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ON THE CONVERGENCE OF SOME FINITE DIFFERENCE SCHEMES FOR A SINGULAR PERTURBATION PROBLEM

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ABSTRACT

A difference schemes was formed with a constant fitting factor on a nonuniform mesh for a singular perturbation problem of the second order and for boundary conditions of the third order. A uniform convergence of the order one was proved with respect to the small parametar of the solution and its first derivative. Emelyanov's technique was used.

We shall consider a problem of the form

$$\text{(1)}\quad \begin{aligned} Lu(x) &= \varepsilon u''(x) + a(x)u'(x) + b(x)u(x) = f(x) \\ u'(0) - \beta u(0) &= \mu_1 , \quad u(1) = \mu_2 \end{aligned}$$

where $\beta \geq 0$ and the functions a,b and f are sufficiently smooth and satisfy $a(x) \geq \alpha > 0$, $b(x) \leq 0$.

By M and M_i we denote the constants which are independent on ε and h (h is determined by (3)).

Emelyanov [2] presented a difference scheme on a uniform mesh for a solving problem (1) and postulated that it is uniformly convergent with the order one. The first derivative which is obtained from scheme [2] has the order one of the uniformly convergence with respect to ε. The result of [7] represent a generalization of the results of [2] on a nonuniform mesh. In this paper we shall show that the fitting factor in [2] and [7], which depends of $a(x_i)$, can be replaced with either constant or "quasi constant" one [4], which depends only on $a(0)=\omega$. In [2] and [7] for obtaining an estimation of the difference between the numerical and asymptotic solution the appearing expressions $\exp[\pm(a(x_i)-a(0))h/\varepsilon]$ create difficulties in a non classical error estimation.

Taking into account the results of [1] and the asymptotic expansions [8] we use an exponentially fitting factor of the form

$$\sigma_i(\rho,\omega) = \omega\,\frac{x_{i+1}-x_i}{\varepsilon}\,\mathrm{cth}\,\omega\,\frac{x_{i+1}-x_i}{\varepsilon}\,.$$

Then the schemes in [2] and [7] become simpler and the conditions of the grid in [7] are relaxed. At the end we shall present some numerical experiments. Further we shall compare the schemes from [2] and [7] and this one. We shall also compare a uniform mesh and a nonuniform one and present a numerical validation of the theoretical order of the uniform convergence for the above schemes. The experimental determination of the order of uniform convergence is based on the General Convergence Principle [1].

Problem (1) can be written in the form

$$\begin{aligned} &L_1(u,v) = \varepsilon v' + av + bu = f \\ (2)\quad &L_2(u,v) = u' - v = 0 \end{aligned}$$

$$v(0) - \beta u(0) = \mu_1,\quad u(1) = \mu_2\,.$$

For solving system (2) we use a nonuniform mesh

$$\begin{aligned} &x_{i+1} = x_i + \sigma_i h,\quad -\eta_{i+1} = \sigma_i,\quad \sigma_i \le M,\quad h^{-1} = -\sum_{i=1}^{n} \eta_i \\ (3)\quad &i = \overline{1,n-1},\quad x_o = 0,\quad x_1 = 1,\quad \sigma_1 = -\eta_1 = 1,\quad \sigma_i \in \mathbb{R} \\ &|\sigma_i + \eta_i| \le M_i x_i\,,\quad x_i \le C_o,\quad C_o \text{ is given, } 0 < C_o \le 1. \end{aligned}$$

and

$$\begin{aligned} (4)\quad &L_1^h(u_o,v_o) = (\varepsilon\omega\rho/2)\,\mathrm{cth}(\omega\rho/2)(v_1-v_o)/2 + \\ &+ b(h/2)(u_I+u_o)/2 = f(h/2)\quad \rho = h/\varepsilon\,. \end{aligned}$$

$$L_2^h(u_o,v_o) = (u_1-u_o)h - (v_1-v_o)/2 = 0$$

(4)

