ZBORNIK RADOVA
Prirodno-matematičkog fakulteta
Univerziteta u Novom Sadu
Serija za matematiku, 15,1 (1985)

REVIEW OF RESEARCH
Faculty of Science
University of Novi Sad
Mathematics Series, 15,1 (1985)

ON COINCIDENCE POINTS IN METRIC AND PROBABILISTIC METRIC SPACES WITH A CONVEX STRUCTURE

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ABSTRACT

The notion of the convexity in metric spaces is introduced by Takahashi in [14] and some fixed point theorems in convex metric spaces are proved in [8],[9],[14].

In this paper we shall prove some theorems on the existence of a coincidence point in metric and probabilistic metric spaces with a convex structure.

1. INTRODUCTION

In [2] B.Fisher proved the following generalization of the contraction principle.

THEOREM A *Let* (X,d) *be a complete metric space and* S,T *continuous mappings from* X *into* X. *Mappings* S *and* T *have a common fixed point if and only if there exists a mapping* $A:X\to SX\cap TX$ *which commutes with* S *and* T *and :*

$$d(Ax,Ay)\leq q\, d(Sx,Ty) \quad , \qquad \textit{for every } x,y\in X$$

where $q\in[0,1)$.

In this paper we investigate the existence of a common fixed point for mappings A and S if :

$$d(Ax,Ay)\leq d(Sx,Sy) \quad ,\text{for every } x,y\in X$$

where (X,d) is a convex metric space in the sense of Takahashi.

The obtained results are closely related to the well known result of Göhde ,if $S = Id$ [3].

AMS Mathematics Subject Classification (1980):47H10.

Key words and phrases: Coincidence points, probabilistic metric spaces, convex structures.

THEOREM B *Let* M *be a star-shaped subset of a Banach space and* $F:M\to M$ *be a nonexpansive mapping with a compact attractor. Then* F *has a fixed point* .

We investigate also the problem of the existence of a coincidence point for A,S and T where A is a multivalued mapping and :

$H(Ax,Ay)\leq d(Sx,Ty)$,for every $x,y\in X$ and

A is (α,S) or (α,T) densifying.

In part 4.of this paper we prove a common fixed point theorem in probabilistic metric spaces with a convex structure.

2. NOTATIONS AND DEFINITIONS

In [14] Takahashi introduced the notion of the convexity in metric spaces .

DEFINITION 1. *Let* (X,d) *be a metric space . A mapping* $W:X\times X\times[0,1]\to X$ *is said to be a convex structure if for every* $(x,y,\lambda)\in X\times X\times[0,1]$:

$$d(u,W(x,y,\lambda))\leq \lambda d(u,x)+(1-\lambda)d(u,y) \text{ ,for every } u\in X .$$

A metric space with a convex structure will be called a convex metric space .

There are many convex metric spaces which cannot be imbedded in any Banach space [14].

DEFINITION 2. *A convex metric space* X *satisfies condition* II *if for all* $(x,y,z,\lambda)\in X\times X\times X\times[0,1]$:

$$d(W(x,z,\lambda),W(y,z,\lambda))\leq \lambda\, d(x,y) \quad ([9]) .$$

DEFINITION 3. *Let* X *be a convex metric space,* $x_o\in X$ *and* $S:X\to X$. *The mapping* S *is said to be* (W,x_o) *-convex if for every* $z\in X$ *and every* $\lambda\in(0,1)$:

$$W(Sz,x_o,\lambda) = S(W(z,x_o,\lambda)) .$$

REMARK If X is a Banach space and $W(x,y,\lambda)=\lambda x+(1-\lambda)y$ for every $(x,y,\lambda)\in X\times X\times[0,1]$ then every homogeneous mapping $S:X\to X$ is $(W,0)$-convex.

By α we shall denote the Kuratowski measure of noncompact - ness .If (X,d) is a metric space then H(A,B) is the Hausdorff metric ,where $A,B\in CB(X)$ (the family of all bounded and closed subsets of X). By 2^X we shall denote the family of all nonempty subsets of X and if $T:X\to 2^X$,we say that T is a closed mapping if from $y_n\in Tx_n (x_n\in X, n\in\mathbb{N})$ and $\lim_{n\to\infty} x_n = x$,$\lim_{n\to\infty} y_n = y$ it follows that $y \in Tx$. A set $M \subset X$ is an attractor for a mapping $F: X \to X$ if for every $x \in X$: $\overline{\bigcup_{n=1}^{\infty} F^n(x)} \cap M \neq \emptyset$.

