. Then $p(\lambda_{n_k}x_{n_k}(\omega) - \lambda x(\omega)) \to 0$, $k \to \infty$ for $\omega \in \Omega_0$, $P(\Omega_0) = 1$ which implies that $\lambda_{n_k}x_{n_k} \to \lambda x$ in the probability, i.e. in the (ε,λ)--topology. Hence every subsequence of the sequence $\{\lambda_n x_n\}$ has a convergent subsequence with the same limit λx. This implies that $\lambda_n x_n \to \lambda x$ in the (ε,λ)-topology.

Let (E,p) be a paranormed space and $K \subseteq E$. In [33] K. Zima introduced a very useful inequality for elements of K which enable us to prove many fixed point results in general topological vector spaces [13], [17], [18].

Definition 3. *Let (E,p) be a paranormed space and K a nonempty subset of E. The set K satisfies the Zima condition if there exists $C(K) > 0$ such that for every $0 \leq \lambda \leq 1$:*

$$p(\lambda(x-y)) \leq C(K)\lambda\, p(x-y), \quad \textit{for every } x,y \in K.$$

In [16] we gave the following example. Let $E = S(0,1)$ and for every $\hat{x} \in E$:

(1) $$p(\hat{x}) = \int_0^1 \frac{|x(t)|}{1 + |x(t)|}\, dt \ , \ \{x(t)\} \in \hat{x} \ .$$

If $s > 0$ let:

$$K_s = \{\hat{x} | \hat{x} \in S(0,1),\ |x(t)| \le s,\ t \in I\}.$$

In [16] it is proved that $C(K) = 1 + 2s$ since:

$$p(\lambda(\hat{x}-\hat{y})) \le (1 + 2s)\lambda p(\hat{x}-\hat{y})$$

for every $\hat{x},\hat{y} \in K_s$ and $0 \le \lambda \le 1$.

Now, we shall introduce the probabilistic Zima condition.

Definition 4. *Let* $(E,\mathcal{F},t)$ *be a random paranormed space and* K *a nonempty subset of* E. *The set* K *satisfies the probabilistic Zima condition if there exists* $C(K) > 0$ *so that:*

$$F_{\lambda(x-y)}(\lambda\varepsilon) \ge F_{x-y}(\varepsilon/C(K))$$

for every $\varepsilon > 0$ *and every* $x,y \in K$.

It is obvious that every subset K of a paranormed space E, which satisfies the Zima condition in the sense of Definition 3 satisfies the probabilistic Zima condition as well. Namely, if $F_{x-y}(\varepsilon/C(K)) = 1$ $(x,y \in K,\ \varepsilon > 0)$ then $p(x-y) < \varepsilon/C(K)$. This implies that for $\lambda \in [0,1]$:

$$p(\lambda(x-y)) \le C(K)\lambda p(x-y) < \lambda\varepsilon$$

which means that $F_{\lambda(x-y)}(\lambda\varepsilon) = 1$.

Let (Ω,A,P) be a probability measure space and X be the space of all the equivalence classes of measurable mappings $x : \Omega \to S(0,1)$. Further, let $s > 0$ and

$$\tilde{K}_s = \{\hat{x} | \hat{x} \in X,\ \hat{x}(\omega) \in K_s, \text{ for every } \omega \in \Omega\}.$$

Then $\tilde{K}_s$ satisfies the probabilistic Zima condition with F defined by:

$$F_{\hat{x}}(\varepsilon) = P\{\omega | p(\hat{x}(\omega)) < \varepsilon\} \quad (\varepsilon > 0,\ \hat{x} \in X),$$

and p is defined by (1).

Then for every $\omega \in \Omega$:

$$p(\lambda(\hat{x}(\omega) - \hat{y}(\omega))) \leq (1+2s)\lambda p(\hat{x}(\omega) - \hat{y}(\omega))$$

If $\omega \in \Omega$ is such that $p(\hat{x}(\omega) - \hat{y}(\omega)) < \varepsilon/(1+2s)$ then $p(\lambda(\hat{x}(\omega) - \hat{y}(\omega))) < \lambda\varepsilon$ and so:

$$P\{\omega | p(\hat{x}(\omega) - \hat{y}(\omega)) < \varepsilon/(1+2s)\} \leq$$

$$\leq P\{\omega | p(\lambda(\hat{x}(\omega) - \hat{y}(\omega))) < \varepsilon\lambda\}$$

which means that:

$$F_{\lambda(\hat{x}-\hat{y})}(\varepsilon\lambda) \geq F_{\hat{x}-\hat{y}}(\varepsilon/(1+2s)).$$

It is known [16] that a subset K of a paranormed space which satisfies the Zima condition is an admissible subset in the sense of V. Klee [25] (Definition 5 given below). The notion of an admissible subset is very important in the fixed point theory in topological vector spaces [22].

