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SOME COMMON FIXED POINT THEOREMS IN CONVEX  
METRIC SPACES

*Olga Hadzić*

*Institute of Mathematics, University of Novi Sad,  
Dr Ilije Djuričića 4, 21000 Novi Sad, Yugoslavia*

ABSTRACT

In this paper we prove a generalization of Theorem 2 from [3] on the existence of the common fixed point for three mappings  $A$ ,  $S$  and  $T$  in convex metric spaces. A theorem on continuous dependence of the common fixed points on parameter is obtained. As an application a generalization of the Krasnoselski fixed theorem is given.

1. PRELIMINARIES

First, we shall recall some definitions and results which we use in the paper.

A metric space  $(M,d)$  is *convex* if for each  $x,y \in M$  such that  $x \neq y$  there exists  $z \in M$ ,  $x \neq z \neq y$  such that:

$$d(x,z) + d(z,y) = d(x,y).$$

The following result is well known [1]:

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**Proposition 1.** *Let  $K$  be a closed subset of the complete and convex metric space  $(M,d)$ . If  $x \in M$  and  $y \in M$  then there exists a point  $z \in \partial K$  such that:*

$$d(x,z) + d(z,y) = d(x,y).$$

Some fixed point theorems in convex metric spaces are proved in [1], [3], [4], [5], [6], [10], [11], [13], [14].

W. Takahashi introduced in [13] the notion of a convex structure  $W$  on a metric space  $(M,d)$ .

**Definition 1.** *Let  $(M,d)$  be a metric space. A mapping  $W : M \times M \times [0,1] \rightarrow M$  is called a convex structure on  $M$  if for all points  $(x,y) \in M \times M$  and  $t \in [0,1]$ :*

$$d(u,W(x,y,t)) \leq td(u,x) + (1-t)d(u,y)$$

for all  $u \in M$ .

In [14] it is proved that:

$$d(x,W(x,y,t)) = (1-t)d(x,y)$$

$$d(y,W(x,y,t)) = t \cdot d(x,y)$$

for every  $x,y \in M$  and  $t \in [0,1]$ . From this it follows that a metric space with a convex structure is a convex metric space. Every normed space  $(M,\|\cdot\|)$  is a metric space with a convex structure where  $W(x,y,t) = t \cdot x + (1-t)y$ ,  $(x,y,t) \in M \times M \times [0,1]$ . An another example of a non normed metric space with a convex structure is given in [13].

Some fixed point theorems in metric spaces with a convex structure are proved in [3], [4], [10], [11], [13], [14].

In [14] L. Talman introduced a class of metric spaces with a convex structure for which a fixed point theorem of Schauder's type holds.

**Definition 2.** *Let  $(M,d)$  be a metric space and*

$P = \{(t_1, t_2, t_3) \in [0, 1]^3, t_1 + t_2 + t_3 = 1\}$ . A strong convex structure (SCS) on  $M$  is a continuous function  $K : M \times M \times M \times P \rightarrow M$  with the property that for each  $(x_1, x_2, x_3, t_1, t_2, t_3) \in M \times M \times M \times P$ ,  $K(x_1, x_2, x_3, t_1, t_2, t_3)$  is the unique point of  $M$  which satisfies:

$$d(y, K(x_1, x_2, x_3, t_1, t_2, t_3)) \leq \sum_{k=1}^3 t_k d(y, x_k)$$

for every  $y \in M$ .

A metric space  $(M, d)$  with a strong convex structure is called strongly convex. A strongly convex metric space is a metric space with the convex structure  $W_K$ , defined by:

$$W_K(x_1, x_2, t) = K(x_1, x_2, x_3, t, 1-t, 0)$$

$(x_1, x_2, t) \in M \times M \times [0, 1]$ .

If  $H \subset M$  and  $(M, d)$  is a metric space with a convex structure  $W$ ,  $H$  is said to be  $W$  convex if and only if  $W(x, y, t) \in H$ , for every  $(x, y, t) \in H \times H \times [0, 1]$ .

If  $S \subseteq M$ ,  $(M, d)$  is a metric space with a convex structure and  $r > 0$  then  $S_r = \{x \in M, d(x, S) < r\}$ . A convex subset  $S$  of  $M$  is stable if the set  $S_r$  is convex for every  $r > 0$ .