DEFINITION 4. *Let* (X,d) *be a metric space,* $B(X)$ *the family of all bounded subsets of* X, $K \subseteq X$ *and* A *and* S *mappings from* X *into* 2^X. *If for every* $M \subseteq K$ *such that* $S(M), A(M) \in B(X)$ *the implication :* $\alpha(S(M)) < \alpha(A(M)) \Rightarrow \overline{M}$ *is compact holds,* A *is said to be* (α,S)*-densifying on* K.

3. COMMON FIXED POINT THEOREMS IN METRIC SPACES

THEOREM 1. *Let* (X,d) *be a convex, complete metric space which satisfies condition II,* $S, A: X \to X$ *continuous, commutative mappings such that* $AX \subseteq SX$, AX *a bounded set,* $x_o \in X$, S (W,x_o)*-convex and* $d(Ax,Ay) < d(Sx,Sy)$, *for every* $x,y \in X$.

If M *is a nonempty subset of* X *such that* SM *is an attractor for the mapping* A *and* A *is* (α,S) *-densifying on* M *then there exists* $x \in X$ *such that* $Ax = Sx$.

P r o o f : Let $\{k_n\}_{n \in \mathbb{N}}$ be a sequence of real numbers from the interval $(0,1)$ such that $\lim_{n\to\infty} k_n = 1$ and for every $n \in \mathbb{N}$ let $A_n x = W(Ax,x_o,k_n)$, for every $x \in X$. We shall prove that all the conditions of the Theorem from [2] are satisfied for $A_n (n \in \mathbb{N})$, S and $Tx = Sx$, for every $x \in X$. Thus, let us show that for every $n \in \mathbb{N}$, $A_n X \subseteq SX$ and $A_n Sx = SA_n x$, for every $x \in X$ and for every $x,y \in X$:

$$d(A_n x, A_n y) < k_n d(Sx,Sy) .$$

For every $x,y \in X$ and every $n \in \mathbb{N}$ we obtain from condition II that :

$$d(A_n x, A_n y) = d(W(Ax,x_o,k_n), W(Ay,x_o,k_n)) < k_n d(Ax,Ay) < k_n d(Sx,Sy).$$

Since $AX \subseteq SX$ for every $x \in X$ there exists $z_x \in X$ such that $Ax = Sz_x$ and so from (W,x_o)-convexity of the mapping S we obtain that:

$$A_n x = W(Ax,x_o,k_n) = W(Sz_x,x_o,k_n) = S(W(z_x,x_o,k_n)) \in SX$$

for every $n \in \mathbb{N}$. Furthermore, A_n and S commute for every $n \in \mathbb{N}$ since:

$$A_nSx = W(ASx,x_o,k_n) = W(SAx,x_o,k_n) = S(W(Ax,x_o,k_n)) = SA_nx$$

for every $x \in X$. From the Theorem proved in [2] it follows that for every $n \in \mathbb{N}$ there exists $x_n \in X$ such that $x_n = A_nx_n = Sx_n$. Since the set AX is bounded there exists $D \in \mathbb{R}$ such that $D = \sup_{x \in X} d(Ax,x_o)$. Then for every $n \in \mathbb{N}$ we have that:

$$d(x_n,Ax_n) = d(W(Ax_n,x_o,k_n),Ax_n) \leq k_n d(Ax_n,Ax_n) + \\ + (1-k_n)d(Ax_n,x_o) \leq D(1-k_n) \ .$$

Since SM is an attractor for the mapping A it follows that:

(1) $$\overline{\bigcup_{m=1}^{\infty} A^m(x_n)} \cap SM \neq \emptyset \ , \quad \text{for every } n \in \mathbb{N} .$$

From (1) we conclude that for every $n \in \mathbb{N}$ there exists $y_n \in M$ such that:

(2) $$Sy_n \in \overline{\bigcup_{m=1}^{\infty} A^m(x_n)} \ .$$

Relation (2) implies that for every $n \in \mathbb{N}$ there exists $m_n \in \mathbb{N}$ such that:

(3) $$d(Sy_n, A^{m_n}x_n) < 1 - k_n \ .$$

Then we have:

$$d(Sy_n,Ay_n) \leq d(Sy_n,A^{m_n}x_n) + d(A^{m_n}x_n,A^{m_n+1}x_n) + \\ + d(A^{m_n+1}x_n,Ay_n) \leq (1-k_n) + D(1-k_n) + d(A^{m_n}x_n,Sy_n) \leq (1-k_n)(D+2)$$

since:

(4) $$d(A^kx_n,A^{k+1}x_n) \leq d(x_n,Ax_n), \quad \text{for every } n \in \mathbb{N} \text{ and } k \in \mathbb{N} \ .$$

The relation (4) can be proved by induction. For $k = 1$ and every $n \in \mathbb{N}$ we have:

$$d(Ax_n,A^2x_n) \leq d(Sx_n,Ax_n) = d(x_n,Ax_n)$$

and let us suppose that for some $k \in \mathbb{N}$ and every $n \in \mathbb{N}$:

$$d(A^kx_n,A^{k+1}x_n) \leq d(x_n,Ax_n) \ .$$

Then we have:

$$d(A^{k+1}x_n, A^{k+2}x_n) < d(S(A^k x_n), S(A^{k+1}x_n)) =$$

$$= d(A^k(Sx_n), A^{k+1}(Sx_n)) = d(A^k x_n, A^{k+1}x_n) < d(x_n, Ax_n) .$$

From $\lim_{n\to\infty} k_n = 1$ we obtain that $\lim_{n\to\infty} d(Sy_n, Ay_n) = 0$. Let $C = \{y_n \mid n \in \mathbb{N}\}$. Then for every $\varepsilon > 0$ there exists $n_o(\varepsilon) \in \mathbb{N}$ such that:

$$S(\{y_n \mid n > n_o(\varepsilon)\}) \subseteq \bigcup_{y \in AC} L(y, \varepsilon)$$

and so [9]:

$$\alpha(SC) = \alpha(S(\{y_n \mid n > n_o(\varepsilon)\})) < \alpha(AC) + 2\varepsilon .$$

Since ε is an arbitrary positive number we have that $\alpha(SC) < \alpha(AC)$. The mapping A is (α, S)-densifying and so it follows that the set C is relatively compact. Suppose that $\lim_{k\to\infty} y_{n_k} = y$. Since A and S are continuous from $\lim_{n\to\infty} d(Sy_n, Ay_n) = 0$ we obtain that $Sy = Ay$.

In [4] we proved the following coincidence theorem.

THEOREM C *Let* (X,d) *be a complete metric space,* S *and* T *continuous mappings from* X *into* X *,* $A: X \to CB(SX \cap TX)$ *a closed mapping such that the following conditions are satisfied:*

1. $H(Ax, Ay) < q\, d(Sx, Ty)$ *, for every* $x, y \in X$ *where* $q \in [0, 1)$.
2. *For every* $x \in X$, $ATx = \bigcup_{u \in Ax} Tu$ *and* $ASx = \bigcup_{v \in Ax} Sv$.

Then there exists a sequence $\{x_n\}_{n \in \mathbb{N}}$ *from* X *such that :*

a) $Sx_{2n+1} \in Ax_{2n}$, $Tx_{2n} \in Ax_{2n-1}$, *for every* $n \in \mathbb{N}$ *and* $z = \lim_{n\to\infty} Tx_{2n} = \lim_{n\to\infty} Sx_{2n+1}$.

b) $Tz \in Az$, $Sz \in Az$.

THEOREM 2. *Let* (X,d) *be a complete convex metric space,* $x_o \in X$, S *and* T *continuous* (W, x_o)*- convex mappings from* X *into* X, $A: X \to 2_c^{SX \cap TX}$ *(the family of nonempty, closed subsets of* $SX \cap TX$) *a closed mapping such that* $\overline{AX}$ *is compact and* W *satisfy condition II . Further, let*

$$\bigcup_{z\in Ax} Sz = ASx, \quad \bigcup_{v\in Ax} Tv = ATx, \quad \textit{for every } x\in X.$$

If for every $x,y\in X$:

$$H(Ax,Ay) < d(Sx,Ty)$$

and A *is* (α,S) *or* (α,T) *densifying on* X *then there exists* $z\in X$ *such that* $Sz\in Az$ *and* $Tz\in Az$.