Definition 5. *Let* E *be a Hausdorff topological vector space and* M *a nonempty subset of* E. *The set* M *is admissible if and only if for every compact subset* K *of* M *and every neighbourhood of zero* V *in* E *there exists a continuous mapping* h : K → M *such that:*

(a) $\dim \mathit{lin}(h(K)) < \infty$ *(*$\mathit{lin}(h(K))$ *is the linear hull of* h(K))

(b) $x - hx \in V$, *for every* $x \in K$.

Every nonempty, convex subset of a locally convex space is an admissible set [27]. In [24] it is proved that every compact mapping f defined on an admissible subset M of a Hausdorff topological vector space so that $f(M) \subseteq M$ has a fixed point.

2. A FIXED POINT THEOREM IN RANDOM PARANORMED SPACES

In this section we shall use the following notation where t is a T-norm:

$$t_n(x) = \underbrace{t(t(\ldots t(t(x,x),\ldots,x)}_{n\text{-times}},\ n \in N,\ x \in [0,1].$$

First, we shall prove the following Lemma.

Lemma. *Let* $(E,\mathcal{F},t)$ *be a random paranormed space with continuous T-norm* t *and* K *a nonempty convex subset of* E *which satisfies the probabilistic Zima condition. If the family* $\{t_n(x)\}_{n \in N}$ *is equicontinuous at the point* $x = 1$ *then* K *is admissible.*

Proof. Let A be a compact subset of K, $\varepsilon > 0$ and $\lambda \in (0,1)$. We have to prove that there exists a continuous mapping $h_{\varepsilon,\lambda} : A \to K$ such that for every $x \in A$:

(2) $$F_{x-h_{\varepsilon,\lambda}(x)}(\varepsilon) > 1 - \lambda,\ \dim \operatorname{lin}(h_{\varepsilon,\lambda}(A)) < \infty.$$

Let $\delta(\lambda) \in (0,1)$ be such that:

$$u > 1 - \delta(\lambda) \Rightarrow t_n(u) > 1 - \lambda,$$

for every $n \in N$.

Since the set A is compact there exists a finite set $\{u_1,u_2,\ldots,u_m\} \subseteq A$ such that:

$$A \subseteq \bigcup_{r=1}^{m} V_{u_r}\left(\frac{\varepsilon}{C(K)},\delta(\lambda)\right).$$

Further, let $\eta_r : A \to R^+$ $(r \in \{1,2,\dots,m\})$ be such a family of functions that:

$$\eta_r(x) \neq 0 \Rightarrow F_{x-u_r}(\varepsilon/C(K)) > 1 - \delta(\lambda)$$

and

$$\sum_{r=1}^{m} \eta_r(x) = 1, \ x \in A.$$

Since E is metrizable such a family exists. Then $h_{\varepsilon,\lambda} : A \to K$ is defined in the following way:

$$h_{\varepsilon,\lambda}(x) = \sum_{i=1}^{m} \eta_i(x)u_i, \ x \in A.$$

Since K is convex and $h_{\varepsilon,\lambda}(A) \subseteq co\{u_1, u_2,\dots,u_m\} \subseteq \mathit{Lin}\,\{u_1, u_2,\dots,u_m\}$ it follows that $\dim(\mathit{Lin}(h_{\varepsilon,\lambda}(A)) < \infty$. Suppose that $x \in A$ and that $\eta_{i_r}(x) \neq 0$ for $r \in \{1,2,\dots,s\}$ and $\eta_i(x) = 0$ for $i \in \{1,2,\dots,m\} \setminus \{i_1,i_2,\dots,i_s\}$. Then we have that:

$$F_{x-h_{\varepsilon,\lambda}(x)}(\varepsilon) = F_{\sum_{r=1}^{s} \eta_{i_r}(x)\cdot x - \sum_{r=1}^{s} \eta_{i_r}(x)u_{i_r}} \cdot\left(\sum_{r=1}^{s} \eta_{i_r}(x)\varepsilon\right)$$

