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A FIXED POINT THEOREM FOR MULTIVALUED QUASI-CONTRACTIONS IN PROBABILISTIC METRIC SPACES

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ABSTRACT

In this note, we first generalize Minh's theorem for spaces with a family of pseudo-metrics, next we introduce the notion of the probabilistic largest distance between two sets and establish some of its simple properties, and finally, we prove an analogue to Minh's theorem for PM-spaces.

1. INTRODUCTION

In |1| Ćirić has proved a fixed point theorem for multivalued quasi-contractions, i.e. for mappings satisfying the following condition

$$(1)\quad \rho(Tx,Ty) \le k.\max\{d(x,y),\ \rho(x,Tx),\ \rho(y,Ty),\ d(x,Ty),\ d(y,Tx)\}$$

where $k < 1$, d is a metric defined in X, ρ stands for the largest distance between two sets defined by

$$\rho(A,B) = \sup\{d(x,y):\ x \in A,\ y \in B\}$$

and finally, as usual,

$$d(x,A) = \inf\{d(x,y):\ y \in A\}.$$

Then, in |2| N.A. Minh generalized Ćirić's result by showing that in (1) all the d's can be replaced by ρ .

Note that up to now there have been a lot of fixed point theorems concerning multivalued mappings of the contractive type in metric spaces, but none of their analogues in probabilistic matric spaces (in abbreviation, PM-spaces) has appeared even for the simplest case such as for Nadler's theorem |3|. The main difficulty is that the topology in a PM-space depends on a parameter $\lambda \in (0,1)$, so we cannot choose an iterative sequence independent of λ without additional assumptions. Recently on the way to getting an analogue to Nadler's theorem in PM-spaces Hadžić proved a theorem whose assumptions assure the existence of such a sequence |4|.

2. MULTIVALUED QUASI-CONTRACTIONS IN SPACES WITH A FAMILY OF PSEUDO-METRICS

First let us recall the following definition. Let X be an arbitrary set, a mapping $d : X \times X \to R_+$ (the set of all non-negative numbers) is called a pseudo-metric if it satisfies the following conditions

$$d(x,x) = 0,$$
$$d(x,y) = d(y,x),$$
$$d(x,y) \leq d(x,z) + d(z,y)$$

for every x,y,z in X.

Let Λ be an index set. A pair (X,d_λ), $\lambda \in \Lambda$, is called a space with a family of pseudo-metrics if d_λ is a pseudo-metric for each $\lambda \in \Lambda$. In the sequel we always assume that the family $\{d_\lambda\}$ satisfies the following "separation condition"

$$d_\lambda(x,y) = 0 \quad (\forall\lambda \in \Lambda) \Rightarrow \quad x = y .$$

In |5| such a space is called uniformizable and some fixed point theorems for singlevalued mappings of the contractive type in these spaces are established.

The notions and results in this section mostly repeat that of |2| for each fixed λ, but for convenience to readers we shall develop them here briefly (the proof is partially similar to that of |1|).

DEFINITION 2. *Let Y be an arbitrary set. A mapping $\rho : X \times X \to R_+$ is called a semimetric if it satisfies the following conditions*

$$\rho(x,y) = \rho(y,x),$$
$$\rho(x,y) \le \rho(x,z) + \rho(z,y)$$

for every x,y,z *in* Y.

Let (Y,ρ_λ) be a space with a family of semimetrics $\{\rho_\lambda : \lambda \in \Lambda\}$. A sequence $\{x_n\} \subseteq Y$ is called a Cauchy sequence if $\lim_{n,m\to\infty} \rho_\lambda(x_n,x_m) = 0$ for each λ. A sequence $\{x_n\}$ is said to be convergent to $x \in Y$ (symbolically, $x_n \to x$) if $\lim_{n\to\infty} \rho_\lambda(x_n,x) = 0$ for each λ. The space Y is said to be complete if every Cauchy sequence is convergent. Note that these notions are formal because a semimetric does not generate any topology, this is easily seen when we consider a stationary sequence $x_n \equiv x$ with $\rho(x,x) > 0$.

In what follows we assume that the family of semimetrics $\{\rho_\lambda\}$ has the following property

$$\rho_\lambda(x,y) = 0 \quad (\forall \lambda \in \Lambda) \implies x = y.$$

Obviously each pseudo-metric is a semimetric. The largest distance between two bounded sets in a metric space is a nontrivial example of semimetrics.

