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ASSOCIATE TRIPLES OF HIGHER DEGREE

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ABSTRACT

Associate triples of higher degree are defined and studied. In particular, we establish the relationships between this property and the existence of functors in categories of algebras and Kleisli categories (of triples of higher degree) which commute with the functors of Eilenberg-Moore and Kleisli respectively.

INTRODUCTION

In [4] Manes proved that for triples $\mathbb{T}$ and $\mathbb{T}'$ in a category K, there exists a bijective correspondence between the theory maps λ from $\mathbb{T}'$ to $\mathbb{T}$ and liftings $S(\lambda): K^{\mathbb{T}} \longrightarrow K^{\mathbb{T}'}$ such that $U^{\mathbb{T}'}S(\lambda) = U^{\mathbb{T}}$ being $U^{\mathbb{T}}: K^{\mathbb{T}} \longrightarrow K$ the forgetful functor. In the similar way, there exists a bijective correspondence between inverse-state transformations [1] $\lambda: \mathbb{T}' \longrightarrow \mathbb{T}$ and liftings $S(\lambda): K_{\mathbb{T}'} \longrightarrow K_{\mathbb{T}}$ such that $F_{\mathbb{T}} = S(\lambda) F_{\mathbb{T}'}$.

The equivalence between the existence of a functor, which commute with the two Eilenberg-Moore functors or the existence of a functor which commute with the two Kleisli functors and the condition for being associate triples, was proved in [2] .

The aim of the present paper is to extend these results to triples of higher degree.

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DEFINITION 1. [3] (see also [5]) *A triple of higher degree in a category* K, $\bar{\mathbb{T}} = (T, M, \eta, \mu)$, *is defined by two endofunctors* T *and* M *such that* $TM = MT$ *together with two natural transformations* $\eta: id_K \longrightarrow TM$ *and* $\mu: TMT \longrightarrow T$ *satisfying:*

$$\text{THD 1} \quad \mu\cdot T\eta = \mu\cdot\eta T = id_T$$

$$\text{THD 2} \quad \mu\cdot TM\mu = \mu\cdot\mu MT$$

In the first part we shall prove the existence of a bijective correspondence between morphisms (dual morphisms) of triples of higher degree and some functors defined in the Kleisli category (category of algebras).

The second part deals with the associate triples of higher degree and we apply the results of the first part to give some properties.

FUNCTORS DEFINED BY MORPHISMS OF TRIPLES

DEFINITION 2. [3] *A* $\bar{\mathbb{T}}$*-algebra is a pair* (A,ξ) *where* A *is an object of* K *and* $\xi: TMA \longrightarrow A$ *is a* K*-morphism, called the structure map of* (A,ξ), *subject to the following conditions:*

$$\text{AG 1} \quad \xi\cdot\eta_A = id_A$$

$$\text{AG 2} \quad \xi\cdot\mu_{MA} = \xi\cdot TM\xi$$

If (A,ξ) and (B,θ) are $\bar{\mathbb{T}}$-algebras, a $\bar{\mathbb{T}}$-morphism from (A,ξ) to (B,θ) is a K-morphism $f:A \longrightarrow B$ such that $\theta\cdot TMf = f\cdot\xi$. The category of $\bar{\mathbb{T}}$-algebras and $\bar{\mathbb{T}}$-morphisms is denoted $K^{\bar{\mathbb{T}}}$. There exists a pair of adjoint functors $U^{\bar{\mathbb{T}}}: K^{\bar{\mathbb{T}}} \longrightarrow K$ and $F^{\bar{\mathbb{T}}}: K \longrightarrow K^{\bar{\mathbb{T}}}$ defined by $U^{\bar{\mathbb{T}}}(f:(A,\xi) \longrightarrow (B,\theta)) = f:A \longrightarrow B$ and $F^{\bar{\mathbb{T}}}(f:A \longrightarrow B) = Tf:(TA,\mu_A) \longrightarrow (TB,\mu_B)$.

Let K and L be categories and let $\bar{\mathbb{T}} = (T, M, \eta,\mu)$ and $\bar{\mathbb{T}}' = (T', M', \eta',\mu')$ be triples of higher degree in K and L respectively.