**Definition 3.** A strongly convex metric space  $(M, d)$  is stable if the set  $\{W(x, y, t), t \in [0, 1]\}$  is stable for every pair  $(x, y) \in M$ .

In [14] it is proved that in a stable strongly convex metric space the convex hull of any precompact subset of  $M$  is precompact.

From Theorem 4.2 [14] we have the following result.

**Proposition 2.** Let  $(M, d)$  be a complete, stable strongly convex metric space and  $F : M \rightarrow M$  a compact mapping. Then  $F$  has a fixed point.

## 2. COMMON FIXED POINT THEOREMS

The following theorem is a generalization of Theorem 2 from [3] and of the well known result of Assad and Kirk [1] for the single-valued mapping.

**Definition 4.**[12] *Let  $(M,d)$  be a metric space,  $K$  a nonempty subset of  $M$  and  $f, S : K \rightarrow M$ . The pair  $(f,S)$  is weakly commutative if for every  $x \in K$  the implication:*

$$fx, Sx \in K \Rightarrow d(fSx, Sfx) \leq d(fx, Sx)$$

*holds.\**

There are examples of weakly commutative pairs  $(f,S)$  which are not commutative [7].

**Theorem 1.** *Let  $(M,d)$  be a complete, convex metric space,  $K$  a nonempty, closed subset of  $M$ ,  $f, S, T : K \rightarrow M$  continuous mappings so that  $\partial K \subseteq SK \cap TK$ ,  $f(K) \cap K \subseteq SK \cap TK$  and:*

$$Tx \in \partial K \Rightarrow fx \in K, Sx \in \partial K \Rightarrow fx \in K.$$

*If  $(f,S)$  and  $(f,T)$  are weakly commutative and there exists a nondecreasing function  $q : [0, \infty) \rightarrow [0, 1)$  such that:*

$$d(fx, fy) \leq q(d(Sx, Ty))d(Sx, Ty)$$

*then there exists  $z \in K$  so that:*

$$z = fz \in \{Tz, Sz\}.$$

*If  $S, T : M \rightarrow M$  then there exists one and only one  $z \in K$  such that  $z = fz = Tz = Sz$ .*

**Proof.** As in [3] it can be proved that there exist two sequences  $\{p_n\}$  and  $\{p'_n\}$  such that  $p'_{n+1} = f(p_n)$ , for every  $n \in \mathbb{N}$  and:

\*If  $S: M \rightarrow M$  the implication is:  $Sx \in K \Rightarrow d(fSx, Sfx) \leq d(fx, Sx)$ .

(i) For every  $n \in \mathbb{N}$ :

$$P'_{2n} \in K \Rightarrow P'_{2n} = TP_{2n}$$

$$P'_{2n} \notin K \Rightarrow TP_{2n} \in \partial K$$

and

$$d(Sp_{2n-1}, TP_{2n}) + d(TP_{2n}, fp_{2n-1}) = d(Sp_{2n-1}, fp_{2n-1}).$$

(ii) For every  $n \in \mathbb{N}$ :

$$P'_{2n+1} \in K \Rightarrow P'_{2n+1} = SP_{2n+1}$$

$$P'_{2n+1} \notin K \Rightarrow SP_{2n+1} \in \partial K$$

and

$$d(TP_{2n}, SP_{2n+1}) + d(SP_{2n+1}, fp_{2n}) = d(TP_{2n}, fp_{2n}).$$

For the completeness we shall give the proof of (i) and (ii). Let  $x \in \partial K$ . From  $\partial K \subseteq T(K)$  it follows that there exists  $p_0 \in K$  such that  $x = Tp_0 \in \partial K$ . Since  $Tp_0 \in \partial K \Rightarrow fp_0 \in K$  we have that  $fp_0 \in f(K) \cap K \subseteq S(K)$ . Hence there exists  $p_1 \in K$  so that  $Sp_1 = fp_0 = p'_1$ . Let  $p'_2 = fp_1$ . If  $fp_1 \in K$  then  $fp_1 \in f(K) \cap K \subseteq T(K)$  and so there exists  $p_2 \in K$  such that  $Tp_2 = fp_1$ . If  $fp_1 \notin K$  then there exists  $q \in \partial K$  so that:

$$(1) \quad d(Sp_1, q) + d(q, fp_1) = d(Sp_1, fp_1).$$

From  $\partial K \subseteq T(K)$  it follows that there exists  $p_2 \in K$  so that  $q = Tp_2$  and hence (1) gives:

$$d(Sp_1, Tp_2) + d(Tp_2, fp_1) = d(Sp_1, fp_1).$$

If we continue in this way we can prove (i) and (ii).

