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ON ONE SPECIAL TWO-DIMENSIONAL FINSLER-OTSUKI SPACE

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ABSTRACT

In this paper we shall consider thoroughly the Finsler--Otsuki $F\text{-}O_2$ space whose theory for the n-dimension is based on [3] (§1 and §2). The intention of our work is the calculation of some basic formulae of the two-dimensional Finsler-Otsuki spaces. Using these formulae and the ideas of the determination of the coefficients of connection $'\Gamma_j{}^i{}_k$ and $''\Gamma_j{}^i{}_k$ from [3], we shall give the explicit formulae of $'\Gamma_j{}^i{}_k$ and $''\Gamma_j{}^i{}_k$ in $F\text{-}O_2$ spaces (Cf. (2.13) and (2.16)) and we shall prove that in the observed space, $''\Gamma_j{}^i{}_k$ are of Cartain's type.

PRELIMINARIES

The fundamental manifold of an $F\text{-}O_n$ space is the manifold of line-elements $(x,\dot{x})$ with metric function $F(x,\dot{x})$ and with tensor $P^i_j(x,\dot{x})$ defined over it. We shall suppose that the metric tensor $g_{ij}(x,\dot{x})$ is defined by the relation

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$$g_{ij}(x,\dot{x}) := \frac{1}{2}\,\frac{\partial^2 F(x,\dot{x})}{\partial \dot{x}^i\,\partial \dot{x}^j} \tag{0.1}$$

with $det\,(g_{ij}) \neq 0$, i. e. $F(x,\dot{x})$ defines the Finsler metric. For the tensor $P^{\,i}_{j}$ we shall suppose that it has the inverse tensor $Q^{\,t}_{i}$, such that

$$P^{\,i}_{j}\, Q^{\,t}_{i} = \delta^{\,t}_{j} \tag{0.2}$$

holds. The invariant differential is defined by the relation

$$D\, T^{\,i}_{j} := P^{\,i}_{a}\, P^{\,b}_{j}\, \bar{D}\, T^{\,a}_{b} \tag{0.3}$$

and the definition of the basic covariant differential $\bar{D}$ is

$$\bar{D}\, T^{\,a}_{b} := \overset{*}{\nabla}_k\, T^{\,a}_{b}\, \bar{\omega}^k(d) + \overset{\circ}{\nabla}_k\, T^{\,a}_{b}\, dx^k \tag{0.4}$$

where

$$\bar{\omega}^k(d) := \bar{D}\, \ell^k, \tag{0.5}$$

$$\overset{*}{\nabla}_k\, T^{\,a}_{b} := T^{\,a}_{b\parallel k} + A_{r\,\,k}^{\,\,a}\, T^{\,r}_{b} - A_{b\,\,k}^{\,\,r}\, T^{\,a}_{r}, \tag{0.6}$$

$$\overset{\circ}{\nabla}_k\, T^{\,a}_{b} := \partial_k\, T^{\,a}_{b} - T^{\,a}_{b\parallel r}\, {}'\Gamma_{o\,\,k}^{\,\,r} + {}'\Gamma_{r\,\,k}^{\,\,a}\, T^{\,r}_{b} - {}''\Gamma_{b\,\,k}^{\,\,r}\, T^{\,a}_{r} \tag{0.7}$$

and $A_{r\,\,k}^{\,\,a}$ is the torsion tensor of the Finslerian metric, ${}'\Gamma_{r\,\,k}^{\,\,a}$ and ${}''\Gamma_{r\,\,k}^{\,\,a}$ are the connection parameters of the F-0_n spaces (see [3] (2.15) - (2.23)). From (0.7) it obviously follows that in our space the Leibnitz formula for the covariant derivative $\overset{\circ}{\nabla}_k$ of mixed tensors does not hold.