P r o o f. Let us prove that all the conditions of the Theorem C are satisfied for A_n, S and T, for every $n\in \mathbb{N}$, where $\lim_{n\to\infty} k_n = 1$ $(k_n\in(0,1),\ n\in\mathbb{N})$ and $A_n x = \bigcup_{z\in Ax} W(z,x_o,k_n)$ $(x\in X)$.

We have that:

$$A_n Sx = \bigcup_{z\in ASx} W(z,x_o,k_n) = \bigcup_{z\in SAx} W(z,x_o,k_n) =$$

$$= \{W(Sy,x_o,k_n)\,|\,y\in Ax\} = \{S(W(y,x_o,k_n))\,|\,y\in Ax\} = SA_n x$$

and similarly $A_n Tx = TA_n x$ $(n\in\mathbb{N})$ for every $x\in X$.

Further, if $u\in A_n x$ then there exists $y\in Ax$ such that $u = W(y,x_o,k_n)$. Since $Ax\subseteq SX\cap TX$ it follows that there exists $z_y\in X$ so that $y = Sz_y$ which implies that:

$$u = W(y,x_o,k_n) = W(Sz_y,x_o,k_n) = S(W(z_y,x_o,k_n)) .$$

Thus we have that $A_m X\subseteq SX\cap TX$, for every $m\in\mathbb{N}$. Since Ax is closed and $\overline{AX}$ is compact, from the continuity of the mapping W in the first variable we have that $A_n x$ is compact (for every $n\in\mathbb{N}$) and so $A_n x\in CB(SX\cap TX)$. Let us prove that the mapping A_n is closed for every $n\in\mathbb{N}$. Let $y_n\in A_m x_n$ $(n\in\mathbb{N})$, $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$. From $A_m x_n = \bigcup_{z\in Ax_n} W(z,x_o,k_m)$ it follows that $y_n = W(z_n, x_o,k_m)$, where $z_n\in Ax_n$, $n\in\mathbb{N}$. Since $\overline{AX}$ is compact it follows that there exists a convergent subsequence $\{z_{n_k}\}_{k\in\mathbb{N}}$ and let $\lim_{k\to\infty} z_{n_k} = z$. From the closedness of the mapping A we obtain that $y=W(z,x_o,k_m)$, where $z\in Ax$. Hence $y\in A_m x$ which means that the mapping A_m is closed. From :

$$H(A_m x,A_m y) = H\Big(\bigcup_{z\in Ax} W(z,x_o,k_m),\ \bigcup_{z\in Ay} W(z,x_o,k_m)\Big)$$

$$< k_m H(Ax,Ay) < k_m d(Sx,Ty), \quad \text{for every } x,y\in X$$

it follows that all the conditions of the Theorem C are satisfied. and so, for every $m \in \mathbb{N}$ there exists $x_m \in X$ such that $Sx_m \in A_m x_m$ and $Tx_m \in A_m x_m$. Let $Sx_m = y_m = W(u_m, x_o, k_m)$, $Tx_m = z_m = W(v_m, x_o, k_m)$, where $u_m \in Ax_m$ and $v_m \in Ax_m$.

Then:

$$d(Sx_m, u_m) = d(W(u_m, x_o, k_m), u_m) < (1-k_m)d(u_m, x_o)$$

and since $\{u_m | m \in \mathbb{N}\} \subseteq AX$ we obtain that $\lim_{m\to\infty} d(Sx_m, u_m) = 0$.

Similarly we can prove that $\lim_{m\to\infty} d(Tx_m, v_m) = 0$ and let $L = \{x_n | n \in \mathbb{N}\}$. Suppose that A is (α, S)-densifying. From $\lim_{m\to\infty} d(Sx_m, u_m) = 0$ it follows that:

(5) $\quad \alpha(SL) < \alpha(\{u_m | m \in \mathbb{N}\})$

and since $\{u_m | m \in \mathbb{N}\} \subseteq AL$, using (5) we conclude that:

(6) $\quad \alpha(SL) < \alpha(AL)$.