$$L_1^h(u_i,v_i) = (\sigma_i(\rho,\omega)/\rho)(\bar{A}_i v_{i-1} + \bar{B}_i v_i + \bar{C}_i v_{i+1}) +$$
$$+ (\bar{\alpha}_i v_{i-1} + \bar{\beta}_i v_{i+1}) a(x_i) + b(x_i)(\bar{\alpha}_i u_{i-1} + \bar{\beta}_i u_{i+1}) = f(x_i)$$
$$L_2^h(u_i,v_i) = (\bar{A}_i u_{i-1} + \bar{B}_i u_i + \bar{C}_i u_{i+1})/h - (\bar{\alpha}_i v_{i-1} \pm \beta_i v_{i+1}) = g_i$$
$$(i = \overline{1,n-1})$$

where

$$\sigma_i(\rho,\omega) = \Omega_i \mathrm{cth}\Omega_i, \quad \Omega_i = \omega\sigma_i\rho$$

$$\bar{A}_i = \sigma_i/\eta_i(\sigma_i-\eta_i) \quad \bar{B}_i = -(\eta_i+\sigma_i)/\sigma_i\eta_i, \quad \bar{C}_i = \eta_i/\sigma_i(\eta_i-\sigma_i)$$

$$\bar{\alpha}_i = \sigma_i/(\sigma_i-\eta_i), \quad \bar{\beta}_i = -\eta_i/(\sigma_i-\eta_i), \quad g_i = 0.$$

We use the solution of system (4) for an approximation of the exact values $u(x)$ and $u'(x)$ in the points of the grid

$$u'(x_i) \approx v_i, \quad u(x_i) \approx u_i \quad (i=0,n)$$

It can be shown that a solution of system (4) can be obtained from the recurrent relations

(5)

$$u_{i+1} = t_i u_o + s_i$$
$$v_{i+1} = p_i u_o + q_i, \quad (i=\overline{0,n-1})$$

similar to those in |7|.

Here

$$t_o = K_o[1 + (B_o + \beta + A_o\beta)h/2]$$
$$s_o = K_o h[(1 + A_o)\mu_1 + r_o + 2g_o]$$
$$p_o = K_o[(A_o + B_o h/2)\mu_1 + r_o + hB_o g_o]$$
$$K_o = (1 - B_o h/2)^{-1}$$
$$A_o = (\omega \mathrm{cth}(\omega\rho/2) - a(h/2))/\psi, \quad B_o = -b(h/2)/\psi$$
$$r_o = 2f(h/2)/\psi, \quad \psi = \omega\mathrm{cth}(\omega\rho/2) + a(h/2)$$
$$t_i = K_i(R_i p_{i-2} + S_i t_{i-2} + B_i\beta_i p_{i-1} + \delta_i t_{i-1})$$
$$s_i = K_i(R_i q_{i-2} + S_i s_{i-2} + \beta_i B_i q_{i-1} + \delta_i s_{i-1} + N_i)$$
$$p_i = K_i(F_i p_{i-2} + Q_i t_{i-2} + B_i p_{i-1} + \delta_i D_i t_{i-1})$$
$$q_i = K_i(F_i q_{i-2} + Q_i s_{i-2} + B_i q_{i-1} + \delta_i D_i s_{i-1} + M_i)$$
$$(i=1,2,\ldots,n-1)$$

$$p_{-1} = \beta, \quad q_{-1} = \mu_1, \quad t_{-1} = 1, \quad s_{-1} = 0$$
$$K_i = (1-\beta_i D_i)^{-1}$$
$$R_i = \beta_i A_i + \alpha_i, \qquad Q_i = \gamma_i D_i + C_i$$
$$S_i = C_i\beta_i + \gamma_i, \qquad M_i = r_i + \rho_i D_i$$
$$F_i = \alpha_i D_i + C_i, \qquad N_i = \rho_i + r_i\beta_i$$
$$A_i = \sigma_i(\omega\sigma_i \mathrm{cth}\Omega_i + a_i\eta_i)/\eta_i^2\phi, \quad a_i = a(x_i)$$
$$\phi = \omega \mathrm{cth}\Omega_i + a_i$$
$$B_i = -(\sigma_i^2-\eta_i^2)\omega\mathrm{cth}\Omega_i/\eta_i^2\,\phi, \quad C_i = b_i\sigma_i/\eta_i\phi$$
$$D_i = -b_i/\phi, \quad r_i = f_i(\sigma_i-\eta_i)/\eta_i\phi$$
$$\alpha_i = -h\sigma_i^2/\eta_i, \quad \beta_i = h\sigma_i, \quad \gamma_i = \sigma_i^2/\eta_i^2$$
$$\delta_i = (\eta_i^2-\delta_i^2)/\eta_i^2, \quad \rho_i = hg_i .$$

LEMMA 1. *Let* u_i *and* v_i *be a solution of system (4). Let* $|K_i| \le M$, $|t_{n-1}^{-1}| \le M$.