LEMMA 1. *Let* (Y,ρ_λ) *be a complete space with a family of semimetrics,* $T : Y \to Y$ *a mapping satisfying the following condition: for each* $\lambda \in \Lambda$ *there is a* $q_\lambda < 1$ *such that*

(2) $$\rho_\lambda(Tx,Ty) \le q_\lambda \cdot \max\{\rho_\lambda(x,y),\ \rho_\lambda(x,Tx),\ \rho_\lambda(y,Ty),\ \rho_\lambda(x,Ty),\rho_\lambda(y,Tx)\}$$

for every x,y *in* Y.

Then there exists a unique fixed point x^* *of* T. *Moreover, we have* $\rho_\lambda(x^*,x^*) = 0$ *for each* $\lambda \in \Lambda$ *and* $T^n x \to x^*$ *for every* x *in* Y.

P r o o f. We shall use the following notations

$$\theta(x,n) = \{T^i x : i = 0,1,\dots,n\}, \text{ where } T^0 x = x,$$

$$\theta(x,\infty) = \{T^i x : i = 0,1,2,\dots\},$$

$$\delta_\lambda(A) = \sup\{\rho_\lambda(x,y) : x,y \in A\}.$$

Fixing $\lambda \in \Lambda$ we have

a) if $1 \le i,j \le n$ then $\rho_\lambda(T^i x, T^j x) \le q_\lambda \delta_\lambda(\theta(x,n))$. From this it follows that $\delta_\lambda(\theta(x,n)) = \rho_\lambda(x, T^k x)$ with some $k \le n$.

b)

$$\delta_\lambda(\theta(x,n)) \le \max\{\frac{\rho_\lambda(x,Tx)}{1-q_\lambda}, \rho_\lambda(x,x)\} .$$

Indeed, fixing n, by a) we have $\delta_\lambda(\theta(x,n)) = \rho_\lambda(x,T^k x)$ with some $k \le n$. If $k \ge 1$ then

$$\delta_\lambda(\theta(x,n)) = \rho_\lambda(x,T^k x) \le \rho_\lambda(x,Tx) + \rho_\lambda(Tx,T^k x) \le$$

$$\le \rho_\lambda(x,Tx) + q_\lambda \delta_\lambda(\theta(x,n)) .$$

From this b) follows.

c)

$$\rho_\lambda(T^n x, T^{n+m} x) \le q_\lambda^n \cdot \max\{\frac{\rho_\lambda(x,Tx)}{1-q_\lambda}, \rho_\lambda(x,x)\} .$$

Indeed, from a) and b) we get

$$\rho_\lambda(T^n x, T^{n+m} x) \le q_\lambda \delta_\lambda(\theta(T^{n-1}x, m+1)) = q_\lambda \rho_\lambda(T^{n-1}x, T^{n-1+m_1}x)$$

$$\le q_\lambda^2 \delta_\lambda(\theta(T^{n-2}x, m_1+1)) = \ldots \le q_\lambda^n \delta_\lambda(\theta(x, m_{n-1}+1)) \le$$

$$\le q_\lambda^n \delta_\lambda(\theta(x,\infty)) \le q_\lambda^n \cdot \max\{\frac{\rho_\lambda(x,Tx)}{1-q_\lambda}, \rho_\lambda(x,x)\} .$$

This implies that $\{T^n x\}$ is a Cauchy sequence and hence $T^n x \to x^*$. Since $\rho_\lambda(x^*,x^*) \le \rho_\lambda(x^*,T^n x) + \rho_\lambda(T^n x, x^*)$, we get $\rho_\lambda(x^*,x^*) = 0$.

d) We have

$$\rho_\lambda(x^*,Tx^*) \le \rho_\lambda(x^*,T^{n+1}x) + \rho_\lambda(T^{n+1}x,Tx^*) \le \rho_\lambda(x^*,T^{n+1}x) +$$

$$+ q_\lambda \max\{\rho_\lambda(T^n x,x^*), \rho_\lambda(T^n x,T^{n+1}x), \rho_\lambda(x^*,Tx^*),$$

$$\rho_\lambda(T^n x,Tx^*), \rho_\lambda(x,T^{n+1}x)\} .$$

Letting $n \to \infty$ we obtain

$$\rho_\lambda(x^*,Tx^*) \le q_\lambda \rho_\lambda(x^*,Tx^*) ,$$

from this $\rho_\lambda(x^*,Tx^*) = 0$ for each λ and hence $x^* = Tx^*$. The uniqueness of x^* is proved in the usual way and the lemma follows.