Let:

$$P_0 = \{p_{2i} \mid i \in \mathbb{N}, p'_{2i} = TP_{2i}\}$$

$$P_1 = \{p_{2i} \mid i \in \mathbb{N}, p'_{2i} \neq TP_{2i}\},$$

$$Q_0 = \{p_{2i+1} \mid i \in \mathbb{N}, p'_{2i+1} = SP_{2i+1}\}.$$

$$Q_1 = \{p_{2i+1} \mid i \in \mathbb{N}, p'_{2i+1} \neq Sp_{2i+1}\}.$$

Let us prove that for every  $n \in \mathbb{N}$ :

$$(p_{2n}, p_{2n+1}) \notin P_1 \times Q_1 \text{ and } (p_{2n-1}, p_{2n}) \notin Q_1 \times P_1.$$

Suppose that  $p_{2n} \in P_1$  which means that  $p'_{2n} \neq Tp_{2n}$ . Then (i) implies that  $Tp_{2n} \in \partial K$  and so  $fp_{2n} \in K$ . Then  $p'_{2n+1} = Sp_{2n+1} = fp_{2n}$  and so  $p_{2n+1} \in Q_0$ . Similarly we can prove that

$$(p_{2n-1}, p_{2n}) \notin Q_1 \times P_1.$$

We shall prove that for every  $n \geq 2$ .

$$d(Tp_{2n}, Sp_{2n+1}) \leq \begin{cases} q(d(Sp_{2n-1}, Tp_{2n}))d(Sp_{2n-1}, Tp_{2n}) \\ \text{or} \\ q(d(Tp_{2n-2}, Sp_{2n-1}))d(Tp_{2n-2}, Sp_{2n-1}) \end{cases}$$

$$d(Sp_{2n-1}, Tp_{2n}) \leq \begin{cases} q(d(Tp_{2n-2}, Sp_{2n-1}))d(Tp_{2n-2}, Sp_{2n-1}) \\ \text{or} \\ q(d(Tp_{2n-2}, Sp_{2n-3}))d(Tp_{2n-2}, Sp_{2n-3}). \end{cases}$$

Let:

$$1. (p_{2n}, p_{2n+1}) \in P_0 \times Q_0.$$

Then:

$$\begin{aligned} d(Tp_{2n}, Sp_{2n+1}) &= d(fp_{2n-1}, fp_{2n}) \leq \\ &\leq q[d(Sp_{2n-1}, Tp_{2n})]d(Sp_{2n-1}, Tp_{2n}). \end{aligned}$$

Let:

$$2. (p_{2n}, p_{2n+1}) \in P_0 \times Q_1.$$

We have that:

$$d(Tp_{2n}, Sp_{2n+1}) \leq d(Tp_{2n}, fp_{2n}) = d(fp_{2n-1}, fp_{2n}) \\ \leq q[d(Sp_{2n-1}, Tp_{2n})]d(Sp_{2n-1}, Tp_{2n}).$$

Let:

$$3. (p_{2n}, p_{2n+1}) \in P_1 \times Q_0.$$

We have:

$$d(Tp_{2n}, Sp_{2n+1}) \leq d(Tp_{2n}, fp_{2n-1}) + d(fp_{2n-1}, fp_{2n})$$

since  $p_{2n+1} \in Q_0$  and hence  $Sp_{2n+1} = fp_{2n}$ . Further:

$$d(Tp_{2n}, Sp_{2n+1}) \leq d(Tp_{2n}, fp_{2n-1}) + \\ + q[d(Sp_{2n-1}, Tp_{2n})]d(Sp_{2n-1}, Tp_{2n}) \\ \leq d(Sp_{2n-1}, Tp_{2n}) + d(Tp_{2n}, fp_{2n-1}) = d(Sp_{2n-1}, fp_{2n-1}).$$