Then

$$|u_i| + |v_i| \le M\,(|\mu_1| + |\mu_2| + H),$$ *where* H *is determined by*

$$|f(x_i)|;\ h|g_i| \le H \quad (i=0,1,\dots,n).$$

The proof is completely analogous to that of Lemma 1 [7].

THEOREM 1. *Let* u_i *and* v_i *be a solution of system (4). Then the following estimation holds*

$$|u_i - u(x_i)| + |v_i - u'(x_i)| \le M\left(\frac{h^2}{\varepsilon} + \frac{h^2}{\varepsilon^2} + \frac{h^4}{\varepsilon^4}\right).$$

The proof is obtained by using the techniques outlined in [7].

For a "non classical" convergence we use the asymptotic solution of problem (1) which has the form ([8]):

$$u(x) = \psi_o(x) + \varepsilon\psi_1(x) + \varepsilon u_o(x/\varepsilon) + \varepsilon^2 u_1(x/\varepsilon) + w(x)$$

$$v(x) = \phi_o(x) + \varepsilon\phi_1(x) + v_o(x/\varepsilon) + \varepsilon v_1(x/\varepsilon) + z(x)$$

where ϕ_i and ψ_i are the solutions of the differential equations system, independent on ε .

$$u_o(t) = -\frac{\mu}{\omega} e^{-\omega t}, \quad v_1(t) = \mu e^{-\omega t}$$

$$u_1(t) = p_1(t)e^{-\omega t}, \quad v_1(t) = p_2(t)e^{-\omega t},$$

$p_i(t)$ are polynomials of the order two. According to [2] the following estimations hold:

$$|w(x)| + |z(x)| \le M\varepsilon^2$$
$$|v^{(k-1)}| + |u^{(k)}| \le M\varepsilon^{k-1} \quad (k = 1,2,3,4).$$

We now introduce the notations

$$Tu = \bar{\alpha}_i u_{i-1} + \bar{\beta}_i u_{i+1}$$
$$Du = h^{-1}(\bar{A}_i u_{i-1} + \bar{B}_i u_i + \bar{C}_i u_{i+1})$$
$$w_i^h = u(x_i) - u_i - w(x_i)$$
$$z_i^h = v(x_i) - v_i - z(x_i), \quad t_i = x_i/\varepsilon .$$

By applying the operators L_1^h and L_2^h to the mesh functions $w_i^h(x_i)$ and $z_i^h(x_i)$ we obtain

$$L_1^h(z_i^h, w_i^h) = \sum_{k=1}^{5} \Phi_{ki}, \quad L_2^h(z_i^h, w_i^h) = \sum_{k=1}^{3} H_{ki}$$

$$L^h(1) = -\varepsilon u_o(1/\varepsilon) - \varepsilon^2 u_1(1/\varepsilon)$$

$$z^h(0) - \beta w^h(0) = -\beta y_o\varepsilon^2, \quad \text{where}$$

$$\Phi_{1i} = -(-f_i + a_i T\phi_o + b_i T\psi_o + \varepsilon(\sigma_i(\rho)-1)D\phi_o + \varepsilon D\phi_o +$$
$$+ \varepsilon a_i T\phi_1 + \varepsilon b_i T\psi_1 + \varepsilon^2(\sigma_i(\rho)D\phi_1 + b_i Tu_1 t_i)$$

$$\Phi_{2i} = \varepsilon x_i a'(\xi)\bar{\beta}_i p_2(t_{i+1})e^{-\omega t_{i+1}}$$