Let (X,d_λ) be a space with a family pseudo-metrics. By ρ_λ we denote the largest distance between two sets generated by d_λ, $\delta_\lambda(A)=\rho_\lambda(A,A)$ - the λ-diameter of A, B(X) - the class of all non-empty subsets of X with finite λ-diameter for each λ (these subsets are said to be bounded).

LEMMA 2. *If* (X,d_λ) *is a space with a family of pseudo-metrics then* $(B(X),\rho_\lambda)$ *is a space with a family of semimetrics. Moreover, if* X *is complete, so is* B(X).

P r o o f. The symmetry and the triangle inequality of ρ_λ follow immediately from the corresponding properties of d_λ. Further, if $\rho_\lambda(A,B) = 0$ for each λ then we have $d_\lambda(x,y)=0$ for each λ and for every $x\in A$, $y\in B$, it, in turn, implies that $x=y=A=B$. Now, let $\{A_n\}$ be a Cauchy sequence in B(X). Taking $x_n\in A_n$ for each n we obtain a Cauchy sequence $\{x_n\}$ in X. By the completeness of X, $x_n\to x$. Fixing $\lambda\in\Lambda$ we get

$$\rho_\lambda(x,A_n)\le\rho_\lambda(x,x_n)+\rho_\lambda(x_n,A_n)\le d_\lambda(x,x_n)+\rho_\lambda(A_n,A_n) .$$

Letting $n\to\infty$ we get $\lim\limits_{n\to\infty}\rho_\lambda(x,A_n)=0$ for each λ , i.e. by definition $A_n\to\{x\}$ in B(X). So B(X) is complete and the lemma follows.

THEOREM 1. *Let* (X,d_λ) *be a complete space with a family of pseudo-metrics,* $T:X\to B(X)$ *be a mapping satisfying the following condition: for each* $\lambda\in\Lambda$ *there is a* $q_\lambda<1$ *such that*

$$(3)\qquad \rho_\lambda(Tx,Ty)\le q_\lambda\max\{d_\lambda(x,y),\rho_\lambda(x,Tx),\rho_\lambda(y,Ty),\\ \rho_\lambda(x,Ty),\rho_\lambda(y,Tx)\}$$

for every x,y *in* X .

Then T *has a unique fixed point* x*. *Moreover, in fact, it is a stationary point (i.e.* $Tx^*=\{x^*\}$).

P r o o f. First we show that T maps B(X) into B(X). Indeed, let $A\in B(X)$ fix $x_o\in A$ and take an arbitrary x in A. Then we have

$$\rho_\lambda(Tx,Tx_o)\le q_\lambda\max\{d_\lambda(x,x_o),\rho_\lambda(x,Tx),\rho_\lambda(x_o,Tx_o),\\ \rho_\lambda(x,Tx_o),\rho_\lambda(x_o,Tx)\} .$$

Suppose for example, the maximum is attained at $\rho_\lambda(x,Tx)$. Then we have

$$\rho_\lambda(Tx,Tx_o) \le q_\lambda \rho_\lambda(x,Tx) \le q_\lambda \rho_\lambda(x,Tx_o) + q_\lambda \rho_\lambda(Tx,Tx_o) .$$

Consequently

$$\rho_\lambda(Tx,Tx_o) \le \frac{q_\lambda}{1-q_\lambda} \rho_\lambda(A,Tx_o) .$$

Similarly for the case when the maximum is attained at $\rho_\lambda(x_o,Tx)$ and thus we get

$$\rho_\lambda(TA,Tx_o) \le \max\{q_\lambda \delta_\lambda(A), \frac{q_\lambda}{1-q_\lambda} \rho_\lambda(A,Tx_o)\} .$$

From this $TA \in B(X)$.

Obviously, from (3) we have for every A,B in B(X)

$$\rho_\lambda(TA,TB) \le q_\lambda \max\{\rho_\lambda(A,B), \rho_\lambda(A,TA), \rho_\lambda(B,TB),$$
$$\rho_\lambda(A,TB), \rho_\lambda(B,TA)\} .$$

From Lemmas 1 and 2 it follows that there is a unique $A_o \in B(X)$ such that $TA_o = A_o$, moreover $\rho_\lambda(A_o,A_o) = 0$ for each λ , i.e. A_o is a singleton $\{x^*\}$.