The relation (6) implies that the set $\{x_n | n \in \mathbb{N}\}$ is relatively compact, since the mapping A is (α, S)-densifying. Suppose that $\lim_{k\to\infty} x_{n_k} = x$. Then from the continuity of the mapping S we obtain that $\lim_{k\to\infty} u_{n_k} = Sx$. Since $u_{n_k} \in Ax_{n_k}$ $(k \in \mathbb{N})$ and the mapping A is closed we conclude that $Sx \in Ax$. Similarly, from $\lim_{k\to\infty} v_{n_k} = Tx$ and $v_{n_k} \in Ax_{n_k}$ $(k \in \mathbb{N})$ it follows that $Tx \in Ax$. (*)

3. A COMMON FIXED POINT THEOREM IN PROBABILISTIC METRIC SPACES WITH A CONVEX STRUCTURE

First, let us give some notations and definitions from the theory of probabilistic metric spaces. Some fixed point theorems in probabilistic metric spaces are proved in [5], [6] and [12].

A triplet $(S, \mathcal{F}, t)$ is a Menger space if and only if S is a nonempty set, $\mathcal{F}: S\times S \to \Delta$, where Δ denotes the set of all distribution functions F, and t is a T-norm [11] so that the following conditions are satisfied ($\mathcal{F}(p,q) = F_{p,q}$, for every $p, q \in S$):

(*) From the proof it is obvious that in the case of a normed space X it is enough to suppose that for every $x \in X$, Ax is a closed and bounded subset of $SX \cap TX$, instead of the compactness of $\overline{AX}$.

1. $F_{p,q}(x) = 1$, for every $x \in \mathbb{R}^+$ if and only if $p = q$.
2. $F_{p,q}(0) = 0$, for every $p,q \in S$.
3. $F_{p,q} = F_{q,p}$, for every $p,q \in S$.
4. $F_{p,r}(u+v) \geq t(F_{p,q}(u), F_{q,r}(v))$, for every $p,q,r \in S$ and every $u,v \in \mathbb{R}^+$.

The (ε,λ)-topology is introduced by the (ε,λ)-neighbourhoods of $v \in S$:

$$U_v(\varepsilon,\lambda) = \{u \mid u \in S, F_{u,v}(\varepsilon) > 1-\lambda\}, \quad \varepsilon > 0, \quad \lambda \in (0,1) .$$

DEFINITION 5. *Let* (S,F,t) *be a Menger space. A mapping* $W: S\times S\times[0,1] \to S$ *is said to be a convex structure if for every* $(u,x,y,\lambda) \in S\times S\times S\times(0,1)$:

$$F_{u,W(x,y,\lambda)}(2\varepsilon) \geq t(F_{u,x}(\tfrac{\varepsilon}{\lambda}), F_{u,y}(\tfrac{\varepsilon}{1-\lambda})), \quad \textit{for every}$$

$\varepsilon \in \mathbb{R}^+$ *and* $W(x,y,0) = y$, $W(x,y,1) = x$.

It is well known that a Menger space is a probabilistic metric space. Every random normed space [13] is a probabilistic metric space with a convex structure $W(x,y,\lambda) = \lambda x + (1-\lambda)y$ since for every $\varepsilon > 0$:

$$F_{u,W(x,y,\lambda)}(2\varepsilon) = F_{u-\lambda x-(1-\lambda)y}(2\varepsilon) = F_{\lambda u+(1-\lambda)u-\lambda x-(1-\lambda)y}(2\varepsilon) =$$

$$= F_{\lambda(u-x)+(1-\lambda)(u-y)}(2\varepsilon) \geq t(F_{\lambda(u-x)}(\varepsilon), F_{(1-\lambda)(u-y)}(\varepsilon)) =$$

$$= t(F_{u-x}(\tfrac{\varepsilon}{\lambda}), F_{u-y}(\tfrac{\varepsilon}{1-\lambda})), \quad \text{for every } \lambda \in (0,1) .$$

The following definition is a generalization of Definition 2.