$$\geq \underbrace{t(t \dots t(}_{s\text{-}times} F_{\eta_{i_1}(x)x - \eta_{i_1}(x)u_{i_1}}(\eta_{i_1}(x)\varepsilon),$$

$$F_{\eta_{i_2}(x)x - \eta_{i_2}(x)u_{i_2}}\cdot(\eta_{i_2}(x)\varepsilon), \dots, F_{\eta_{i_s}(x)x - \eta_{i_s}(x)u_{i_s}}$$

$$(\eta_{i_s}(x)\varepsilon)) \geq t_s(\min_{1\le r\le s}\{F_{x-u_{i_r}}(\varepsilon/C(K)\}) > 1 - \lambda$$

since

$$F_{x-u_{i_r}}(\varepsilon/C(K)) > 1 - \delta(\lambda), \text{ for } r \in \{1,2,\dots,s\}.$$

Hence (2) is satisfied.

V. Radu introduced in [28] the following definition.

Definition 6. *A T-norm t is h - T norm if the family $\{t_n(x)\}_{n\in N}$ is equicontinuous at the point $x = 1$.*

It is proved in [28] that a T-norm t is h - T norm if and only if for every $a \in (0,1)$ there exists $b \leq a$ such that $t(b,b) = b < 1$. A nontrivial example of a h - T norm is given in [9] . It is obvious that t = *min* is a h - T norm. In his Ph.D. Thesis V.M. Sehgal introduced the notion of a contraction mapping on a probabilistic metric space.

If $(S,\mathcal{F})$ is a probabilistic metric space and $f : S \to S$ then f is a probabilistic q-contraction on S if [30]:

$$F_{f(x_1),f(x_2)}(\varepsilon) \geq F_{x_1,x_2}(\varepsilon/q), \text{ for every } \varepsilon > 0,$$

for all $x_1, x_2 \in S$, where $q \in (0,1)$.

It is known [29] that in a Menger space $(S,\mathcal{F},t)$ a necessary and sufficient condition that every probabilistic q-contraction has the fixed point is that T-norm t is h-T norm.

Using the Lemma we shall prove the following fixed point theorem.

Theorem 1. *Let $(E,\mathcal{F},t)$ be a complete random paranormed space with continuous T-norm t, M a closed and convex subset of E which satisfies the probabilistic Zima condition, $P : M \to E$ a probabilistic q-contraction, $S : M \to E$ a compact mapping such that $Px + Sy \in M$ for every $x,y \in M$. If T-norm t is h-T-norm then there exists $x \in M$ so that $Px + Sx = x$.*

Proof. Since for every $y \in \overline{SM}$, the mapping $x \mapsto Px + y$ $(x \in M)$ is a probabilistic q-contraction and T-norm t is h-T norm it follows that there exists $Ry \in M$ so that $Ry = PRy + y$. We shall prove that the mapping $y \mapsto Ry \; (y \in \overline{SM})$ is continuous. Denote by $\mathcal{S}(\overline{S(M)},M)$ the set of all continuous mappings from $\overline{S(M)}$ into M and by $\mathcal{S}(\overline{S(M)},E)$ the space of all continuous mappings from $\overline{S(M)}$ into E. Let $\tilde{x} \in \mathcal{S}(S(M),E)$ and $\varepsilon > 0$. Then by the definition:

$$\tilde{F}_{\tilde{x}}(\varepsilon) = \sup_{\delta<\varepsilon} \inf_{y\in\overline{S(M)}} F_{\tilde{x}(y)}(\varepsilon).$$

Then the triple $(\mathcal{S}(S(M),E)\mathcal{F},t)$ is a complete Menger space. $(\mathcal{F}(x,y) = \mathcal{F}_{x-y})$. Let $H : \mathcal{S}(\overline{S(M)},M) \to \mathcal{S}(\overline{S(M)},M)$ be defined by:

$$(H\tilde{x})(y) = P\tilde{x}(y) + y, \quad y \in \overline{S(M)}, \ \tilde{x} \in \mathcal{S}(\overline{S(M)},M).$$