$$\Phi_{4i} = -x_o\varepsilon\left(\frac{\sigma_i(\rho,\omega)}{\rho}\bar{A}_i e^{-\omega t_{i-1}} + \frac{\sigma_i(\rho,\omega)}{\rho}\bar{B}_i e^{-\omega t_i} - a_i\bar{\alpha}_i e^{-\omega t_{i-1}} + \right.$$
$$\left. + \omega\frac{1}{sh\Omega_i}\frac{\eta_i}{\eta_i - \sigma_i} e^{-\omega t_i}\right)$$

$$\Phi_{5i} = A + \frac{\omega_1}{2} B .$$

$$A = -\omega\sigma_i\mu \mathrm{cth}\Omega_i h D e^{-\omega t_i} - \omega\mu T e^{\omega t_i} - a'(\theta) x_i T e^{\omega t_i}$$

$$B = \varepsilon\mu(\bar{A}_i \frac{\sigma_i(\rho,\omega)}{\rho} e^{-\omega t_{i-1}} t_{i-1}^2 + \bar{B}_i \frac{\sigma_i(\rho,\omega)}{\rho} t_i^2 e^{\omega t_i} +$$

$$+ \omega t_{i-1}^2 \frac{e^{-\omega t_i} \eta_i}{\mathrm{sh}\ \Omega_i(\eta_i - \sigma_i)} + a_i t_{i-1}^2 e^{-\omega t_{i-1}})$$

$$H_{1i} = -(D\psi_o - T\phi_o) - \varepsilon(D\psi_1 - T\phi_1)$$

$$H_{2i} = -\varepsilon\frac{\mu}{\omega} De^t + \mu Te^{-\omega t}$$

$$H_{3i} = \varepsilon^2 Du_1(t) - \varepsilon Tv_1(t) .$$

Using the Taylor expansions of the above functions about x_i and the following inequalities ([6],[1]):

$$c'e^t \le \mathrm{sh}t \le c''e^t , \qquad 0 < c < t < \infty$$

$$c't \le \mathrm{sh}t \le c''t \qquad 0 < t < c$$

c, c' and c'' are constants.

$$t^j e^{-t} \le c(\theta)e^{-\theta t}, \quad \theta \in (0,1), \quad 0 \le t < \infty$$

we can prove Lemma 2.

LEMMA 2. *For Φ_{ki} and H_{ki} the following estimations hold*

$$|\Phi_{ki}| \le M(\varepsilon^2 + h)$$

$$|H_{ki}| \le M(\varepsilon^2 + h).$$

We now turn to the "non classical" estimation contained in Theorem 2.

THEOREM 2. *Let u_i and v_i be a solution of system (4) on grid (3). Then the following estimation holds*

$$|u_i - v(x_i)| + |v_i - u'(x_i)| \le M(h + \varepsilon^2) .$$

The proof is obtained by applying Lemma 1 and Lemma 2.

THEOREM 3. *Let u_i and v_i be the solution of system (4) on grid (3). Then the estimation*

$$|u_i - u(x_i)| + |v_i - u'(x_i)| \leq Mh \quad , \textit{holds.}$$

Using Theorem 1 when $h \leq \varepsilon^2$ and Theorem 2 when $h \geq \varepsilon^2$ the proof is immediate.

For choosing η_i, σ_i and c_o in (3) we use the test for the order of uniform convergence [1]. The following theorem transforms the results of [1] on a nonuniform mesh.

THEOREM 4. *Let F^{m_k} be the mesh function defined on the mesh*

$$I_{m_k} = \{x_i, x_i = x_{i-1} + \sigma_i h, (i = 1, \ldots, m_k - 1),\ x_o = 1,\ h^{-1} = \sum_{i=1}^{m_k} \sigma_i\}$$

and p be a positive number. Then

$$|F^{m_k} - F| \leq M_1 m_k^{-p} \qquad \textit{for all} \quad m_k \geq m_o$$

if and only if the following two conditions hold:

a) $|F^{m_k} - F| = o(1)$, *as* $m_k \to \infty$

b) $|F^{m_k} - F^{m_{k+1}}| \leq M_2\, m_k^{-p}$ *for all* $m_k \geq m_o$

$m_{k+1} = \lambda m_k, \quad \lambda > 1.$

The proof is similar to that |2| p. 20.

