ZBORNIK RADOVA
Prirodno-matematičkog fakulteta
Univerziteta u Novom Sadu
Serija za matematiku, 15,1 (1985)

REVIEW OF RESEARCH
Faculty of Science
University of Novi Sad
Mathematics Series, 15,1 (1985)

SOME RESULTS ON THE PRODUCT OF DISTRIBUTIONS DEFINED BY DISTRIBUTION VECTORS

Brian Fisher
Department of Mathematics, The University Leicester, LE1 7RH England

ABSTRACT

The product of two distributions is defined by a distribution vector in order to give more information about the product.

In the following we let N be the neutrix, see van der Corput [1], having domain $N' = \{1,2,\ldots,n,\ldots\}$ and range N'' the real numbers, with negligible functions linear sums of the functions $n^{\lambda}\ln^{r-1}n$, $\ln^r n$ for $\lambda > 0$ and $r = 1,2,\ldots$, and all functions which converge to zero as n tends to infinity.

It follows that if

$$f(n) = 3n^2 + n + n^2\ln n + 2 + 3n^{-1}$$

then the neutrix limit as n tends to infinity of f(n) exists and

---

AMS Mathematics Subject Classification (1980): 46F10
Key words and phrases: Dirac delta-function, distribution, test function, neutrix, neutrix limit.

$$\underset{n\to\infty}{\text{N-lim}}\, f(n) = 2.$$

Now let ρ be a fixed infinitely differentiable function having the properties

(i) $\rho(x) = 0$ for $|x| \geqslant 1$,

(ii) $\rho(x) \geqslant 0$,

(iii) $\rho(x) = \rho(-x)$,

(iv) $\int_{-1}^{1}\rho(x)dx = 1$.

We define the function δ_n by $\delta_n(x) = n\rho(nx)$ for $n = 1,2,\ldots$. It is obvious that $\{\delta_n\}$ is a regular sequence converging to the Dirac delta-function δ.

We now define the locally summable function $\ln x_+$ by

$$\ln x_+ = \begin{cases} \ln x, & x > 0, \\ 0, & x < 0 \end{cases}$$

and we define the locally summable functions $\ln x_-$, $\ln|x|$ and $\text{sgn}x \ln|x|$ by

$$\ln x_- = \ln(x), \qquad \ln|x| = \ln x_+ + \ln x_-,$$

$$\text{sgn}x\cdot\ln|x| = \ln x_+ - \ln x_-.$$

We define the distribution x_+^{-p} by

$$x_+^{-p} = \frac{(-1)^{p-1}}{(p-1)!}\frac{d^p}{dx^p}\ln x_+$$

and we define the distributions x_-^{-p}, $|x|^{-p}$ and $\text{sgn}x\cdot|x|^{-p}$ by

$$x_-^{-p} = (-x)_+^{-p}, \qquad |x|^{-p} = x_+^{-p} + x_-^{-p},$$

$$\text{sgn}x\cdot|x|^{-p} = x_+^{-p} - x_-^{-p}$$

for $p = 1,2,\ldots$.

The following definition for the product of two distributions was given in [2].

DEFINITION 1. *Let* g *and* f *be distributions and let* $g_n = g * \delta_n$. *We say that the neutrix product* $f \cdot g$ *of* f *and* g *exists and is equal to* h *on the open interval* (a,b) *if*

$$\underset{n\to\infty}{\text{N-lim}}\ (f, g_n\phi) = (h, \phi)$$

for all test functions ϕ *with compact support contained in the interval* (a,b) .

In order to give more information about the behaviour of the product $f \cdot g$ the notion of a distribution vector was introduced in [3], where the following definitions, theorems and lemma were given.

DEFINITION 2. *Let* h_r *be a distribution for* $r = 0,1,2,\ldots$. *We say that*

$$\underset{\sim}{h} = [h_0, h_1, \ldots, h_r, \ldots]$$

is a distribution vector.