In [3], ${}'\Gamma_{i\,\,k}^{\,\,j}$ and ${}''\Gamma_{i\,\,k}^{\,\,j}$ are constructed so that

$$\overset{*}{\nabla}_k\, g_{ij} = \overset{\circ}{\nabla}_k\, g_{ij} = 0 \tag{0.8}$$

and

(0.9) $$''\Gamma_{i\ k}^{\ j} = ''\Gamma_{k\ i}^{\ j}$$

hold. The coefficients of connection $'\Gamma_{i\ k}^{\ j}$ and $''\Gamma_{i\ k}^{\ j}$ and the tensor $P^i_j(x,\dot{x})$ satisfy the relation analogous to Otsuki's relations [4] (3.13) which is

(0.10) $$\partial_k P^i_j - P^i_{j\|r}\ '\Gamma_{o\ k}^{\ r} - '\Gamma_{j\ k}^{\ s} P^i_s + ''\Gamma_{s\ k}^{\ i} P^s_j = 0.$$

In [3], it has been proved that by the above conditions A_{ijk} is totally symmetric and

(0.11) a) $A_{ijk} = \frac{1}{2} g_{ij\|k}$, b) $A_{iok} = A_{iko} = A_{oik} = 0.$

From (0.10), it is possible to get that

(0.12) $$'\Gamma_{o\ k}^{\ s}(P^i_s + \ell^b P^i_{b\|s}) = \ell^b \partial_k P^i_b + ''\Gamma_{s\ k}^{\ i} P^s_o$$

where, as in Finsler geometry, the index "o" denotes the contraction by ℓ^i where $\ell^i = \dot{x}^i/F$. In [3], it was supposed that there exists tensor $\overset{*}{Q}{}^t_i$ satisfying the relation

(0.13) $$(P^i_s + \ell^b P^i_{b\|s})\overset{*}{Q}{}^t_i = \delta^t_s,$$

which makes possible the definition of $'\Gamma_{o\ k}^{\ j}$, and

(0.14) $$''\Gamma_{ijk} = [ijk]_g - \{A_{ijt}\ '\Gamma_{o\ k}^{\ t} + A_{jkt}\ '\Gamma_{o\ i}^{\ t} - A_{jkt}\ '\Gamma_{o\ j}^{\ t}\}.$$

It is known, that

(0.15) $$\overset{*}{\Delta}_k \ell^i = \delta^i_k - \ell^i \ell_k$$

and it has been proved in [3], 2, that:

(0.16) $$\overset{\bullet}{\nabla}_k \ell^i = \ell^i('\Gamma_{ook} - ''\Gamma_{ook}).$$

If the observed space is two-dimensional, then it is known that beside vector ℓ^i we have the unit vector h^i, orthogonal to it. In the following we shall use the connections between vectors ℓ^i and h^i which were laid down by L. Bervald in [1]. Now we shall give some important formulae of 2-dimensional Finsler spaces. They are

$$\delta^i_j = \ell^i \ell_j + h^i h_j \tag{0.17}$$

$$\begin{cases} g_{ij} = \ell_i \ell_j + h_i h_j \; ; \quad g^{ij} = \ell^i \ell^j + h^i h^j \\ \varepsilon^{ij} = \ell^i h^j - h^i \ell^j \,. \end{cases} \tag{0.18}$$

From (0.18) one can see, obviously, that

$$\begin{array}{llll} \text{a)} & h^i = - \varepsilon^{ir} \ell_r & \text{b)} & h_i = - \varepsilon_{ir} \ell^r \\ \text{c)} & \ell^i = \varepsilon^{ir} h_r & \text{d)} & \ell_i = \varepsilon_{ir} h^r \end{array} \tag{0.19}$$

(see [1] (4.1) and (4.4)).

§ 1. BASIC FORMULAE OF ℓ^i AND h^i

It is necessary to determine some formulae according to which it is possible to compare simple Finsler and Finsler-Otsuki spaces. At first it is not difficult to see that

$$\overset{\bullet}{\nabla}_k \ell^i = 0. \tag{1.1}$$

From (0.1) and definition (0.7) applied on F^2, using (0.11) and (0.8), we get

$$2F^{-1} \overset{\circ}{\nabla}_k F = (\partial_k g_{ij})\ell^i \ell^j - 2 \,{}'\Gamma_{ook} \,. \tag{1.2}$$