DEFINITION 3. [3] *A dual morphism from* $\bar{\mathbb{T}}$ *to* $\bar{\mathbb{T}}'$ *is a pair* (X,λ), *where* $X:K \longrightarrow L$ *is a functor and* $\lambda : T'M'X \longrightarrow XMT$ *is a natural transformation satisfying the following three axioms:*

$$\text{DM 1} \quad \lambda\cdot\eta' X = X\eta$$

$$\text{DM 2} \quad \lambda\cdot\mu' M' X = X\mu M\cdot\lambda TM\cdot T'M'\lambda$$

$$\text{DM 3} \quad X\mu M\cdot\lambda TM\cdot T'M'X\eta = \lambda$$

PROPOSITION 1. *Let* $\bar{\mathbb{T}} = (T, M, \eta, \mu)$ *and* $\bar{\mathbb{T}}' = (T', M', \eta', \mu')$ *be triples of higher degree in* K *and* L *respectively and let* $X: K \longrightarrow L$ *be a functor. Then, each natural transformation* $\lambda: T'M'X \longrightarrow XMT$ *satisfying the axioms of the definition 3, induces a functor* $S(\lambda): K^{\bar{\mathbb{T}}} \longrightarrow L^{\bar{\mathbb{T}}'}$ *over* X. *Conversely, each functor* $V: K^{\bar{\mathbb{T}}} \longrightarrow L^{\bar{\mathbb{T}}'}$ *over* X *induces a natural transformation* $\lambda: T'M'X \longrightarrow XMT$ *such that* (X, λ) *is a dual morphism from* $\bar{\mathbb{T}}$ *to* $\bar{\mathbb{T}}'$ *and* $S(\lambda) = V$.

P r o o f. If $f:(A,\xi) \longrightarrow (B,\theta)$ is a $\bar{\mathbb{T}}$-morphism, we define $S(\lambda)(f) = Xf: (XA, X\xi\cdot\lambda_A) \longrightarrow (XB, X\theta\cdot\lambda_B)$.

λ to $S(\lambda)$ is well defined:

$$X\theta\cdot\lambda_B\cdot T'M'Xf = Xf\cdot X\xi\cdot\lambda_A = X\theta\cdot XTMf\cdot\lambda_A$$

$$X\xi\cdot\lambda_A\cdot\eta'_{XA} = X\xi\cdot X\eta_A = Id_{XA}$$

$$X\xi\cdot\lambda_A\cdot\mu'_{M'XA} = X\xi\cdot X\mu_{MA}\cdot\lambda_{TMA}\cdot T'M'\lambda_A =$$

$$= X\xi\cdot\lambda_A\cdot T'M'X\xi\cdot T'M'\lambda_A,$$

furthermore $U^{\bar{\mathbb{T}}'}\cdot S(\lambda) = X\cdot U^{\bar{\mathbb{T}}}$.

Let $V: K^{\bar{\mathbb{T}}} \longrightarrow L^{\bar{\mathbb{T}}'}$ be a functor over X and let $V(TMA, \mu_{MA}) = (XTMA, \xi_A)$. We define $\lambda_A = \xi_A\cdot T'M'X\eta_A$.

V to λ is well defined: if $f: A \longrightarrow B$ is a K-morphism,

$$XMTf\cdot\lambda_A = XMTf\cdot\xi_A\cdot T'M'X\eta_A = \xi_B\cdot T'M'XTMf\cdot T'M'X\eta_A =$$

$$= \lambda_B\cdot T'M'Xf$$

$$\lambda_A\cdot\eta'_{XA} = X\eta_A$$

$$\lambda_A\cdot\mu'_{M'XA} = \xi_A\cdot T'M'X\eta_A\cdot\mu'_{M'XA} = \xi_A\cdot\mu'_{M'XTMA}\cdot T'M'T'M'X\eta_A =$$