If α is any real number we define $\alpha\underset{\sim}{h}$ to be the distribution vector

$$\alpha\underset{\sim}{h} = [\alpha h_0, \alpha h_1, \ldots, \alpha h_r, \ldots]$$

and if $\underset{\sim}{k} = [k_0, k_1, \ldots, k_r, \ldots]$ is a second distribution vector we define $\underset{\sim}{h} + \underset{\sim}{k}$ to be the distribution vector

$$\underset{\sim}{h} + \underset{\sim}{k} = [h_0 + k_0, h_1 + k_1, \ldots, h_r + k_r, \ldots] .$$

If $h_{r+i} = 0$ for $i = 1,2,\ldots$ we write

$$\underset{\sim}{h} = [h_0, h_1, \ldots, h_r, 0, \ldots] = [h_0, h_1, \ldots, h_r]$$

and if $h_i = 0$ for $i = 1,2,\ldots$ we write

$$\underset{\sim}{h} = [h_0] = h_0.$$

DEFINITION 3. *Let* $\underset{\sim}{h} = [h_0, h_1, \ldots, h_r, \ldots]$ *be a distribution vector and let* ϕ *be an arbitrary test function with compact support. We define* $(\underset{\sim}{h},\phi)$ *by the sequence of real numbers*

$$(\underset{\sim}{h},\phi) = ((h_0,\phi),(h_1,\phi),\ldots,(h_r,\phi),\ldots).$$

DEFINITION 4. *Let* $h = [h_0, h_1, \ldots, h_r, \ldots]$ *be a distribution vector. We define the derivative* $\underset{\sim}{h}'$ *of* $\underset{\sim}{h}$ *by*

$$\underset{\sim}{h}' = [h_0', h_1', \ldots, h_r', \ldots].$$

DEFINITION 5. *Let* f *and* g *be distributions and let* $g_n = g * \delta_n$. *We say that the neutrix product* $f \cdot g$ *of* f *and* g *exists and is equal to the distribution vector* $\underset{\sim}{h} = [h_0, h_1, \ldots, h_r, \ldots]$ *on the open interval* (a,b) *if*

$$\underset{n\to\infty}{\text{N-lim}}\; n^{-r}(f, g_n\phi) = (h_r, \phi)$$

for $r = 0,1,2,\ldots$ *and all test functions* ϕ *with compact support contained in the interval* (a,b).

DEFINITION 6. *Let* f *and* g *be distributions and suppose that the neutrix product* $f \cdot g$ *exists as the distribution vector* $h = [h_0, h_1, \ldots, h_r, \ldots]$ *on the open interval* (a,b). *We say that* h_0 *is the finite part of* $f \cdot g$ *on the interval* (a,b). *If* $h_r \neq 0$ *for some* $r \geqslant 1$ *we write*

$$\text{p.f. } f \cdot g = h_0$$

on the interval (a,b) *and if* $h_r = 0$ *for* $r = 1,2,\ldots$ *we write*

$$f \cdot g = h_0$$

on the open interval (a,b).

THEOREM 1. *Let* $\underset{\sim}{h} = [h_0, h_1, \ldots, h_r, \ldots]$ *be a distribution vector and let* ϕ *be an arbitrary test function with compact support. Then*

$$(\underset{\sim}{h}', \phi) = -(\underset{\sim}{h}, \phi').$$

THEOREM 2. *Let* f *and* g *be distributions and suppose that the neutrix products* $f \cdot g$ *and* $f' \cdot g$ *(or* $f \cdot g'$*) exist as distribution vectors on the open interval* (a,b). *Then the neutrix product* $f \cdot g'$ *(or* $f' \cdot g$*) exists as a distribution vector and*

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

on the interval (a,b).

LEMMA 1.

$$\int_0^{1/n} x^p \delta_n^{(q)}(x)dx = (-1)^{p+1} n^{q-p} p! \, \rho_{q-p-1}$$

for $p = 0,1,\ldots, q-1 \quad q = 1,2,\ldots$,

$$\int_0^{1/n} x^p \delta_n^{(p)}(x)dx = \tfrac{1}{2}(-1)^p p!$$

for $p = 0,1,2,\ldots$ *and*

$$\int_0^{1/n} |x^p \delta_n^{(q)}(x)|dx = O(n^{q-p})$$

for $q = 0,1,\ldots, p-1$ *and* $p = 1,2,\ldots$, *where*

$$\rho_{2r} = \rho^{(2r)}(0), \quad \rho_{2r+1} = \rho^{(2r+1)}(0) = 0$$

for $r = 0,1,2,\ldots$.

We now prove the following lemma.