From  $p_{2n} \in P_1$  it follows that  $p_{2n-1} \in Q_0$  and so  $Sp_{2n-1} = fp_{2n-2}$ . This implies that:

$$d(Tp_{2n}, Sp_{2n+1}) \leq d(fp_{2n-2}, fp_{2n-1}) \leq \\ \leq q[d(Tp_{2n-2}, Sp_{2n-1})]d(Tp_{2n-2}, Sp_{2n-1}).$$

We can prove in a similar way that the following implications hold:

$$(p_{2n-1}, p_{2n}) \in Q_0 \times P_0 \Rightarrow d(Sp_{2n-1}, Tp_{2n}) \leq \\ \leq q[d(Tp_{2n-2}, Sp_{2n-1})]d(Tp_{2n-2}, Sp_{2n-1}) \\ (p_{2n-1}, p_{2n}) \in Q_1 \times P_0 \Rightarrow d(Sp_{2n-1}, Tp_{2n}) \leq \\ \leq q[d(Tp_{2n-2}, Sp_{2n-3})]d(Tp_{2n-2}, Sp_{2n-3})$$

$$\begin{aligned} (P_{2n-1}, P_{2n}) \in Q_0 \times P_1 &\Rightarrow d(SP_{2n-1}, TP_{2n}) \leq \\ &\leq q[d(TP_{2n-2}, SP_{2n-1})]d(TP_{2n-2}, SP_{2n-1}). \end{aligned}$$

Let  $\delta = \max\{d(TP_2, SP_3), d(TP_2, SP_1)\}$ . We shall prove that:

$$(2) \quad d(TP_{2n}, SP_{2n+1}) \leq [q(\delta)]^{n-1} \delta$$

$$(3) \quad d(SP_{2n+1}, TP_{2n+2}) \leq [q(\delta)]^n \delta$$

for every  $n \in \mathbb{N}$ . For  $n = 1$  we have that  $d(TP_2, SP_3) \leq \delta$  and:

$$d(SP_3, TP_4) \leq q[d(TP_2, SP_3)]d(TP_2, SP_3) \leq q(\delta)\delta$$

or:

$$d(SP_3, TP_4) \leq q[d(TP_2, SP_1)]d(TP_2, SP_1) \leq q(\delta)\delta.$$

Suppose that (2) and (3) are satisfied for  $n = k$  and prove that:

$$(4) \quad d(TP_{2k+2}, SP_{2k+3}) \leq [q(\delta)]^k \delta$$

$$(5) \quad d(TP_{2k+4}, SP_{2k+3}) \leq [q(\delta)]^{k+1} \delta$$

We have that:

$$\begin{aligned} d(TP_{2k+2}, SP_{2k+3}) &\leq q[d(SP_{2k+1}, TP_{2k+2})]d(SP_{2k+1}, TP_{2k+2}) \\ &\leq q[(q(\delta))^k \delta][q(\delta)]^k \delta \leq [q(\delta)]^{k+1} \delta \end{aligned}$$

or:

$$\begin{aligned} d(TP_{2k+2}, SP_{2k+3}) &\leq q[(TP_{2k}, SP_{2k+1})]d(TP_{2k}, SP_{2k+1}) \\ &\leq q[(q(\delta))^{k-1} \delta][q(\delta)]^{k-1} \delta \leq [q(\delta)]^k \delta, \end{aligned}$$

which proves (4). Inequality (5) can be proved similarly. Hence (2) and (3) are satisfied for every  $n \in \mathbb{N}$ . From (2) and (3) it



is obvious that  $\{Tp_{2n}\}$  and  $\{Sp_{2n+1}\}$  are Cauchy sequences in  $K$ . Since  $M$  is complete we obtain that there exists  $z \in K$  so that  $z = \lim_{n \rightarrow \infty} Tp_{2n} = \lim_{n \rightarrow \infty} Sp_{2n+1}$ . There exists at least one subsequence  $\{Tp_{2n_k}\}$  or  $\{Sp_{2n_k+1}\}$  such that for every  $k \in \mathbb{N}$ ,  $p_{2n_k} \in P_0$  or  $p_{2n_k+1} \in Q_0$ . We shall suppose that  $p_{2n_k} \in P_0$  ( $k \in \mathbb{N}$ ). Then  $fp_{2n_k-1} \in K$  and  $Sp_{2n_k} = fp_{2n_k-1}$  for every  $k \in \mathbb{N}$ .