$$= \xi_A\cdot T'M'\xi_A\cdot T'M'T'M'X\eta_A.$$

But $\xi_A = \xi_A\cdot T'M'X\mu_{MA}\cdot T'M'X\eta_{TMA} = X\mu_{MA}\cdot\xi_{TMA}\cdot T'M'X\eta_{TMA}$ because $X\mu_{MA}$ is a $\bar{\mathbb{T}}'$-morphism. Hence

$$X\mu_{MA}\cdot\lambda_{TMA}\cdot T'M'\lambda_A = X\mu_{MA}\cdot\xi_{TMA}\cdot T'M'X\eta_{TMA}\cdot T'M'\xi_A\cdot T'M'T'M'X\eta_A =$$

$$= \xi_A\cdot T'M'\xi_A\cdot T'M'T'M'X\eta_A$$

Moreover,

$$X\mu_{MA}\cdot\lambda_{TMA}\cdot T'M'X\eta_A = X\mu_{MA}\cdot\xi_{TMA}\cdot T'M'X\eta_{TMA}\cdot T'M'X\eta_A =$$

$$= \xi_A\cdot T'M'X\mu_{MA}\cdot T'M'X\eta_{TMA}\cdot T'M'X\eta_A = \xi_A\cdot T'M'X\eta_A = \lambda_A.$$

λ to $S(\lambda)$ to $\bar{\lambda}$, $\bar{\lambda} = \lambda$. The result follows from $\bar{\lambda}_A = X\mu_{MA}\cdot\lambda_{TMA}\cdot T'M'X\eta_A = \lambda_A$ since $S(\lambda)(TMA, \mu_{MA}) = (XTMA, X\mu_{MA}\cdot\lambda_{TMA})$.

V to λ to $S(\lambda)$, $S(\lambda) = V$. Let (A,α) be a $\bar{\mathbb{T}}$-algebra and let $V(A,\alpha) = (XA, \alpha')$, $V(TMA, \mu_{MA}) = (XTMA, \xi_A)$. As $\alpha:(TMA, \mu_{MA}) \longrightarrow (A,\alpha)$ is a $\bar{\mathbb{T}}$-morphism, $V(\alpha) = X(\alpha):(XTMA,\xi_A) \longrightarrow (XA,\alpha')$ is a $\bar{\mathbb{T}}'$-morphism and $X\alpha\cdot\xi_A\cdot T'M'X\eta_A = \alpha'\cdot T'M'X\alpha\cdot T'M'X\eta_A = \alpha'$. But $S(\lambda)(A,\alpha) = (XA,X\alpha\cdot\lambda_A) = (XA,X\alpha\cdot\xi_A\cdot T'M'X\eta_A)$ and therefore $S(\lambda) = V$.

If $\bar{\mathbb{T}}$ is a triple of higher degree in K, we define a category $K_{\bar{\mathbb{T}}}$, called the Kleisli Category of $\bar{\mathbb{T}}$. The category $K_{\bar{\mathbb{T}}}$ have the same objects as K; for arrows $K_{\bar{\mathbb{T}}}(A,B) \subset K(A,MTB)$ and $f:A \longrightarrow MTB \in K_{\bar{\mathbb{T}}}(A,B)$ if and only if $f = M\mu_B\cdot\eta_{MTB}\cdot f$. If $f \in K_{\bar{\mathbb{T}}}(A,B)$ and $g \in K_{\bar{\mathbb{T}}}(B,C)$, then $g\circ f \in K_{\bar{\mathbb{T}}}(A,C)$ is defined by the composite $M\mu_C\cdot MTg\cdot f$. The functors $U_{\bar{\mathbb{T}}} : K_{\bar{\mathbb{T}}} \longrightarrow K$ and $F_{\bar{\mathbb{T}}}:K \longrightarrow K_{\bar{\mathbb{T}}}$ given by $U_{\bar{\mathbb{T}}}(\alpha:X \longrightarrow MTY) = \mu_Y\cdot T\alpha:TX \longrightarrow TY$ and $F_{\bar{\mathbb{T}}}(a:A \longrightarrow B) = \eta_B\cdot a:A \longrightarrow MTB$ form a pair of adjoint functors.