LEMMA 2.

$$\text{(1)}\qquad \int_0^{1/n} x^p \ln x \delta_n^{(q)}(x)dx = (-1)^p n^{q-p} p!\,(c_{q-p} - \rho_{q-p-1} a_p) + $$

$$+ (-1)^p n^{q-p} \ln n\, p!\, \rho_{q-p-1}$$

for $p = 0,1,\dots, q-1$ *and* $q = 1,2,\dots,$

$$\text{(2)}\qquad \int_0^{1/n} x^p \ln x \delta_n^{(p)}(x)dx = (-1)^p p!(c_0 + \tfrac{1}{2}a_p) - \tfrac{1}{2}(-1)^p \ln n p!$$

for $p = 0,1,2,\dots$ *and*

$$\text{(3)}\qquad \int_0^{1/n} |x^p \ln x \delta_n^{(q)}(x)|dx = O(n^{q-p}\ln n)$$

for $q = 0,1,\dots, p-1$ *and* $p = 1,2,\dots$, *where*

$$c_s = \int_0^1 \ln t \rho^{(s)}(t)dt$$

$s = 0,1,2,\dots$

$$a_s = \begin{cases} 0, & s = 0, \\ \sum_{i=1}^{s} \frac{1}{i}, & s = 1,2,\dots . \end{cases}$$

PROOF. Putting $nx = t$ we have

$$\text{(4)}\qquad \int_0^{1/n} x^p \ln x \delta_n^{(q)}(x)dx = n^{q-p} \int_0^1 t^p \ln(t/n)\rho^{(q)}(t)dt =$$

$$= n^{q-p} \int_0^1 t^p \ln t \rho^{(q)}(t)dt - \ln n \int_0^{1/n} x^p \delta_n^{(q)}(x)dx.$$

We now prove by induction that

$$\int_0^1 t^p \ln t \rho^{(q)}(t)dt = (-1)^p p!(c_{q-p} - \rho_{q-p-1} a_p) \tag{5}$$

for $p = 0,1,\ldots, q-1$ and $q = 1,2, \ldots$. This is true by definition for $p = 0$ and $q = 1,2,\ldots$. We therefore assume that equation (5) holds for some $p < q$ and $q = 1,2,\ldots$.

Integrating by parts we then have

$$\int_0^1 t^{p+1} \ln t \rho^{(q+1)}(t)dt = -(p+1) \int_0^1 t^p \ln t \rho^{(q)}(t)dt -$$
$$- \int_0^1 t^p \rho^{(q)}(t)dt = (-1)^{p+1}(p+1)!(c_{q-p} - \rho_{q-p-1} a_p) -$$
$$- (-1)^{p+1} p!\, \rho_{q-p-1} = (-1)^{p+1}(p+1)!(c_{q-p} - \rho_{q-p-1} a_{p+1})$$

on using our assumption and lemma 1. Equation (5) follows by induction.

Equation (1) now follows from equation (4) and lemma 1 for $p = 0,1,2,\ldots, q-1$ and $q = 1,2,\ldots$.

Next we prove by induction that

$$\int_0^1 t^p \ln t \rho^{(p)}(t)dt = (-1)^p p!(c_0 + \tfrac{1}{2}a_p) \tag{6}$$

for $p = 0,1,2,\ldots$. This is also true by definition when $p = 0$. We therefore assume that equation (6) holds for some p.

Integrating by parts we then have

$$\int_0^1 t^{p+1} \ln t \rho^{(p+1)}(t)dt = -(p+1) \int_0^1 t^p \ln t \rho^{(p)}(t)dt -$$
$$- \int_0^1 t^p \rho^{(p)}(t)dt = (-1)^{p+1}(p+1)!(c_0 + \tfrac{1}{2}a_p) - \tfrac{1}{2}(-1)^p p! =$$
$$= (-1)^{p+1}(p+1)!(c_0 + \tfrac{1}{2}a_{p+1})$$

on using our assumption and lemma 1. Equation (6) follows by induction.

Equation (2) now follows from equation (4) and lemma 1 for $p = 0,1,2,\ldots$.

Finally we have

$$\int_0^{1/n} |x^p \ln x \delta_n^{(q)}(x)|dx \leq n^{q-p} \int_0^1 |t^p \ln t \rho^{(q)}(t)|dt + $$
$$+ n^{q-p} \ln n \int_0^1 |t^p \rho^{(q)}(t)|dt$$

and equation (3) follows. This completes the proof of the lemma.