The uniqueness of x^* is proved as follows. Let y^* be another fixed point of T, i.e. $y^* \in Ty^*$, By (3) we have

$$\rho_\lambda(Ty^*,Ty^*) \le q_\lambda \max\{d_\lambda(y^*,y^*), \rho_\lambda(y^*,Ty^*)\} \le q_\lambda \rho_\lambda(Ty^*,Ty^*).$$

From this $\rho_\lambda(Ty^*,Ty^*) = 0$, it implies that $Ty^* = \{y^*\}$ and $\rho_\lambda(y^*,y^*) = 0$. Now again by (3) we get

$$d_\lambda(x^*,y^*) = d_\lambda(Tx^*,Ty^*) \le q_\lambda d_\lambda(x^*,y^*)$$

for each λ , hence $x^* = y^*$. The proof of the theorem is complete.

3. THE PROBABILISTIC LARGEST DISTANCE BETWEEN TWO SETS.

Let us recall some definitions. A function $F : R \to [0,1]$ (here R denotes the set of all real numbers) is called a distribution function if it is non-decreasing, left-continuous, inf F = 0 and sup F = 1. The family of all distribution functions is denote by $\mathcal{D}$. A PM-space is a pair $(X,\mathcal{F})$, where X is a certain set and $\mathcal{F} : X \times X \to \mathcal{D}$ satisfying the following conditions (here instead of $F(x,y)$ we write F_{xy} for simplicity of notation)

$$F_{xy}(t) = 1 \quad (\forall t > 0) \iff x = y \ ,$$
$$F_{xy}(0) = 0,$$
$$F_{xy} = F_{yx} \ ,$$
$$F_{xy}(t+s) \geq \min \{F_{xz}(t), F_{zy}(s)\}$$

for every x,y,z in X and $t,s > 0$ (see |7| for details).

Similarly to the notion of probabilistic diameter introduced in |6|, here we define

$$\delta_{AB}(t) = \sup_{s<t} \inf_{x\in A, y\in B} F_{xy}(s) \ ,$$

where A and B are probabilistic bounded sets, i.e. they satisfy the condition $\sup_{t>o} D_C(t) = 1$ with $D_C(t) = \delta_{CC}(t)$.

Here are some simple properties of δ_{AB}.

1. δ_{AB} is a distribution function.

Evidently, δ_{AB} is non-decreasing and left-continuous, $\delta_{AB}(0) = 0$. To prove $\sup\delta_{AB} = 1$ we now show that it satisfies the probabilistic triangle inequality, i.e.

$$\delta_{AB}(t+s) \geq \min\{\delta_{AC}(t), \delta_{CB}(s)\}$$

for every A,B,C probabilistic bounded and $t,s > 0$.

Suppose on the contrary that there are A,B,C,t,s such that

$$a = \delta_{AB}(t+s) < b < \min\{\delta_{AC}(t), \delta_{CB}(s)\} \ . \tag{4}$$

Since $b < \sup_{v<t} \inf_{x\in A, y\in B} F_{xz}(v)$, there is $v_o < t$ such that $\inf_{x\in A, z\in C} F_{xz}(v_o) \geq b$. Analogously, there exists $w_o < s$ such that $\inf_{z\in C, y\in B} F_{zy}(w_o) \geq b$. By the probabilistic triangle inequality for F_{xy} we have

$$F_{xy}(v_o+w_o) \geq \min\{F_{xz}(v_o), F_{zy}(w_o)\}$$

for every x,y,z in X. In particular, fixing $x \in A$, $y \in B$ in the left side of the above inequality we obtain

$$F_{xy}(v_o+w_o) \geq \min\{ \inf_{x\in A, z\in C} F_{xz}(v_o), \inf_{z\in C, y\in B} F_{zy}(w_o)\} \geq b.$$

From this

$$a = \sup_{u<s+t} \inf_{x\in A, y\in B} F_{xy}(u) \geq b$$

and we get a contradiction to (4), so the probabilistic triangle inequality for δ_{AB} is proved.

Now fixing $x_o \in A$, $y_o \in B$ we have

$$\delta_{AB}(t) \geq \min\{\delta_{Ax_o}(\tfrac{t}{3}), \delta_{x_oy_o}(\tfrac{t}{3}), \delta_{y_oB}(\tfrac{t}{3})\} \geq$$

$$\geq \min\{ D_A(\tfrac{t}{3}), F_{x_oy_o}(\tfrac{t}{3}), D_B(\tfrac{t}{3})\} .$$

Since D_A, D_B and $F_{x_oy_o}$ are distribution functions, from this we get the desired result.