By applying Theorem 3 we form the test for the experimental determination of the order of uniform convergence for difference schemes on a nonuniform mesh. Let $m_{k+1} = 2m_k$, $I_{m_k} \subset I_{m_{k+1}}$, $u_j^{m_k}$ be an approximative solution and $u(x_j)$ the exact solution at the point x_j. We de note

$$z_{k,\varepsilon} = \max_j |u_j^{m_k} - u_j^{m_{k+1}}| \qquad (k=0,1,\ldots)$$

where the maximum is taken over all the points of the coarser one of the two meshes.

If the difference scheme is uniformly convergent, then

$$|u_j^{m_k} - u(x_j)| \le M_1 m_k^{-p}$$

by applying Theorem 3 we can show that

$$|u_j^{m_k} - u(x_j)| \to o(1), \quad m_k \to \infty$$

$z_{k,\varepsilon} \le M_2 m_k^{-p}$ then holds, and that

M_1 is independent of ε if and only if M_2 is. We take the quantities

$$\frac{z_{k,\varepsilon}}{z_{k+1,\varepsilon}} \simeq \frac{m_k^{-p}}{(2m_k)} = 2^p$$, and determine p in

this way

$$p = \log_2(z_{k,\varepsilon}/z_{k+1,\varepsilon})$$

Numerical results. We treat problem |1.|

$$\varepsilon u''(x) + (1+x^2)u'(x) - (x-\tfrac{1}{2})^2 u(x) = 4(3x^2-3x+1)(x+1)^2$$

$$u(0) - u'(0) = 0, \quad u(1) = 0$$

$$\varepsilon = 0.0625 .$$

In the following series of figures and tables we present the results of our computations.

We use the notations

Scheme I ... $\sigma_i(\rho,\omega)$ with a uniform mesh

Scheme II ... $\sigma_i(\rho,a(x_i))$, with a uniform mesh

Scheme III ... $\sigma_i(\rho,a(x_i))$ mesh II

Sdheme IV ... $\sigma_i(\rho,\omega)$ with mesh I:

Mesh I ... 0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.07, 0.09, 0.11, 0.13, 0.16, 0.20, 0.24, 0.08, 0.32, 0.36, 0.40, 0.46, 0.52, 0.58, 0.64, 0.70, 0.75, 0.80, 0.84, 0.88, 0.92, 0.95, 0.97, 0.98, 0.99, 1.00.

Mesh II ... 0.0,0.02,0.04,0.0608,0.082368,0.1046733 0.1276864, 0.1513789, 0.1750714, 0.1997785 0.225500, 0.2522360, 0.2799866, 0.3087516, 0.3385311, 0.3693251, 0.4011336, 0.4339656,

0.467794, 0.50266460, 0.5385124, 0.5753934
0.6132888, 0.6521987, 0.6921232, 0.7330621
0.7750155, 0.8179834, 0.8619658, 0.90699628
0.9529741, 1.00

For computing p we start with $m_o = 32$ and we put $m_{k+1} = 2m_k$ (k=0,1,2,3).