THEOREM 3. *The neutrix products* $\ln x_+ \cdot \delta^{(q)}$ *and* $\delta^{(q)} \cdot \ln x_+$ *exist as distribution vectors and*

(7) $$\ln x_+ \cdot \delta^{(q)} = \underset{\sim}{h}(q) = [h_0(q), h_1(q), \ldots, h_q(q)]$$

for $q = 0,1,2,\ldots,$ *where*

$$h_i(q) = \begin{cases} (c_0 + \frac{1}{2}a_q)\delta^{(q)}, & i = 0, \\ (c_i - \rho_{i-1}a_{q-i})\delta^{(q-i)}, & 1 \leq i \leq q \end{cases}$$

and

(8) $$\delta^{(q)} \cdot \ln x_+ = \underset{\sim}{k}(q) = [k_0(q), k_1(q), \ldots, k_q(q)]$$

for $q = 0,1,2,\ldots,$ *where*

$$k_i(q) = \begin{cases} c_0\delta^{(q)}, & i = 0, \\ \binom{q}{i} c_i\delta^{(q-i)}, & 1 \leq i \leq q. \end{cases}$$

PROOF. Let ϕ be an arbitrary test function with compact support.

Then

$$\phi(x) = \sum_{j=0}^{q} \frac{x^j}{j!} \phi^{(j)}(0) + \frac{x^{q+1}}{(q+1)!} \phi^{(q+1)}(\xi x),$$

where $0 < \xi < 1$.

On using lemma 2 it follows that

$$(\ln x_+, \delta_n^{(q)}\phi) = \sum_{j=0}^{q} \frac{\phi^{(j)}(0)}{j!} \int_0^{1/n} x^j \ln x \delta_n^{(q)}(x)dx +$$

$$+ \frac{1}{(q+1)!} \int_0^{1/n} x^{q+1} \ln x \delta_n^{(q)}(x)\phi^{(q+1)}(\xi x)dx =$$

$$= \sum_{j=0}^{q-1} \phi^{(j)}(0)(-1)^j n^{q-j}(c_{q-j} - \rho_{q-j-1}a_j) +$$

$$+ \sum_{j=0}^{q-1} \phi^{(j)}(0)(-1)^j n^{q-j} \ln n j! \rho_{q-j-1} +$$

$$+ \phi^{(q)}(0)(-1)^q(c_0 + \tfrac{1}{2}a_q) - \phi^{(q)}(0)(-1)^q \ln n q! +$$

$$+ O(n^{-1}\ln n).$$

Thus

$$\underset{n\to\infty}{\text{N-lim}}\, n^{-i}(\ln x_+, \delta_n^{(q)}\phi) = \begin{cases} (c_0 + \tfrac{1}{2}a_q)(\delta^{(q)}, \phi), & i = 0, \\ (c_i - \rho_{i-1}a_{q-i})(\delta^{(q-i)}, \phi), & 1 \leqslant i \leqslant q \end{cases}$$

and equation (7) follows.

We now put

$$f_n(x) = \ln x_+ * \delta_n(x)$$

so that

$$f_n(x) = \int_{-1/n}^{x} \ln(x - t)\delta_n(t)dt$$

for $|x| \leqslant 1/n$. Then

$$f_n^{(j)}(0) = \int_{-1/n}^{0} \ln(-t)\delta_n^{(j)}(t)dt = (-1)^j \int_0^{1/n} \ln t\delta_n^{(j)}(t)dt =$$

$$= \begin{cases} c_0 - \frac{1}{2}\ln n, & j = 0, \\ (-1)^j n^j(c_j + \ln n\rho_{j-1}) & j \geqslant 1 \end{cases}$$

on using lemma 2. Thus

$$(\delta^{(q)}, f_n\phi) = (-1)^q \sum_{j=0}^{q} \binom{q}{j} f_n^{(j)}(0)\phi^{(q-j)}(0) =$$

$$= (-1)^q(c_0 - \tfrac{1}{2}\ln n)\phi^{(q)}(0) +$$

$$+ (-1)^q \sum_{j=1}^{q} \binom{q}{j}(-1)^j n^j(c_j + \ln n\rho_{j-1})\phi^{(q-j)}(0)$$

and so

$$\underset{n\to\infty}{\text{N-lim}}\ n^{-i}(\delta^{(q)}, f_n\phi) = \begin{cases} c_0(\delta^{(q)},\phi), & i = 0, \\ \binom{q}{i}c_i(\delta^{(q-i)},\phi), & 1 \leqslant i \leqslant q. \end{cases}$$

Equation (8) follows. This completes the proof of the theorem.