2. $\delta_{AB}(t) = 1 \quad (\forall t > 0) \implies A = B.$

Indeed, since $\sup\limits_{s<t} \inf\limits_{x\in A, y\in B} F_{xy}(s) = 1$ for each $t > 0$ then for every $\lambda \in (0,1)$ and $t > 0$ there exists $s_o < t$ such that $F_{xy}(s_o) > 1 - \lambda$ for every $x \in A$, $y \in B$. It follows that $F_{xy}(t) = 1$ for every $x \in A$, $y \in B$ and $t > 0$, it in turn implies $A = B$ and moreover, they are a singleton.

By the facts indicated in 1. and 2. and the obvious property $\delta_{AB} = \delta_{BA}$, we can call δ_{AB} a probabilistic semimetric in the space of all probabilistic bounded subsets of X.

3. It is well-known that the (ε,δ)-topology in a PM-space can be described by a family of pseudo-metrics

$$d_\lambda(x,y) = \sup\{t : F_{xy}(t) \leq 1 - \lambda\} \quad , \quad (\lambda \in (0,1)).$$

The λ-larges distance between two bounded sets is defined as usual

$$\rho_\lambda(A,B) = \sup\{d_\lambda(x,y) : x \in A, y \in B\} .$$

Now we are interested in the following problem: In which case have we

$$\rho_\lambda(A,B) = \sup\{t : \delta_{AB}(t) \leq 1 - \lambda\} ? \quad . \tag{5}$$

First we show that

$$\sup\{t : \delta_{AB}(t) \le 1-\lambda\} = \sup\{t : \inf_{x\in A, y\in B} F_{xy}(t) \le 1-\lambda\} .$$

For simplicity of notation we denote for the moment

$$G = \inf_{x\in A, y\in B} F_{xy},\ Z_1 = \{t : G(t) \le 1-\lambda\},\ Z_2 = \{t : \sup_{s<t} G(s) \le 1-\lambda\}.$$

So we must prove that $\sup Z_1 = \sup Z_2$.

Since $\sup_{s<t} G(s) \le G(t)$, we have $\sup Z_1 \le \sup Z_2$. In order to prove the converse inequality, we take arbitrary $t \in Z_2$ and note that if $s < t$ then $G(s) \le 1-\lambda$ and hence $s \in Z_1$. This means that $t \le \sup Z_1$, consequently, $\sup Z_2 \le \sup Z_1$ as desired.

Thus we now must only answer the question: When does the equality

$$(6) \quad \sup_{x\in A, y\in B} \sup\{t : F_{xy}(t) \le 1-\lambda\} = \sup\{t : \inf_{x\in A, y\in B} F_{xy}(t) \le 1-\lambda\}$$

hold ?

Note that the inequality " $\le$ " always holds since $\inf_{x\in A, y\in B} F_{xy} \le F_{xy}$ for every $x \in A$, $y \in B$. The equality is not true in general, so we must restrict ourselves to a certain subclass of $B(X)$, namely the class of all nonempty compact (in the (ε,δ)-topology) subsets of X, denoted by $K(X)$.

First we claim that if $A,B \in K(X)$ then $\inf_{x\in A, y\in B} F_{xy}(t) = \min_{x\in A, y\in B} F_{xy}(t)$, i.e. for each t there are $x_t \in A$, $y_t \in B$ such that $\inf_{x\in A, y\in B} F_{xy}(t) = F_{x_t y_t}(t)$. For this it suffices to verify the lower semi-continuity of $F_{xy}(t)$ considered as a function from $X \times X$ into $[0,1]$ for fixed t. Thus, fixing $(x_o, y_o) \in X \times X$ we have to show that for any $\varepsilon > 0$ there exist a neighbourhood $U_{x_o}(\alpha,\delta)$ of x_o aud a neighbourhood $U_{y_o}(\beta,\delta)$ of y_o such that

$$F_{x_o y_o}(t) - \varepsilon \le F_{xy}(t)$$

for every $x \in U_{x_o}(\alpha,\delta)$ and $y \in U_{y_o}(\beta,\delta)$. Obviously, we may assure

$$\varepsilon < F_{x_o y_o}(t), \quad \text{so} \quad a = F_{x_o y_o}(t) - \varepsilon > 0 \quad .$$