In [3], it has been proved that

$$(\partial_k g_{ij})\ell^i \ell^j = 2 \,{}''\Gamma_{ook} \,. \tag{1.3}$$

From (1.2) and (1.3), we get

$$\overset{\circ}{\nabla}_k F = -F({}'\Gamma_{ook} - {}''\Gamma_{ook}). \tag{1.4}$$

Now it is possible to formulate

LEMMA 1. *$\overset{\circ}{\nabla}_k F$ vanishes iff ${}'\Gamma_{ook} = {}''\Gamma_{ook}$.*

Now we shall apply the formula (0.7) on ℓ_j. Using the relation $\ell_j = g_{ij}\ell^i$, the Leibniz formula for partial derivatives and (0.11), we get

$$\overset{\circ}{\nabla}_k \ell_j = (\ell_j \ell^r - \delta_j^r)({}'\Gamma_{ork} - {}''\Gamma_{ork}). \tag{1.5}$$

The above formulae hold in n-dimensional spaces. In the following, we shall suppose that the observed space is 2-dimensional. Substituting (0.17) in (1.5), we get that in a 2-dimensional Finsler-Otsuki space

$$\overset{\circ}{\nabla}_k \ell_j = -({}'\Gamma_{ork} - {}''\Gamma_{ork})h^r h_j, \tag{1.6}$$

and there holds

THEOREM 1. *In an F-O_2 space $\overset{\circ}{\nabla}_k \ell_j$ vanishes iff* ${}'\Gamma_{ork}h^r = {}''\Gamma_{ork}h^r$.

Very useful are the formulae holding for the ε-tensor. It is known that

$$\varepsilon^{ij} = \begin{pmatrix} 0 & 1/\sqrt{g} \\ -1/\sqrt{g} & 0 \end{pmatrix}. \tag{1.7}$$

Using the definition of the operation denoted by "$''$", the simple quality of the partial derivative and the usual notation [2] (P. 67)

(1.8) $$A_s := \frac{1}{2} F \frac{\partial \ln g}{\partial \dot{x}^s} \equiv A_s{}^t{}_t,$$

we get

(1.9) $$\varepsilon^{ij}{}_{\| s} = - \varepsilon^{ij} A_s.$$

According to the relation $\varepsilon^{ij}{}_{\| s}\, \ell^s = 0$, we can write

(1.10) $$A_s := \sqrt{A}\, h_s.$$

Substituting (1.10) in (1.9), we get

(1.11) $$\varepsilon^{ij}{}_{\| s} = - \varepsilon^{ij} \sqrt{A}\, h_s.$$

From (1.7), using relations (0.8), (0.9) and (1.10), it follows that

(1.12) $$\partial_k \varepsilon^{ir} = - \varepsilon^{ir}(\sqrt{A}\, h_t\, {}'\Gamma_o{}^t{}_k + {}''\Gamma_{ook} + h^a h^b\, {}''\Gamma_{abk}).$$

Using the connections (0.19.a) and (0.19.c) between the vectors ℓ^i and h^i and (0.17), we have that

$$h^i{}_{\| s} = - (\varepsilon^{ik})_{\| s}\, \ell_k - \ell^i h_s.$$

Substituting (1.11), we get

(1.13) $$h^i{}_{\| s} = - (\sqrt{A}\, h^i + \ell^i) h_s.$$

From $h_i = g_{ij} h^j$, (0.11) and (1.13), it follows that

(1.14) $$h_{i \| s} = 2A_{ijs} h^j - (\sqrt{A}\, h_i + \ell_i) h_s.$$

Using the fact that we can apply the Leibniz formula on the partial derivatives, from (0.19.a) according to relations (1.12), (0.18), (0.8) and (0.17), it follows that

(1.15) $$\partial_k h^i = -(\sqrt{A}\, h_s {}'\Gamma_o{}^s{}_k h^i + h^a {}''\Gamma_a{}^i{}_k + \ell^i h^b {}''\Gamma_{obk}).$$

Finally, we shall calculate the covariant derivative $\overset{\circ}{\nabla}_k h^i$. Using relation (0.19.a) and the definitions (0.7), (0.17) and (0.18), according to results (1.15) and (1.13), we get

(1.16) $$\overset{\circ}{\nabla}_k h^i = (\ell^i \ell^s h_t + \delta^i_t h^s)\overset{\circ}{\nabla}_k \delta^t_s .$$

THEOREM 2. *In the observed space* $\overset{\circ}{\nabla}_k h^i = 0$ *iff* $\overset{\circ}{\nabla}_k \delta^t_s = 0$.