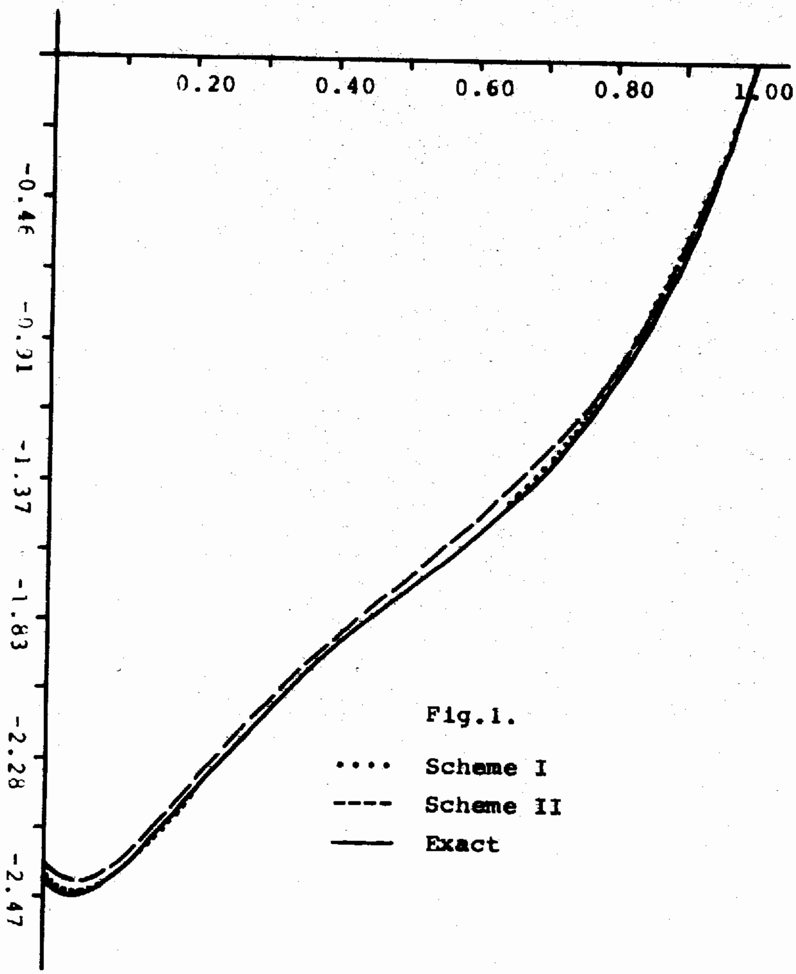

Fig.1.

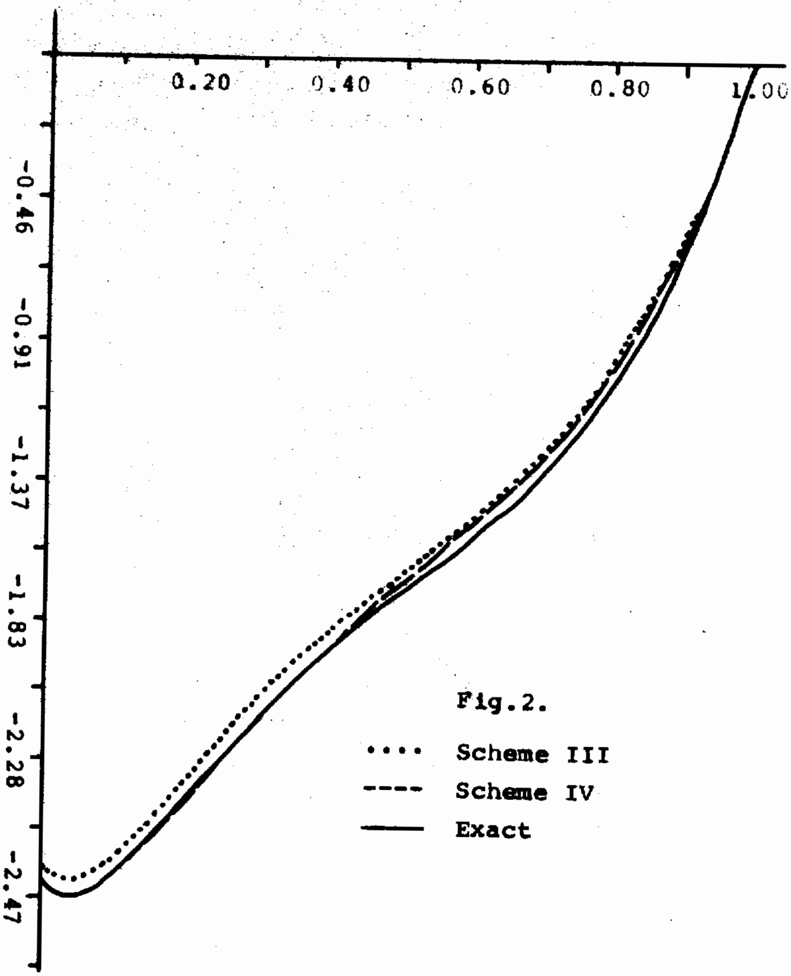
0.20
0.40
0.60
0.80
1.00
-0.46
-0.91
-1.37
-1.83
-2.28
-2.47
Scheme III
Scheme IV
Exact

Fig.2.