Suppose on the contrary that for any neighbourhood U of x_o and any neighbourhood V of y_o there exist $\bar{x} \in U$ and $\bar{y} \in V$ such that $a > F_{\bar{x}\bar{y}}(t)$. Letting $\delta = 1 - a$, by the left-continuity of $F_{x_o y_o}$ there is $s < t$ such that $F_{x_o y_o}(s) > a = 1 - \delta$. Taking r so that $s < r < t$ we have

$$F_{xy}(t) \geq \min\{F_{xx_o}(t-r),\ F_{x_o y_o}(s),\ F_{y_o y}(r-s)\}$$

for every $x, y \in X$. Put

$$U_{x_o}(t-r,\delta) = \{x \in X;\ F_{xx_o}(t-r) > 1 - \delta\},$$
$$U_{y_o}(r-s,\delta) = \{y \in X;\ F_{yy_o}(r-s) > 1 - \delta\}$$

and $\bar{x} \in U_{x_o}$, $\bar{y} \in U_{y_o}$ such that $F_{\bar{x}\bar{y}}(t) < a = 1 - \delta$. On the other hand, from the above arguments we get $F_{\bar{x}\bar{y}}(t) > 1 - \delta$. This contradiction proves our assertion.

Now we are in a situation to prove equality (6). Suppose on the contrary that there exists s such that

$$\sup_{x \in A, y \in B} \sup\{t : F_{xy}(t) \leq 1 - \lambda\} < s < \sup\{t : \min_{x \in A, y \in B} F_{xy}(t) \leq 1 - \lambda\} .$$

The first inequality shows that $F_{xy}(s) > 1 - \lambda$ for every $x \in A$, $y \in B$, but this contradicts the second inequality showing that $F_{x_s y_s}(s) \leq 1 - \lambda$ for some $x_s \in A$, $y_s \in B$. Thus, (6) is proved and from this (5) follows.

From (5) and the left-continuity of $\delta_{AB}(t)$ we get an important inequality for our purpose

$$\delta_{AB}(\rho_\lambda(A,B)) \leq 1 - \lambda \tag{7}$$

for every $A, B \in K(x)$.

4. MULTIVALUED QUASI-CONTRACTIONS IN PM-SPACES

Now we can state the main result of this paper.

THEOREM 2. *Let* (X, F) *be a complete PM-space,* $T : X \to K(X)$ *a mapping satisfying the following condition: there exists* $k < 1$ *such that*

$$\delta_{TxTy}(kt) \geq \min\{F_{xy}(t), \delta_{xTx}(t), \delta_{yTy}(t), \delta_{xTy}(t), \delta_{yTx}(t)\} \tag{8}$$

for every x,y *in* X, t > 0 .

Then there is a unique fixed point of T , *moreover it is a stationary point.*

P r o o f. Put

$$d_\lambda(x,y) = \sup\{t : F_{xy}(t) \leq 1-\lambda\} , \quad (\lambda \in (0,1)).$$

Then (X,d_λ) is a complete space with a family of pseudo-metrics. We have only to show that T satisfies the conditions in Theorem 1.

Suppose the contrary that there are $x,y \in X$ and $\lambda \in (0,1)$ such that

$$\rho_\lambda(Tx,Ty) > k\cdot\max\{d_\lambda(x,y),\rho_\lambda(x,Tx),\rho_\lambda(y,Ty),\rho_\lambda(x,Ty),\rho_\lambda(y,Tx)\}.$$

Put $t = \dfrac{\rho_\lambda(Tx,Ty)}{k}$. Then $t > \max\{d_\lambda(x,y),\dots,\rho_\lambda(y,Tx)\}$ and by (7) we obtain

$$\min\{F_{xy}(t),\ \delta_{xTx}(t),\dots,\ \delta_{yTx}(t)\} > 1-\lambda \quad .$$

But then from (8) we get

$$\delta_{TxTy}(\rho_\lambda(Tx,Ty)) = \delta_{TxTy}(kt) > 1-\lambda \quad ,$$

a contradiction to (7).

Applying Theorem 1, Theorem 2 follows.

Remark that this theorem is a partial analogue of Minh's theorem in |2|.

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REZIME

TEOREMA O NEPOKRETNOJ TAČKI ZA VIŠEZNAČNE KVAZIKONTRAKCIJE U VEROVATNOSNIM METRIČKIM PROSTORIMA

U ovom radu dokazano je uopštenje teoreme Minha |2| u verovatnosnim metričkim prostorima.