PROOF. Condition $\overset{\circ}{\nabla}_k \delta^t_s = 0$ is obviously sufficient. To prove that it is necessary we shall suppose that $\overset{\circ}{\nabla}_k h^i = 0$. Contracting this relation by ℓ_i, according to (1.16), we get

$$(\ell^s h_t + \ell_t h^s)\overset{\circ}{\nabla}_k \delta^t_s = 0$$

Since $(\ell^s h_t + \ell_t h^s)(\ell^t h_i + \ell_i h^t) = \delta^t_i$ (cf. (0.17)), it follows that $\overset{\circ}{\nabla}_k \delta^t_s = 0$. The theorem is proved.

In an exactly F-O_2 space there is $\overset{\circ}{\nabla}_k \delta^t_s \neq 0$, because in case $\overset{\circ}{\nabla}_k \delta^t_s = 0$ we have

$$\overset{\circ}{\nabla}_k \delta^t_s = {}'\Gamma_s{}^t{}_k - {}''\Gamma_s{}^t{}_k = 0$$

and so the F-O_2 space will be a 2-dimensional ordinary Finsler space (cf. [1]).

§ 2. ONE SPECIAL 2-DIMENSIONAL FINSLER-OTSUKI SPACE

In this paragraph we shall observe a special F-O_2. The idea of F-O_n spaces comes from A. Moór. In [3], he considered, among others, a special case characterised by the relation $P^i_o = \ell^i$. Here we shall observe the 2-dimensional Finsler-Otsuki space. The basic tensor of it, by our supposition, satisfies the relation

(2.1) $$P^i_o = h^i .$$

From this it obviously follows that

$$(2.2)\qquad P^i_j = \ell^i h_j + \ell_j h^i + \alpha h^i h_j, \quad \alpha = \alpha(x,\dot{x})$$

is the most general form of P^i_j, if (2.1) holds, and P_{ij} is symmetric in i,j. It is not difficult to prove

LEMMA 2. *From (2.2) it follows that*

$$(2.3)\qquad Q^i_j = \ell^i h_j + \ell_j h^i - \alpha\ell^i\ell_j$$

and

$$Q^i_o = h^i - \alpha\ell^i .$$

PROOF. From (2.2) and (2.3), it follows that $P^i_j Q^j_k = \delta^i_k$, i.e. (2.3) is the inverse tensor of (2.2). Contracting (2.3) by ℓ^j, we get Q^i_o. So the theorem is proved.

In the following, in the observed special $F\text{-}O_2$, we shall calculate the tensor $\overset{*}{Q}_i$ defined by (0.13) and the coefficients of connection $''\Gamma_{ijk}$ from (0.14). Now it is necessary to calculate some expressions. At first, from (2.1), it follows that $P^i_{o\|s} = h^i{}_{\|s}$ and

$$P^i_{b\|s}\ell^b = h^i{}_{\|s} - P^i_b(\ell^b{}_{\|s}).$$

Substituting (1.13) and (2.2), we get

$$(2.4)\qquad P^i_{b\|s}\ell^b = - h_s[(\sqrt{A} + \alpha)h^i + 2\ell^i].$$

In an analogous way, from (2.1) it follows that

$$(\partial_k P^i_a)\ell^a = \partial_k h^i - P^i_a(\partial_k \ell^a).$$

Substituting (1.15) and (2.2), using that according to (0.7), (0.8) and (0.11.b)

$$\partial_k \ell^a = \dot{x}^a \partial_k (g_{ij}\dot{x}^i\dot{x}^j)^{-1/2} = -\tfrac{1}{2}(g_{ij\|t}{}'\Gamma_o{}^t{}_k + {}'\Gamma_{ijk} +$$

$$+ {}''\Gamma_{jik})\ell^i\ell^j\ell^a = -{}''\Gamma_{ook}\ell^a,$$

we get

$$(2.5) \qquad (\partial_k P^i_a)\ell^a = -\sqrt{A}h_t{}'\Gamma_o{}^t{}_k h^i - h^r{}''\Gamma_r{}^i{}_k + (\ell^r h^i - \ell^i h^r){}''\Gamma_{ork}.$$