TABELE 1

ε	k	P_k	P_v	ε	P_u	P_v
SCHEME I						
2^{-1}	0	.1913569E+01	.2000908E+01	2^{-6}	.1059774E 01	.1729662E 01
	1	.1978778E+01	.1998136E+01		.190528 E 01	.195587 E 01
	2	.1982236E+01	.2010407E+01		.1979763E 01	.1976815E 01
2^{-3}	0	.1715132E+01	.1997260E+01	2^{-8}	.3478056E 00	.1105804E 01
	1	.2006710E+01	.1999607E+01		.1410312E 01	.1338642E 01
	2	.1922706E+01	.1998428E+01		.1842019E 01	.1711334E 01
2^{-4}	0	.1448952E+01	.1881095E+01	2^{-9}	.2882550E 00	.1062844E 01
	1	.2007009E+01	.1995805E+01		.1041304E 01	.1068558E 01
	2	.1895981E+01	.1886913E+01		.1420031E 01	.1329972E 01
SCHEME IV, MESH I						
2^{-1}	0	.1926638E 01	.1996914E 01	2^{-6}	.1422633E 01	.1413524E 01
	1	.2004060E 01	.1998076E 01		.2208937E 01	.1784494E 01
	2	.1935634E 01	.2004372E 01		.2183209E 01	.1919046E 01
2^{-3}	0	.2124112E 01	.1963680E 01	2^{-8}	.1218086E 01	.1779681E 01
	1	.1997517E 01	.1992213E 01		.2218703E 01	.1158596E 01
	2	.1706424E 01	.2000217E 01		.2287345E 01	.1480441E 01
2^{-4}	0	.2414846E 01	.1900175E 01	2^{-9}	.7186019E 00	.2143383E 01
	1	.1973113E 01	.1974578E 01		.1858415E 01	.1053521E 01
	2	.1871747E 01	.1994568E 01		.2217098E 01	.1129259E 01
SCHEME II						
2^{-1}	0	.2953125E 01	.2027054E 01	2^{-6}	.7675155E 00	.1293319E 01
	1	.1893125E 01	.1976741E 01		.1777995E 01	.1625615E 01
	2	.1799886E 01	.1990415E 01		.1936217E 01	.1800102E 01
2^{-3}	0	.2110542E 01	.1940582E 01	2^{-8}	-.3318977E-01	.9576272E 01
	1	.1993218E 01	.1972463E 01		.1244720E 01	.1024618E 01
	2	.2018799E 01	.1987117E 01		.1847666E 01	.1311047E 01
2^{-4}	0	.1787984E 01	.1831006E 01	2^{-9}	-.6986086E-01	.9262857E 01
	1	.1980410E 01	.1917159E 01		.9985099E 00	.9649740E 01
	2	.1995338E 01	.1958870E 01		.1262807E 01	.1034051E 01
SCHEME III MESH II						
2^{-1}	0	.2213184E 01	.1981158E 01	2^{-6}	.133265E 01	.9237934E 01
	1	.1884465E 01	.1992462E 01		.1540363E 01	.1210781E 01
	2	.1030667E 01	.1999636E 01		.1775866E 01	.1628408E 01
2^{-3}	0	.1774796E 01	.183530E 01	2^{-8}	.1508032E 01	.8960677E 00
	1	.1956434E 01	.1950804E 01		.1990338E 01	.9521322E 00
	2	.2006448E 01	.1987238E 01		.1719587E 01	.1003877E 01

2^{-4}	0	.1418320E 01	.156033E 01	2^{-9}	.1067063E 01	.8662433E 00
	1	.1896836E 01	.1855237E 01		.1488140E 01	.8518415E 01
	2	.1966221E 01	.1960452E 01		.1999397E 01	.975875E 01

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REZIME

O KONVERGENCIJI NEKIH KONAČNIH DIFERENCNIH ŠEMA ZA SINGULARNE PERTURBACIONE PROBLEME

U radu je data jedna diferencna šema sa konstantnim "fitting" faktorom za rešavanje konturnih problema za diferencijalne jednačinę drugog reda sa malim parametrom uz najveći izvod i konturne uslove trećeg reda. Dokazana je uniformna konvergencija po malom parametru za rešenje i njegov prvi izvod. Koristi se neekvidistantna mreža.