COROLLARY 1. *The neutrix products* $\ln x_- \cdot \delta^{(q)}$, $\delta^{(q)} \cdot \ln x_-$, $\ln|x| \cdot \delta^{(q)}$, $\delta^{(q)} \cdot \ln|x|$, $(\mathrm{sgn}x \cdot \ln|x|) \cdot \delta^{(q)}$ *and* $\delta^{(q)} \cdot (\mathrm{sgn}x \cdot \ln|x|)$ *exist as distribution vectors and*

(9) $$\ln x_- \cdot \delta^{(q)} = [h_0(q), -h_1(q), \ldots, (-1)^q h_q(q)],$$

(10) $$\delta^{(q)} \cdot \ln x_- = [k_0(q), -k_1(q), \ldots, (-1)^q k_q(q)],$$

$$\ln|x| \cdot \delta^{(q)} = [2h_0(q), 0, 2h_2(q), \ldots, h_q(q) + (-1)^q h_q(q)],$$

$$\delta^{(q)} \cdot \ln|x| = [2k_0(q), 0, 2k_2(q), \ldots, k_q(q) + (-1)^q k_q(q)],$$

$$(\mathrm{sgn}x\cdot\ln|x|)\cdot\delta^{(q)} = [0,\ 2h_1(q),\ 0,\dots,\ h_q(q) - (-1)^q h_q(q)],$$

$$\delta^{(q)}\cdot(\mathrm{sgn}x\cdot\ln|x|) = [0,\ 2k_1(q),\ 0,\dots,\ k_q(q) - (-1)^q k_q(q)],$$

for $q = 0,1,2,\dots$.

PROOF. Equations (9) and (10) follow from equations (7) and (8) on replacing x by -x. The remaining results follow from the definitions of $\ln|x|$ and $\mathrm{sgn}x\cdot\ln|x|$.

COROLLARY 2.

$$\text{p.f. } \ln x_+\cdot\delta^{(q)} = \text{p.f. } \ln x_-\cdot\delta^{(q)} = (c_0 + \tfrac{1}{2}a_q)\delta^{(q)},$$

$$\text{p.f.}\,\delta^{(q)}\cdot\ln x_+ = \text{p.f. }\delta^{(q)}\cdot\ln x_- = c_0\delta^{(q)},$$

$$\text{p.f. } \ln|x|\cdot\delta^{(q)} = (2c_0 + a_q)\delta^{(q)},$$

$$\text{p.f. } \delta^{(q)}\cdot\ln|x| = 2c_0\delta^{(q)},$$

$$\text{p.f. } (\mathrm{sgn}x\cdot\ln|x|)\cdot\delta^{(q)} = \text{p.f.}\,\delta^{(q)}\cdot(\mathrm{sgn}x.\ln|x|) = 0$$

for $q = 0,1,2,\dots$.

THEOREM 4. *The neutrix products* $x_+^{-p}\cdot\delta^{(q)}$ *and* $\delta^{(q)}\cdot x_+^{-p}$ *exist as distribution vectors for* $p = 1,2,\dots$ *and* $q = 0,1,2,\dots$. *In particular*

$$(11)\qquad x_+^{-1}\cdot\delta^{(q)} = \underset{\sim}{u}(q) = [u_0(q),\ u_1(q),\dots,\ u_{q+1}(q)],$$

for $q = 0,1,2,\dots$, *where*

$$u_i(q) = \begin{cases} -\dfrac{1}{2(q+1)}\,\delta^{(q+1)}, & i = 0, \\ \dfrac{\rho_i}{q-i+1}\,\delta^{(q-i+1)}, & 1 \leqslant i \leqslant q, \\ -c_{q+1}\,\delta, & i = q+1 \end{cases}$$

and

$$(12) \qquad \delta^{(q)} \cdot x_+^{-} = \underset{\sim}{v}(q) = [0, v_1(q), \ldots, v_{q+1}(q)],$$

$q = 0,1,2,\ldots,$

$$v_i(q) = \begin{cases} -\binom{q}{i-1}\,\delta^{(q-i+1)}, & 1 \leqslant i \leqslant q, \\ -c_{q+1}\delta, & i = q+1. \end{cases}$$

PROOF. Using theorem 2 we see that the neutrix product $x_+^{-} \cdot \delta^{(q)}$ exists as a distribution vector and

$$x_+^{-1} \cdot \delta^{(q)} = (\ln x_+ \cdot \delta^{(q)})' - \ln x_+ \cdot \delta^{(q+1)} = \underset{\sim}{h}'(q) - h(q+1)$$

for $q = 0,1,2,\ldots$. Equation (11) follows.

Next, assuming that $x_+^{-p} \cdot \delta^{(q)}$ exists for some positive integer p and $q = 0,1,2,\ldots$, it again follows from theorem 2 that $x_+^{-p-1} \cdot \delta^{(q)}$ exists for $q = 0,1,2,\ldots$. It follows by induction that $x_+^{-p} \cdot \delta^{(q)}$ exists for $p = 1,2,\ldots$ and $q = 0,1,2,\ldots$.