Then for every $\varepsilon > 0$ and $x_1,x_2 \in \mathcal{S}(\overline{S(M)},M)$:

$$\tilde{F}_{H\tilde{x}_1-H\tilde{x}_2}(\varepsilon) = \sup_{\delta<\varepsilon} \inf_{y\in\overline{S(M)}} F_{(H\tilde{x}_1)(y)-(H\tilde{x}_2)(y)}(\delta) \geq$$

$$\geq \sup_{\delta<\varepsilon} \inf_{y\in\overline{S(M)}} F_{P\tilde{x}_1(y)-P\tilde{x}_2(y)}(\delta) \geq$$

$$\geq \sup_{\delta<\varepsilon} \inf_{y\in\overline{S(M)}} F_{\tilde{x}_1(y)-\tilde{x}_2(y)}(\delta/q) =$$

$$= \tilde{F}_{\tilde{x}_1-\tilde{x}_2}(\varepsilon/q).$$

Hence, there exists one and only one element $\tilde{x} \in \mathcal{S}(\overline{S(M)},M)$ such that $H\tilde{x} = \tilde{x}$ and so:

$$(H\tilde{x})(y) = \tilde{x}(y), \quad \text{for every} \quad y \in \overline{S(M)}.$$

This means that $\tilde{x}(y) = Ry$, $y \in \overline{S(M)}$ and since $\tilde{x}$ is continuous we obtain that R is continuous. Then the mapping $RS : M \to M$ satisfies all the conditions of Hahn's and Pötter's fixed point theorem. This implies that there exists $z \in M$ such that $RSz = z$ which means that $z = Pz + Sz$.

For the next fixed point theorem in a random paranormed space $(S,\mathcal{F},min)$ we shall need some notions introduced in [32]. First, let us remark that in a random paranormed space $(S,\mathcal{F},min)$ the (ε,λ)-topology can be introduced by the family of functions $\{p_\lambda\}_{\lambda\in(0,1)}$ with the following properties:

1. $p_\lambda(x) = 0, \ \forall\lambda \in (0,1) \Leftrightarrow x = 0.$
2. $p_\lambda(-x) = p_\lambda(x)$, for every $x \in S$.

3. $p_\lambda(x+y) \le p_\lambda(x) + p_\lambda(y)$, for every $x,y \in S$.

4. If $\delta_n \to \delta(\delta_n,\delta \in \mathbb{R})$ and $x_n \to x(x_n,x \in S)$ in the (ε,λ)-topology then for every $\lambda \in (0,1)$: $p_\lambda(\delta_n x_n - \delta x) \to 0$.

As in the case of a random normed spaces we have that $p_\lambda(x) = \sup\{u|F_x(u) \le 1-\lambda\}$, $x \in S$, $\lambda \in (0,1)$. We shall prove only property 4. Suppose that $\delta_n \to \delta$ and $x_n \to x$, in the (ε,λ)-topology. Then from the definition of a random paranormed space it follows that for every $u > 0$ and every $\lambda \in (0,1)$ there exists $n_0(u,\lambda) \in \mathbb{N}$ so that:

$$F_{\delta_n x_n - \delta x}(u) > 1 - \lambda \ , \text{ for every } n \ge n_0(u,\lambda)$$

which means that $p_\lambda(\delta_n x_n - \delta x) < u$. Hence $p_\lambda(\cdot)$ has property 4. For every two probabilistic bounded subsets A and B let

$$h_{AB}(u) = \sup_{s<u} \inf_{x\in A} \sup_{y\in B} F_{x-y}(s) \quad [32] \quad (u \in \mathbb{R}).$$

The probabilistic inner measure of noncompactness of A, $b_A(\cdot)$ is defined by [32]:

$$b_A(u) = \sup\{\rho|\rho > 0, \text{ there is a finite set } A_f \subseteq A \text{ such that } h_{AA_f}(u) \ge \rho\}.$$

The function $b_A(\cdot)$ is strict if $u < v \Rightarrow b_A(u) < b_A(v)$. $(u,v \in [0,\infty))$.

Theorem 2. *Let* $(S,\mathcal{F},\min)$ *be a complete random paranormed space,* G *a probabilistic bounded, closed and convex subset of* S, $T : G \to R(G)$ *(the family of all nonempty, closed and convex subsets of* G*) an upper semicontinuous mapping,* b_A *be strict for every* $A \subseteq G$ *and there exists* $q \in (0,1)$ *such that for every* $u > 0$ *and every* $A \subseteq G$:

$$b_{T(A)}(u) \ge b_A(u/q).$$

If G *satisfies the probabilistic Zima conditions and* $qC(G) < 1$ *there exists* $x \in G$ *so that* $x \in Tx$.