DEFINITION 4. [3] *A morphism from* $\bar{\mathbb{T}}$ *to* $\bar{\mathbb{T}}'$ *is a pair* (X,λ), *where* $X:K \longrightarrow L$ *is a functor and* $\lambda:XMT \longrightarrow M'T'X$ *is a natural transformation which satisfies M1, M2 and M3 below:*

$$M\,1 \quad \lambda\cdot X\acute{\eta} = \eta' X$$

$$M\,2 \quad \lambda\cdot XM\mu = M'\mu' X\cdot M'T'\lambda\cdot\lambda MT$$

$$M\,3 \quad M'\mu' X\cdot M'T'\lambda\cdot\eta' XMT = \lambda$$

PROPOSITION 2. *Let* $\bar{\mathbb{T}}$ *and* $\bar{\mathbb{T}}'$ *be triples of higher degree in* K *and* L *respectively and let* $X:K \longrightarrow L$ *be a functor. Then, each natural transformation* $\lambda:XMT \longrightarrow M'T'X$ *satisfying the axioms of definition 4 induces a functor* $S(\lambda):K_{\bar{\mathbb{T}}} \longrightarrow K_{\bar{\mathbb{T}}'}$ *lifting of* X. *If* id_{MTA} *is a* $K_{\bar{\mathbb{T}}}$*-morphism for every* A *in* K, *each such functor induces a natural transformation* λ *such that* (X,λ) *is a morphism from* $\bar{\mathbb{T}}$ *to* $\bar{\mathbb{T}}'$. *In this case, the two passages are mutually inverse bijections.*

P r o o f. Suppose $\alpha \in K_{\bar{\mathbb{T}}}(A,B)$, we define $S(\lambda)(\alpha : A \longrightarrow MTB) = \lambda_B \cdot X\alpha : XA \longrightarrow M'T'XB$.

λ to $S(\lambda)$ is well defined:

$$M'\mu'_{XB} \cdot \eta'_{M'T'XB} \cdot \lambda_B \cdot X\alpha = M'\mu'_{XB} \cdot M'T'\lambda_B \cdot \eta'_{XMTB} \cdot X\alpha = \lambda_B \cdot X\alpha$$

If $\alpha \in K_{\bar{\mathbb{T}}}(A,B)$ and $\beta \in K_{\bar{\mathbb{T}}}(B,C)$,

$$S(\lambda)(\beta \circ \alpha) = \lambda_C \cdot X(M\mu_C \cdot MT\beta\alpha) = M'\mu'_{XC} \cdot M'T'\lambda_C \cdot \lambda_{MTC} \cdot XMT\beta \cdot X\alpha =$$

$$= M'\mu'_{XC} \cdot M'T'S(\lambda)(\beta) \cdot S(\lambda)(\alpha) = S(\lambda)(\beta) \circ S(\lambda)(\alpha)$$

Furthermore, by the axiom 1 of λ, $F_{\bar{\mathbb{T}}'} \cdot X = S(\lambda) \cdot F_{\bar{\mathbb{T}}}$.

Let V be a lifting of X. Define $\lambda_A = V(id_{MTA})$ and $V(\alpha : A \longrightarrow MTB) = V(id_{MTB} \circ F_{\bar{\mathbb{T}}}\alpha) = \lambda_B \circ F_{\bar{\mathbb{T}}'} X\alpha = M'\mu'_{XB} \cdot M'T'\lambda_B \cdot \eta'_{XTMB} \cdot X\alpha = \lambda_B \cdot X\alpha$

V to λ is well defined:

If $f : A \longrightarrow B$ is a K-morphism,

$$\lambda_B \cdot XMTf = V(M\mu_B \cdot MT\eta_B \cdot MTf \cdot id_{MTA}) = V(\eta_B f \circ id_{MTA}) =$$

$$= V(\eta_B f) \circ V(id_{MTA}) = M'\mu'_{XB} \cdot M'T'\lambda_B \cdot M'T'X\eta_B \cdot M'T'Xf \cdot \lambda_A =$$

$$= M'\mu'_{XB} \cdot M'T'\eta'_{XB} \cdot M'T'Xf \cdot \lambda_A = M'T'Xf \cdot \lambda_A$$

and

$$\lambda_A \cdot XM\mu_A = V(M\mu_A) = V(id_{MTA}) \circ V(id_{MTMTA}) = \lambda_A \circ \lambda_{MTA} =$$

$$= M'\mu'_{XA} \cdot M'T'\lambda_A \cdot \lambda_{MTA}.$$

The remaining details are clear.