The pair  $(f, S)$  is weakly commutative which implies that  $Sz = fz$ , which can be easily proved [3].

Further for every  $k \in \mathbb{N}$ :

$$(6) \quad d(fp_{2n_k}, fSp_{2n_k-1}) \leq q[d(Tp_{2n_k}, S(Sp_{2n_k-1})) \cdot d(Tp_{2n_k}, S(Sp_{2n_k-1}))]$$

and

$$(7) \quad d(fp_{2n_k}, fp_{2n_k-1}) \leq q[d(Tp_{2n_k}, Sp_{2n_k-1})] \cdot d(Tp_{2n_k}, Sp_{2n_k-1}) \leq d(Tp_{2n_k}, Sp_{2n_k-1}).$$

Since  $\lim_{k \rightarrow \infty} Tp_{2n_k} = \lim_{k \rightarrow \infty} fp_{2n_k-1} = z$  from (7) we obtain that

$z = \lim_{k \rightarrow \infty} fp_{2n_k-1} = \lim_{k \rightarrow \infty} fp_{2n_k}$ . On the other side

$d(Tp_{2n_k}, S(Sp_{2n_k-1})) \leq M$  ( $k \in \mathbb{N}$ ) since  $\lim_{k \rightarrow \infty} d(Tp_{2n_k}, S(Sp_{2n_k-1})) = d(Tp_{2n_k}, S(Sp_{2n_k-1})) = d(z, Sz)$ . Hence (6) implies that:

$$(8) \quad d(fp_{2n_k}, fSp_{2n_k-1}) \leq q(M)d(Tp_{2n_k}, S(Sp_{2n_k-1}))$$

and since  $\lim_{k \rightarrow \infty} fSp_{2n_k-1} = \lim_{k \rightarrow \infty} Sfp_{2n_k-1}$  we obtain from (6) that:

$$d(z, Sz) \leq q(M)d(z, Sz).$$

Suppose that  $T : M \rightarrow M$  and prove that  $Tz = fz$ . Then from:

$$\begin{aligned} d(fp_{2n_k}, fp_{2n_k-1}) &= d(fp_{2n_k}, Tp_{2n_k}) \leq \\ &\leq q[d(Tp_{2n_k}, Sp_{2n_k-1})]d(Tp_{2n_k}, Sp_{2n_k-1}) \leq \end{aligned}$$

$$\leq d(Tp_{2n_k}, Sp_{2n_k-1})$$

it follows that  $\lim_{k \rightarrow \infty} fp_{2n_k} = z$  and so:

$$Tz = \lim_{k \rightarrow \infty} T(fp_{2n_k}) = \lim_{k \rightarrow \infty} f(Tp_{2n_k}) = fz.$$

The following theorem is a theorem on continuous dependence of the common fixed points on the parameter.

**Theorem 2.** *Let  $(M, d)$  be a complete, convex metric space,  $K$  a nonempty closed subset of  $M$ ,  $U$  a topological space,  $f : K \times U \rightarrow M$  such that for every  $u \in U$ ,  $f(\cdot, u)$  is continuous on  $K$  and for every  $x \in K$   $f(x, \cdot)$  is continuous on  $U$ ,  $S$  and  $T$  continuous mappings from  $M$  into  $M$  so that  $\partial K \subseteq SK \cap TK$ ,  $f(K, U) \cap K \subseteq SK \cap TK$  and for every  $u \in U$ :*

$$Tx \in \partial K \Rightarrow f(x, u) \in K, \quad Sx \in \partial K \Rightarrow f(x, u) \in K$$

where  $x \in K$ .