Now we can determine the tensor $\overset{*}{Q}{}^t_i$ from relation (0.13). Substituting (2.4) and (2.2) in (0.13), we get

$$(2.6) \qquad (h^i\ell_s - \ell^i h_s - \sqrt{A}h^i h_s)\overset{*}{Q}{}^t_i = \delta^t_s .$$

A contraction by ℓ^s gives

$$h^i \overset{*}{Q}{}^t_i = \ell^t,$$

and it follows that it must be $\overset{*}{Q}{}^t_i = \varepsilon^t_i + \varphi^t_i$, where $\varphi^t_i h^i = 0$. It is not difficult to see that $\varphi^t_i = (\alpha\ell^t + \beta h^t)\ell_i$, i.e. according to (0.18),

$$\overset{*}{Q}{}^t_i = \ell^t h_i - \ell_i h^t + (\alpha\ell^t + \beta h^t)\ell_i .$$

Substituting in (2.6), we get that $\alpha = -\sqrt{A}$; $\beta = 0$ and after all

$$(2.7) \qquad \overset{*}{Q}{}^t_i = \ell^t h_i - \ell_i h^t - \sqrt{A}\,\ell^t\ell_i .$$

Now we shall observe (0.12). Substituting (2.5),(2.2) and contracting by $\overset{*}{Q}{}^t_i$ according to (0.17), we get

$${}'\Gamma_o{}^t{}_k = {}''\Gamma_o{}^t{}_k - \sqrt{A}\,\ell^t h_r({}'\Gamma_o{}^r{}_k - {}''\Gamma_o{}^r{}_k),$$

or

$$({}'\Gamma_o{}^r{}_k - {}''\Gamma_o{}^r{}_k)(\delta^t_r + \sqrt{A}\,\ell^t h_r) = 0 .$$

Hence there holds

THEOREM 3. *From (2.1) it follows that*

$$ {}'\Gamma_{o\ k}^{\ r} = {}''\Gamma_{o\ k}^{\ r} \tag{2.8}$$

PROOF. Since the tensor $\delta_t^i - \sqrt{A}\,\ell^i h_t$ is the inverse of $\delta_t^i + \sqrt{A}\,\ell^t h_r$, the statement of the theorem follows after a contraction with $\delta_t^i - \sqrt{A}\,\ell^i h_t$. This theorem guarantees that the observed F-O_2 space has an analogous structure as the special F-O_n space in [3]] characterised by $P_o^i = \ell^i$ (cf. [3] Satz 1, especially [3] (2.21)).

COROLLARY 1. *In the observed* F-O_2 *spaces*

$$ \mathring{\nabla}_k \ell^i = 0; \quad \mathring{\nabla}_k F = 0; \quad \mathring{\nabla}_k \ell_i = 0; \tag{2.9}$$

$$ \mathring{\nabla}_k h^i = h^r({}'\Gamma_{r\ k}^{\ i} - {}''\Gamma_{r\ k}^{\ i}) \tag{2.10}$$

hold.

Using (2.8), it is possible to determine the coefficients of connection ${}''\Gamma_{ijk}$. Substituting (2.8) in (0.14) according to condition (0.11.b), we get

$$ {}''\Gamma_{ijk} = [ijk]_g - \{A_{ijt}\,{}''\Gamma_{o\ k}^{\ t} + \{ijk\}\}.^{*} \tag{2.11}$$

From this relation we can see

COROLLARY 2. *The coefficients of connection* ${}''\Gamma_{ijk}$ *are of Cartain's kind and do not depend on* P, *respectively (cf. (2.2)).*

Contracting (2.11) by $g^{js}\ell^i$ and $g^{js}\ell^i\ell^k$, respectively, we get

$$ {}''\Gamma_{o\ k}^{\ s} = \{{}_{o\ k}^{\ s}\} - g^{js} A_{jkt}\{{}_{o\ o}^{\ t}\}\ ; \quad {}''\Gamma_{o\ o}^{\ t} = \{{}_{o\ o}^{\ t}\}\ . \tag{2.12}$$