ASSOCIATE TRIPLES

DEFINITION 5. *Let* $\bar{\mathbb{T}} = (T, M, \eta, \mu)$ *and* $\bar{\mathbb{T}}' = (T, M', \eta', \mu')$ *be triples of higher degree in a same category* K. $\bar{\mathbb{T}}$ and $\bar{\mathbb{T}}'$ *are said to be associate if* $\mu \cdot \mu' MT = \mu' \cdot TM'\mu$.

The next result is easily proved.

PROPOSITION 3. *The following three conditions are equivalent:*

(1) $\bar{\mathbb{T}}$ *and* $\bar{\mathbb{T}}'$ *are associate*

(2) $\mu' = \mu \cdot \mu' MT \cdot TM'\eta T$

(3) $\mu = \mu' \cdot TM'\mu \cdot T\eta' MT$.

In [2] it is proved that two triples $\mathbb{T} = (T, \eta, \mu)$ and $\mathbb{T}^* = (T, \eta^*, \mu^*)$ are associate if and only if $\bar{\mathbb{T}} = (T, T, \eta\eta^*, \mu^*\cdot\mu T)$ is a triple of higher degree or if and only if $\bar{\mathbb{T}}' = (T, T, \eta\eta^*, \mu\cdot T\mu^*)$ is a triple of higher degree. We can assert

PROPOSITION 4. *If* $\mathbb{T}$ *and* $\mathbb{T}^*$ *are associate triples, then the triples of higher degree which define are also associate.*

Two associate triples of higher degree $\bar{\mathbb{T}}$ and $\bar{\mathbb{T}}'$ define two natural transformations $f = M'\mu\cdot\eta' MT$ and $g = \mu' M\cdot TM'\eta$ which are a morphism and a dual morphism respectively from $\bar{\mathbb{T}}$ to $\bar{\mathbb{T}}'$. Moreover f and g satisfy the following law:

$$\mu\cdot TM\mu\cdot gTMT = \mu'\cdot TM'\mu\cdot TM'Tf.$$

PROPOSITION 5. *If* $\bar{\mathbb{T}}$ *and* $\bar{\mathbb{T}}'$ *are associate triples and* $g = \mu' M\cdot TM'\eta$ *then* $S(g)\cdot F^{\bar{\mathbb{T}}} = F^{\bar{\mathbb{T}}'}$ *and* $U^{\bar{\mathbb{T}}'}\cdot S(g) = U^{\bar{\mathbb{T}}}$ *hold. Conversely, if a functor* $V\colon K^{\bar{\mathbb{T}}} \longrightarrow K^{\bar{\mathbb{T}}'}$ *exists such that* $U^{\bar{\mathbb{T}}'}\cdot V = U^{\bar{\mathbb{T}}}$ *and* $V\cdot F^{\bar{\mathbb{T}}} = F^{\bar{\mathbb{T}}'}$*, the triples* $\bar{\mathbb{T}}$ *and* $\bar{\mathbb{T}}'$ *are associate and* $V = S(\mu' M\cdot TM'\eta)$.

P r o o f. $U^{\bar{\mathbb{T}}'}\cdot S(g) = U^{\bar{\mathbb{T}}}$ follows from proposition 1.

$$S(g)\cdot F^{\bar{\mathbb{T}}}(a\colon A \longrightarrow B) = Ta\colon (TA, \mu_A\cdot\mu'_{MTA}\cdot TM'\eta_{TA}) \longrightarrow (TB, \mu_B\cdot\mu'_{MTB}\cdot TM'\eta_{TB}) =$$
$$= Ta\colon (TA, \mu'_A) \longrightarrow (TB, \mu'_B) = F^{\bar{\mathbb{T}}'}(a\colon A \longrightarrow B).$$