*If there exists a nondecreasing function  $q : [0, \infty) \rightarrow [0, 1)$  such that for every  $(x, y, u) \in K \times K \times U$ :*

$$d(f(x, u), f(y, u)) \leq q(d(Sx, Ty))d(Sx, Ty),$$

*the set  $f(K, U) \cap K$  is bounded and for every  $u \in U$  the pairs  $(f(\cdot, u), S)$  and  $(f(\cdot, u), T)$  are weakly commutative then there exists the unique continuous mapping  $z : u \mapsto z(u)$  ( $u \in U$ ), from  $U$  into  $K$  such that:*

$$z(u) = f(z(u), u) = Sz(u) = Tz(u), \quad u \in U.$$

**Proof.** It is obvious that for every  $u \in U$  there exists one and only one element  $z(u) \in K$  such that  $z(u) = f(z(u), u) = Sz(u) = Tz(u)$ . We shall prove that the mapping  $u \mapsto z(u)$  is continuous at every point  $u_0 \in U$ . Let  $\varepsilon > 0$ . We have to prove that there exists a neighbourhood  $V(u_0) \subseteq U$  of  $u_0$  so that the following implication holds:

$$(9) \quad u \in V(u_0) \Rightarrow d(z(u), z(u_0)) < \varepsilon.$$

Since  $z(u) \in f(K, U) \cap K$  and the set  $f(K, U) \cap K$  is bounded, there exists  $P > 0$  such that:

$$d(z(u), z(u_0)) \leq P, \quad \text{for every } u \in U.$$

Then we have:

$$\begin{aligned} d(z(u), z(u_0)) &\leq d(z(u), f(z(u_0), u)) + \\ &\quad + d(f(z(u_0), u), z(u_0)) \\ &= d(f(z(u), u), f(z(u_0), u)) + d(f(z(u_0), u), z(u_0)) \leq \\ &\leq q[d(Sz(u), Tz(u_0))]d(Sz(u), Tz(u_0)) + \\ &\quad + d(f(z(u_0), u), z(u_0)) = \\ &= q[d(z(u), z(u_0))] \cdot d(z(u), z(u_0)) + \\ &\quad + d(f(z(u_0), u), z(u_0)) \leq \\ &\leq q(P)d(z(u), z(u_0)) + d(f(z(u_0), u), f(z(u_0), u_0)). \end{aligned}$$

This implies that:

$$d(z(u), z(u_0)) \leq \frac{d(f(z(u_0), u), f(z(u_0), u_0))}{1 - q(P)}$$

and since for every  $z \in K$ , the mapping  $u \mapsto f(z, u)$  is continuous it is obvious that there exists  $V(u_0)$  so that (9) holds.

Using Theorem 2 we shall prove a generalization of the Krasnoselski fixed point theorem and the Melvin fixed point theorem in convex metric spaces.

**Theorem 3.** *Let  $(M, d)$  be a complete strongly convex*

metric space whose SCS is stable,  $K$  a nonempty, closed convex subset of  $M$ ,  $Q : K \rightarrow M$  a compact mapping,  $G : K \times \overline{Q(K)} \rightarrow M$ ,  $S, T : M \rightarrow M$  so that all the conditions of Theorem 2 are satisfied for  $U = \overline{Q(K)}$  and  $f(x, u) = G(x, u)$  ( $x \in K, u \in \overline{Q(K)}$ ). Then there exists at least one element  $x \in K$  such that  $x = G(x, Q(x)) = Sx = Tx$ .

Proof. From Theorem 2 it follows that there exists one and only one continuous mapping  $R : \overline{Q(K)} \rightarrow K$  so that:

$$Ru = G(Ru, u) = SRu = TRu.$$

Define the mapping  $\tilde{R} : K \rightarrow K$  in the following way:  $\tilde{R}x = RQx$  for every  $x \in K$ . Then  $\tilde{R}$  is a compact mapping and from Proposition 2 it follows that there exists  $x \in K$  such that  $\tilde{R}x = x = RQx = G(RQx, Qx) = G(x, Qx) = Sx = Tx$ .

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## REZIME

NEKE TEOREME O NEPOKRETNOSTI TAČKI U KONVEKSNIM  
METRIČKIM PROSTORIMA

U ovom radu se dokazuje jedno uopštenje teoreme 2 iz [3] o postojanju zajedničke nepokretne tačke za tri preslikavanja. Dobijena je teorema o neprekidnoj zavisnosti zajedničkih nepokretnih tačaka od parametra. Kao primena dato je jedno uopštenje teoreme Krasnoseljskog.

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