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* *{ijk} denotes the cyclic permutation of the indices with the change of the sign in the last part.*

Substituting in (2.11), finally we get

$$(2.13)\quad {}''\Gamma_{ijk} = [ijk]_g - \{A_{ijt}\{{}_o{}^t{}_k\} - A_{ijt}g^{tr}A_{rku}\{{}_o{}^u{}_o\} + \{ijk\}\}.$$

We can determine the coefficients of connection ${}'\Gamma_i{}^j{}_k$ from relation [3] (2.22), i.e. from

$$(2.14)\qquad {}'\Gamma_i{}^j{}_k = {}''\Gamma_i{}^j{}_k + Q^j_s\,{}''\nabla_k P^s_i,$$

since in our space (2.8) holds. At first we shall calculate ${}''\nabla_k P^s_i$. According to [3], Theorem 1., the fact that from [3] (2.14) and [3] (2.15), it follows that ${}''\Gamma_j{}^i{}_k = \Gamma^{*i}_j{}_k$, since (2.8) holds, we get

$${}''\nabla_k P^s_i = \partial_k P^s_i - (\dot\partial_t P^s_i)\,{}''\Gamma_o{}^t{}_k - {}''\Gamma_i{}^t{}_k P^s_t + {}''\Gamma_t{}^s{}_k P^t_i$$

where "$_{|k}$" denote the Cartan-covariant derivative. Substituting P^s_i from (2.2) and using the fact that the Leibniz formula for ${}''\nabla_k \equiv {}_{|k}$ holds, according to the relations $\ell^i{}_{|k} = 0$, $h^i{}_{|k} = 0$ (see [1] (2.19) and (4.5)), we get in the observed space

$$(2.15)\qquad {}''\nabla_k P^i_j = ({}''\nabla_k\alpha)h^i h_j = \alpha_{|k}h^i h_j$$

since (2.8) holds.

Contracting (2.15) with Q^j_s and using (2.3), we get

$$Q^s_i\,{}''\nabla_k P^i_j = \ell^s h_j \overset{\circ}{\nabla}_k\alpha \; .$$

Finally there holds

COROLLARY 3. *In the observed* F-O_2 *space there is*

$$(2.16)\qquad {}'\Gamma_i{}^j{}_k = {}''\Gamma_i{}^j{}_k + \ell^j h_i \overset{\circ}{\nabla}_k\alpha$$

Now we can see that in the observed F-O_2 space, ${}'\Gamma_i{}^j{}_k$ depend on the choice of the scalar α.

From (2.16) it is not difficult to see that the coefficients of connection ${}'\Gamma_i{}^j{}_k$ are generally not symmetric at under indices. Substituting ${}''\Gamma_{ijk}$ in (2.15) from (2.13), we get that

$$(2.17)\qquad {}'\Gamma_{ijk} = [ijk]_g - \{A_{ijt}\{{}_o{}^t{}_k\} - A_{ijt}g^{tr}A_{rku}\{{}_o{}^u{}_o\} + + \{ijk\}\} + \ell_j h_i \overset{\circ}{\nabla}_k \alpha \,.$$

With (2.11) and (2.16) the coefficients of connections ${}''\Gamma_i{}^j{}_k$ and ${}'\Gamma_i{}^j{}_k$ of F-O_2 space are determined explicitly. It is interesting question to consider 3-dimensional and n-dimensional Finsler-Otsuki spaces where supposition (2.1) is satisfied.

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REZIME

O JEDNOM SPECIJALNOM DVO-DIMENZIONALNOM FINSLER-OTSUKIJEVOM PROSTORU

U radu su prvo date neke osnovne formule dvodimenzionalnog Finsler-Otsukijevog prostora. Zatim su na osnovu ideje date u [3] eksplicitno (u (2.13) i (2.16)) dati koeficijenti koneksije ${}'\Gamma_j{}^i{}_k$ i ${}''\Gamma_j{}^i{}_k$ jednog specijalnog F - O_2 prostora.