Proof. Let for every $\lambda \in (0,1)$, $\varepsilon > 0$ and $x \in S$:

$$B_\lambda(x,\varepsilon) = \{y \mid y \in S,\ p_\lambda(x-y) < \varepsilon\}.$$

For every $A \subseteq S$ which is probabilistic bounded and every $\lambda \in (0,1)$ the Hausdorff measure of noncompactness $\chi(A)$ is defined by:

$$\chi_\lambda(A) = \inf\{\varepsilon \mid \varepsilon > 0, \text{ there exists a finite set } \{x_1,x_2,\ldots,x_n\} \subseteq A \text{ so that } A \subseteq \bigcup_{i=1}^{n} B_\lambda(x_i,\varepsilon)\}.$$

As in [19] it can be shown that:

a) $\chi_\lambda(A) = 0,\ \forall \lambda \in (0,1) \Leftrightarrow \bar{A}$ is compact.

b) $\chi_\lambda(\overline{co}A) \le C(G)\chi_\lambda(A)$ for every $\lambda \in (0,1)$ and every $A \subseteq G$.

Since b_A is strict it follows that $\chi_\lambda(A) = \beta_\lambda(A)$ [32] ($A \subseteq G$) where:

$$\beta_\lambda(A) = \sup\{u \mid b_A(u) \le 1-\lambda\} \quad (\lambda \in (0,1)).$$

Furthermore,

$$\{u \mid b_{T(A)}(u) \le 1-\lambda\} \subset \{u \mid b_A(u/q) \le 1-\lambda\} = q\{u \mid b_A(u) \le 1-\lambda\}$$

and so:

$$\chi_\lambda(T(A)) = \beta_\lambda(T(A)) = \sup\{u \mid b_{T(A)}(u) \le 1-\lambda\} \le q\sup\{u \mid b_A(u) \le 1-\lambda\} = q\beta_\lambda(A) = q\chi_\lambda(A).$$

From this it is easy to prove that there exists a nonempty, convex and compact subset K of G such that $T(K) \subseteq K$. Using the probabilistic Zima condition for the set G we obtain that for every $x,y \in G$, every $\delta \in [0,1]$, and every $\lambda \in (0,1)$:

$$(3) \qquad p_\lambda(\delta(x-y)) \le \delta C(G)\, p_\lambda(x-y).$$

From (3) it follows that K is σ-admissible [16], [22]. Then [22] there exists $x \in K$ such that $x \in Tx$.

REFERENCES

[1] Gh. Bocşan, *On the Kuratowski function in random normed spaces*, Sem. Teor. Funct. Mat. Apl., A. Spaţii metrice probabiliste, Timişoara, 8 (1974).

[2] Gh. Bocşan, *On some fixed theorems in random normed spaces*, Sem. Teor. Funct. Mat. Apl., A. Spaţii metrice probabiliste, Timişoara, 13 (1974).

[3] Gh. Bocşan, *A computational formula of Kuratowski's function and applications to random normed spaces*, Sem. Teor. Funct. Mat. Apl., A. Spaţii metrice probabiliste, Timişoara, 31 (1975).

[4] G.L. Cain and R.H. Kasriel, *Fixed and periodic points of local contraction mappings on probabilistic metric spaces*, Math. Syst. Theory, 9 (1976), 289 - 297.

[5] Shih-sen Chang, *On some fixed point theorems in probabilistic metric space and its applications*, Z. Warscheinlichkeitstheorie Verw. Geb., 63 (1983), 463 - 474.

[6] Gh. Constantin and Gh. Bocşan, *The Kuratowski function and some applications to the probabilistic metric spaces*, Sem. Teor. Funct. Math. Apl., A. Spaţii metrice probabiliste, Timişoara, 1 (1973).

[7] Gh. Constantin and Gh. Bocşan, *On some measure of noncompactness in probabilistic metric spaces*, Sem. Teor. Funct. Math., Apl., A. Spaţii metrice probabiliste, Timişoara, 11 (1974).

[8] O. Hadžić, *A fixed point theorem in random normed spaces*, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak. Ser. Mat., 7 (1977), 23 - 27.

[9] O. Hadžić, *On the (ε,λ)-topology of probabilistic locally convex spaces*, Glas. Mat., Vol. 13 (33), (1978), 293 - 297.

[10] O. Hadžić, *Fixed point theorems for multivalued mappings in probabilistic metric spaces*, Mat. Vesnik, 3 (16)(31) (1979), 125 - 133.