DEFINITION 6. *A probabilistic metric space* (S,F,t) *with a convex structure* W *satisfies condition* PII *if for all* $(x,y,z,\lambda) \in S\times S\times S\times(0,1)$:

$$F_{W(x,z,\lambda),W(y,z,\lambda)}(\lambda\varepsilon) \geq F_{x,y}(\varepsilon) \quad , \quad \textit{for every } \varepsilon \in \mathbb{R}^+.$$

It is easy to see that every random normed space satisfies condition PII .

If (S,F,t) is a Menger space with a convex structure W and $x_o \in S$, we say that $T:S \to S$ is (W,x_o) - convex if, as in Definition 3, $W(Tz,x_o,\lambda) = T(W(z,x_o,\lambda))$ for every $\lambda \in (0,1)$ and every $z \in S$. In the next Theorem we suppose that condition PII is satisfied.

THEOREM 3. *Let (X,F,t) be a probabilistic metric space with a convex structure W and continuous T norm t, A and S continuous, commutative mappings from X into X such that AX is probabilistic bounded subset of SX, $x_o \in X$ and S (W,x_o) -convex so that:*

$$F_{Ax,Ay}(\varepsilon) \geq F_{Sx,Sy}(\varepsilon) \quad , \text{for every } x,y \in X \text{ and every } \varepsilon \in \mathbb{R}^+$$

If there exists a compact set $M \subseteq X$ such that SM is an attractor for A, then there exists $z \in X$ such that $Az = Sz$.

P r o o f. Let $k_n \in (0,1)$ $(n \in \mathbb{N})$, $\lim_{n\to\infty} k_n = 1$ and for every $x \in X$, $A_n x = W(Ax,x_o,k_n)$ $(n \in \mathbb{N})$. Similarly as in Theorem 1 it follows that all the conditions of Theorem 2 from [5] are satisfied for A_n, S and $I_X = T$ and so there exists $x_n \in X$ such that $x_n = A_n x_n = Sx_n$, for every $n \in \mathbb{N}$. Further, we have for every $n \in \mathbb{N}$ and every $\varepsilon > 0$ that:

$$F_{x_n,Ax_n}(\varepsilon) = F_{A_n x_n,Ax_n}(\varepsilon) = F_{W(Ax_n,x_o,k_n),Ax_n}(\varepsilon) \geq$$

$$\geq t\left(F_{Ax_n,Ax_n}\left(\frac{\varepsilon}{2k_n}\right),F_{Ax_n,x_o}\left(\frac{\varepsilon}{2(1-k_n)}\right)\right) = t\left(1,F_{Ax_n,x_o}\left(\frac{\varepsilon}{2(1-k_n)}\right)\right) =$$

$$= F_{Ax_n,x_o}\left(\frac{\varepsilon}{2(1-k_n)}\right) .$$

Since $\overline{\bigcup_{m\in\mathbb{N}} A^m(x_n)} \cap SM \neq \emptyset$ for every $n \in \mathbb{N}$, it follows that there exists $y_n \in M$ $(n \in \mathbb{N})$ such that $Sy_n \in \overline{\bigcup_{m\in\mathbb{N}} A^m(x_n)}$. Let us prove that for every $\varepsilon > 0$: $\lim_{n\to\infty} F_{Sy_n,Ay_n}(\varepsilon) = 1$. Let $\varepsilon > 0$ and $\lambda \in (0,1)$. We have to prove that there exists $n(\varepsilon,\lambda) \in \mathbb{N}$ so that

$$F_{Sy_n,Ay_n}(\varepsilon) > 1 - \lambda \text{ , for every } n \geq n(\varepsilon,\lambda) .$$

As in [5] it can be proved that for every n and every k from $\mathbb{N}$:

$$F_{A^k x_n,A^{k+1}x_n}(\varepsilon) \geq F_{x_n,Ax_n}(\varepsilon), \quad \text{for every} \quad \varepsilon > 0.$$