Similarly, theorem 2 proves that $\delta^{(q)} \cdot x_+^{-1}$ exists and

$$\delta^{(q)} \cdot x_+^{-1} = (\delta^{(q)} \cdot \ln x_+)' - \delta^{(q+1)} \cdot \ln x_+ = \underset{\sim}{k}'(q) - \underset{\sim}{k}(q+1)$$

for $q = 0,1,2,\ldots$. Equation (12) follows.

The existence of $\delta^{(q)} \cdot x_+^{-p}$ for $p = 1,2,\ldots$ and $q = 0,1,2,\ldots$ now follows by induction on using theorem 2. This completes the proof of the theorem.

COROLLARY 1. *The neutrix products* $x_-^{-p}\cdot\delta^{(q)}$, $\delta^{(q)}\cdot x_-^{-p}$, $|x|^{-p}\cdot\delta^{(q)}$, $\delta^{(q)}\cdot|x|^{-p}$, $(\mathrm{sgn}x.|x|^{-p})\cdot\delta^{(q)}$ *and* $\delta^{(q)}\cdot(\mathrm{sgn}x.|x|^{-p})$ *exist as distribution vectors for* $p = 1,2,\ldots$ *and* $q = 0,1,2,\ldots$ *In particular*

$$x_-^{-1}\cdot\delta^{(q)} = [-u_0(q),\ u_1(q),\ldots,\ (-1)^q u_{q+1}(q)],$$

$$\delta^{(q)}\cdot x_-^{-1} = [0,\ v_1(q),\ \ldots,\ (-1)^q v_{q+1}(q)],$$

$$|x|^{-1}\cdot\delta^{(q)} = [0,\ 2u_1(q),\ 0,\ \ldots,\ u_{q+1}(q) + (-1)^q u_{q+1}(q)],$$

$$\delta^{(q)}\cdot|x|^{-1} = [0,\ 2v_1(q),\ 0,\ldots,\ v_{q+1}(q) + (-1)^q v_{q+1}(q)],$$

$$x^{-1}\cdot\delta^{(q)} = [2u_0(q),\ 0,\ 2u_2(q),\ldots,\ u_{q+1}(q) - (-1)^q u_{q+1}(q)],$$

$$\delta^{(q)}\cdot x^{-1} = [0,\ 0,\ 2v_2(q),\ldots,\ v_{q+1}(1) - (-1)^q v_{q+1}(q)]$$

for $q = 0,1,2,\ldots$, *where*

$$x^{-1} = \mathrm{sgn}x.|x|^{-1} = x_+^{-1} - x_-^{-1}.$$

COROLLARY 2.

$$\text{p.f. } x_+^{-1}\cdot\delta^{(q)} = -\frac{1}{2(q+1)}\delta^{(q+1)},$$

$$\text{p.f. } x_-^{-1}\cdot\delta^{(q)} = \frac{1}{2(q+1)}\delta^{(q+1)},$$

$$\text{p.f.}\,\delta^{(q)}\cdot x_+^{-1} = \text{p.f. }\delta^{(q)}\cdot x_-^{-1} = \text{p.f. }|x|^{-1}\cdot\delta^{(q)}$$

$$= \text{p.f.}\,\delta^{(q)}\cdot|x|^{-1} = \delta^{(q)}\cdot x^{-1} = 0,$$

$$\text{p.f. } x^{-1}\cdot\delta^{(q)} = -\frac{1}{q+1}\delta^{(q+1)}$$

for $q = 0,1,2,\ldots$.

The proofs of these corollaries are immediate.

REFERENCES

[1] J.G. van der Corput, *Introduction to the neutrix calculus*, J. Analyse Math., 7 (1959-60), 291 - 398.

[2] B. Fisher, *A non-commutative neutrix product of distributions*, Math. Nachr., 108 (1982), 117 - 127.

[3] B. Fisher, *Product of distributions defined by a vector*, Proc. International Conference on Complex Analysis and Applications, Varna, Bulgaria (1985), to appear.

Received by the editors September 1, 1985.

REZIME

NEKI REZULTATI O PROIZVODU DISTRIBUCIJA DEFINISANOM POMOĆU DISTRIBUCIONIH VEKTORA

Definisan je proizvod dve distribucije pomoću distribucionog vektora i ispitane osobine ovako definisanog proizvoda.