If $U^{\bar{\mathbb{T}}'}\cdot V = U^{\bar{\mathbb{T}}}$, by proposition 1 $V = S(\lambda)$ where $\lambda\colon T'M' \longrightarrow TM$ is a dual morphism and $\lambda_A = \xi_A\cdot TM'\eta_A$ being $V(TMA, \mu_{MA}) = (TMA, \xi_A)$. Moreover $U\cdot F^{\bar{\mathbb{T}}}MA = F^{\bar{\mathbb{T}}'}MA$, thus $V(TMA, \mu_{MA}) = (T'MA, \mu'_{MA})$ and $\lambda_A = \mu'_{MA}\cdot TM'\eta_A$.

On the other hand $V\cdot F^{\bar{\mathbb{T}}}A = F^{\bar{\mathbb{T}}'}A$, so $(TA, \mu_A\cdot\lambda_{TA}) = (TA, \mu_A\cdot\mu'_{MTA}\cdot TM'\eta_{TA}) = (T'A, \mu'_A)$, therefore $\mu'_A = \mu_A\cdot\mu'_{MTA}\cdot TM'\eta_{TA}$, $T = T'$ and $\bar{\mathbb{T}}$ and $\bar{\mathbb{T}}'$ are associate.

PROPOSITION 6. *If* $\bar{\mathbb{T}}$ *and* $\bar{\mathbb{T}}'$ *are associate triples and* $f = M'\mu\cdot\eta' MT$, *then* $S(f)\cdot F_{\bar{\mathbb{T}}} = F_{\bar{\mathbb{T}}'}$ *and* $U_{\bar{\mathbb{T}}'}\cdot S(f) = U_{\bar{\mathbb{T}}}$ *hold. Conversely, if a functor* $V\colon K_{\bar{\mathbb{T}}} \longrightarrow K_{\bar{\mathbb{T}}'}$ *exists, such that* $U_{\bar{\mathbb{T}}'}\cdot V = U_{\bar{\mathbb{T}}}$ *and* $V\cdot F_{\bar{\mathbb{T}}} = F_{\bar{\mathbb{T}}'}$*, the triples* $\bar{\mathbb{T}}$ *and* $\bar{\mathbb{T}}'$ *are associate and* $V = S(M'\mu\cdot\eta' MT)$.

P r o o f. $S(f)\cdot F_{\bar{\mathbb{T}}} = F_{\bar{\mathbb{T}}'}$ follows from proposition 2.

$$U_{\bar{\mathbb{T}}}(\alpha\colon X \longrightarrow MTY) = \mu_Y\cdot T\alpha\colon TX \longrightarrow TY =$$
$$= \mu'_Y\cdot TM'\mu_Y\cdot T\eta'_{MTY}\cdot\alpha\colon TX \longrightarrow TY = U_{\bar{\mathbb{T}}'}\cdot S(f)(\alpha\colon X \longrightarrow MTY).$$

Conversely, suppose V such that $V \cdot F_{\bar{\Pi}'} = F_{\bar{\Pi}}$. By the proposition 2, $V = S(\lambda)$ with $\lambda = V(\mathrm{id}_{MTA})$. Moreover if $U_{\bar{\Pi}}(A) = U_{\bar{\Pi}'} \cdot V(A)$, $TA = T'A$ for every A in K, therefore $T = T'$; $U_{\bar{\Pi}}(\mathrm{id}_{MTA}) = U_{\bar{\Pi}'}(\lambda_A)$ so $\mu = \mu' \cdot T'\lambda$, $M'\mu \cdot \eta' MT =$

$= M'\mu' \cdot \eta' M'T \cdot \lambda = \lambda$ and $\mu = \mu' \cdot TM'\mu \cdot T\eta' MT$.

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REZIME

ASOCIRANE TROJKE VIŠEG STEPENA

Asocirane trojke višeg stepena su definisane i izučene. Specijalno, utvrdjena je veza izmedju ove osobine i postojanja funktora u kategoriji algebri i Kleislievih kategorija (trojke višeg stepena) koji komutiraju s funktorima Eilenberg-Moora i Kleislija.

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