[11] O. Hadžić, *Fixed points theorems in probabilistic metric and random normed spaces*, Math. Sem. Notes, Kobe Univ., Vol. 7 (1979), 261 - 270.

[12] O. Hadžić, *A generalization of the contraction principle in probabilistic metric spaces*, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak.,Ser. Mat., 10(1980), 13 - 21.

[13] O. Hadžić, *Some fixed point and almost fixed point theorems for multivalued mappings in topological vector spaces*, Nonlinear Anal. Theory, Methods, Appl. Vol. 5, No. 9 (1981), 1009 - 1019.

[14] O. Hadžić, *A generalization of Kakutani's fixed point theorem in paranormed spaces*, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak., Ser. Mat., 11(1981), 19 - 28.

[15] O. Hadžić, *On multivalued mappings in paranormed spaces*, Comm. Math. Univ. Carol., 22, 1 (1981), 129 - 136.

[16] O. Hadžić, *On equilibrium point in topological vector spaces*, Comm. Math. Univ. Carol., 23 (1982), 727 - 738.

[17] O. Hadžić, *A fixed point theorem for the sum of two mappings*, Proc. Amer. Math. Soc., 85 (1982), 37 - 41.

[18] O. Hadžić, *Some applications of a fixed point theorem for multivalued mappings in topological vector spaces*, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak. Ser. Mat., 13 (1983), 15 - 29.

[19] O. Hadžić, *Fixed point theory in topological vector spaces*, University of Novi Sad, Institute of Mathematics, 1984, 377 pp.

[20] O. Hadžić, *Fixed points theorems for multivalued mappings in not necessarily locally convex topological vector spaces*, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak., Ser. Mat., 14, 2 (1984), 27 - 40.

[21] O. Hadžić, D. Nikolić-Despotović, *A proof of the admissibility of a class of random normed spaces*, Mat. vesnik, 3 (16) (31) (1979), 267 - 271.

[22] S. Hahn, *A remark on a fixed point theorem for condensing set-valued mappings*, TU Dresden, Informationen, 07-5-77.

[23] S. Hahn, *A fixed point theorem for multivalued condensing mappings in general topological vector spaces*, Univ. u Novom Sadu, Zb. Rad. Prirod.-Mat. Fak., Ser. Mat., 15, 1 (1985), 97 - 106.

[24] S. Hahn, F. Pötter, *Über Fixpunkte kompakter Abbildungen in topologischen Vektorräumen*, Studia Math., L (1974), 1 - 16.

[25] V. Klee, *Leray-Schauder theory without local convexity*, Math. Ann., 141 (1960), 286 - 296.

[26] K. Menger, *Statistical metric*, Proc. Nat. Acad. USA, 28 (1942), 535 - 537.

[27] M. Nagumo, *Degree of mapping in convex linear topological spaces*, Am. J. Math., 73 (1981), 497 - 511.

[28] V. Radu, *On the t-norms of Hadžić's type and fixed points in probabilistic metric spaces*, Seminarul de Teoria Probabilităţilor şi Aplicaţii, 66 (1983).

[29] V. Radu, *On the contraction principle in Menger spaces*, Seminarul de Teoria Probabilităţilor şi Aplicaţii, 68 (1983).

[30] V.M. Sehgal and A.T. Bharucha-Reid, *Fixed points of contraction mappings on probabilistic metric spaces* Math. Syst. Theory, 6 (1972), 97 - 102.

[31] A.N. Sherstnev, *The notion of a random normed space*, Dokl. Acad. Nauk.SSSR, 149 (1963), 280 - 283.

[32] Do Hong Tan, *On the probabilistic inner measure of noncompactness*, Univ. u Novom Sadu, Zb. Rad. Prirod. -Mat. Fak. Ser. Mat., 13 (1983), 73 - 79.

[33] K. Zima, *On the Shauder fixed point theorem with respect to paranormed space*, Comm. Math., 19 (1977) 421 - 423.

REZIME

TEOREME O NEPOKRETNOJ TAČKI U SLUČAJNIM PARANORMIRANIM PROSTORIMA

U ovom radu uveden je pojam slučajnog paranormiranog prostora. Dokazana je dopustivost jedne klase slučajnih paranormiranih prostora i dobijene su teoreme o nepokretnoj tački.

Received by the editors March 12, 1986.