Since t is continuous there exists $\eta\in(0,1)$ so that

$$x,y,z \geq \eta \Rightarrow t(x,t(y,z)) > 1-\lambda .$$

Further, for every $\varepsilon>0$ and every $k\in\mathbb{N}$:

$$F_{Sy_n,Ay_n}(\varepsilon) \geq t(F_{Sy_n,A^kx_n}(\tfrac{\varepsilon}{3}), t(F_{A^kx_n,A^{k+1}x_n}(\tfrac{\varepsilon}{3}), F_{A^{k+1}x_n,Ay_n}(\tfrac{\varepsilon}{3}))) \geq$$

$$\geq t(F_{Sy_n,A^kx_n}(\tfrac{\varepsilon}{3}), t(F_{Ax_n,x_o}(\tfrac{\varepsilon}{6(1-k_n)}), F_{A^kx_n,Sy_n}(\tfrac{\varepsilon}{3}))) .$$

Since $Sy_n\in\overline{\bigcup_{m\in\mathbb{N}} A^m x_n}$, for every $n\in\mathbb{N}$ there exists $m_n\in\mathbb{N}$ such that :

$$F_{Sy_n,A^{m_n}x_n}(\tfrac{\varepsilon}{3}) > \eta \quad ,\text{for every } n\in\mathbb{N} .$$

Further, the set AX is probabilistic bounded, which means that $\sup_x D_{AX}(x) = 1$, where :

$$D_{AX}(x) = \sup_{t<x} \inf_{p,q\in AX} F_{p,q}(t) = 1 .$$

Let us prove that there exists $n_o(\varepsilon,\eta)\in\mathbb{N}$ such that

(7) $$F_{Ax_n,x_o}(\tfrac{\varepsilon}{6(1-k_n)}) > \eta \quad ,\text{for every } n\geq n_o(\varepsilon,\eta) .$$

For every $z\in X$ we have that :

$$F_{Ax_n,x_o}(\tfrac{\varepsilon}{6(1-k_n)}) \geq t(F_{Ax_n,Az}(\tfrac{\varepsilon}{12(1-k_n)}), F_{Az,x_o}(\tfrac{\varepsilon}{12(1-k_n)}))$$

and since $\lim_{n\to\infty} k_n = 1$ and AX is probabilistic bounded it follows that there exists $n_o(\varepsilon,\eta)\in\mathbb{N}$ so that (7) holds . Then from :

$$F_{Sy_n,Ay_n}(\varepsilon) \geq t(F_{Sy_n,A^{m_n}x_n}(\tfrac{\varepsilon}{3}), t(F_{Ax_n,x_o}(\tfrac{\varepsilon}{6(1-k_n)}), F_{A^{m_n}x_n,Sy_n}(\tfrac{\varepsilon}{3}))) \geq t(\eta,t(\eta,\eta)) \quad ,\text{for every } n\geq n_o(\varepsilon,\eta)$$

we obtain that :

$$F_{Sy_n,Ay_n}(\varepsilon) > 1-\lambda \; , \text{ for every } n\geq n_o(\varepsilon,\eta)$$

and so :

$$\lim_{n\to\infty} F_{Sy_n,Ay_n}(\varepsilon) = 1 \quad ,\text{for every } \varepsilon>0 .$$

Since $\{y_n\}_{n\in\mathbb{N}} \subseteq M$ and the set M is compact there exists a convergent subsequence $\{y_{n_k}\}_{k\in\mathbb{N}}$ and let $\lim_{k\to\infty} y_{n_k} = z$. We shall prove that $Az = Sz$. This follows from the inequality:

$$F_{Az,Sz}(\varepsilon) \geq t(F_{Az,Ay_{n_k}}(\tfrac{\varepsilon}{3}),t(F_{Ay_{n_k},Sy_{n_k}}(\tfrac{\varepsilon}{3}),F_{Sy_{n_k},Sz}(\tfrac{\varepsilon}{3})))$$

$(k\in\mathbb{N}, \varepsilon > 0)$ since A and S are continuous and $t(1,t(1,1)) = 1$.

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Received by the editors March ,1985.

REZIME

O TAČKAMA KOINCIDENCIJE U METRIČKIM I VEROVATNOSNIM METRIČKIM PROSTORIMA SA KONVEKSNOM STRUKTUROM

Pojam konveksnosti u metričkim prostorima je uveo Takahashi u [14] i neke teoreme o nepokretnoj tački u konveksnim metričkim prostorima su dokazane u [8],[9], [14].

U ovom radu su dokazane teoreme o postojanju tačke koincidencije u metričkim i verovatnosnim metričkim prostorima sa konveksnom strukturom.