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# A LERAY-SCHAUDER PRINCIPLE FOR MULTIVALUED MAPPINGS IN TOPOLOGICAL VECTOR SPACES

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## ABSTRACT

Using a result of Schöneberg |15| and a fixed point theorem from |2| we prove in this paper a fixed point theorem for multivalued mappings in not necessarily locally convex topological vector spaces.

## INTRODUCTION

In recent time there is an increasing interest in the fixed point theory in not necessarily locally convex topological vector spaces and some results for singlevalued and multivalued mappings are obtained in |1|,|2|,|3|,|4|,|5|,|6|,|7|, |8|,|10| and |15| . It is proved in |8| that for admissible (see |10|) topological vector spaces many results from the fixed point theory in locally convex topological vector spaces can be generalized. Since there are many important nonlocally convex topological vector spaces such as $L^p (0< p < 1)$, $S(0,1)$ - the space of measurable functions, Hardy's class, $SL_\phi$, $KL_\phi$ |12|, it is of interest to obtain fixed point theorems for not necessarily locally convex topological vector spaces.

Using a result of Schöneberg |15| and a fixed point theorem from |2|, we shall prove in this paper a fixed point theorem for multivalued mappings in not necessarily locally convex topological vector spaces which are Hausdorff.

## A LERAY-SCHAUDER PRINCIPLE

First, we shall give some known results, notations and definitions.

In |2| we have proved the following fixed point theorem for multivalued mappings in not necessarily locally convex topological vector spaces.

THEOREM A. *Let* $E$ *be a Hausdorff topological vector space,* $\mathcal{U}$ *be the fundamental system of neighborhoods of zero in* $E$, $K$ *be a closed and convex subset of* $E$, $\mathcal{R}(K)$ *be the family of all nonempty, closed and convex subset in* $K$, $F : K \to \mathcal{R}(K)$ *be an upper semi-continuous mapping,* $\overline{F(K)}$ *be compact and the following condition be satisfied: (*) For every* $V \in \mathcal{U}$ *there is* $U \in \mathcal{U}$ *so that the convex hull of* $U \cap (F(K)-F(K))$ *is in* $V$. *Then there exists* $x \in K$ *so that* $x \in Fx$ .

In |2| some applications of the above fixed point theorem are given. If the space $E$ is a locally convex topological vector space the condition (*) from the above theorem is satisfied for every $F : K \to 2^K$ since we can take for $U \in \mathcal{U}$ the neighborhood $V$, if we suppose that co $U = U$ (co-convex hull) for every $U \in \mathcal{U}$.

Now, let us introduce the following definition.

DEFINITION 1. *Let* $E$ *be a topological vector space,* $\mathcal{U}$ *be the fundamental system of neighborhoods of* $0 \in E$, $K \subseteq Z$.$K$ *is said to be of Zima's type if and only if for every* $V \in \mathcal{U}$ *there exists* $U \in \mathcal{U}$ *such that co* $(U \cap (K-K)) \subseteq V$.

If $E$ is a locally convex topological vector space and $K \subseteq E$ then $K$ is of Zima's type. K.Zima has given in |16| a subset $K$ of a paranormed space $E$ which is in fact of Zima's type in the sense of Definition 1. Now, we shall give an example of a subset of Zima's type in a nonlocally convex topological vector space.

Let $E$ be a vector space over the real or complex number field and $\| \|^* : E \to [0, \infty)$ so that the following conditions are satisfied:

1. $\|x\|^* = 0 \Longleftrightarrow x = 0$.

2. $\|-x\|^* = \|x\|^*$, $x \in E$.

3. $\|x+y\|^* \leq \|x\|^* + \|y\|^*$, $x,y \in E$.

4. If $\|x_n - x\|^* \to 0$ and $\lambda_n \to \lambda_o$ then $\|\lambda_n x_n - \lambda_o x_o\|^* \to 0$.

Then $(E, \|\ \|^*)$ is a paranormed space which is also a topological vector space in which the fundamental system V of neighborhoods of zero in E is given by:

$$V = \{V_\varepsilon\}\varepsilon > 0, \quad V_\varepsilon = \{x \mid \|x\|^* < \varepsilon,\ x \in E\}.$$

If $K \subseteq E$, where $(E, \|\ \|^*)$ is a paranormed space, and there exists $C(K) > 0$ such that |16|:

$\|tx\|^* \leq C(K)t\|x\|^*$, for every $x \in K-K$ and every $t \in [0,1]$, in |2| it is proved that K is of Zima's type.

Now, let $(E, \|\ \|^*)$ be the space $(S(0,1), \|\ \|^*)$ ($S(0,1)$ the space of equivalence classes of finite measurable functions on $[0,1]$ with measure $\mu$) and for every $\hat{x} \in S(0,1)$:

$$\|\hat{x}\|^* = \int_0^1 \frac{|x(t)|}{1+|x(t)|}\mu(dt), \quad \{x(t)\} \in \hat{x}.$$

It is known that $S(0,1)$ is an admissible topological vector space |13| and the convergence in the paranorme is the convergence in the measure.

Let $\alpha > 0$ and $K_\alpha = \{\hat{x} \mid \hat{x} \in S(0,1),\ |x(t)| \leq \alpha,\ t \in [0,1]\}$. We shall show that $K_\alpha$ is of Zima's type and that $C(K_\alpha) = 1 + 2\alpha$ which means that:

$$\|s(\hat{x}-\hat{y})\|^* \leq (1+2\alpha)s\|\hat{x}-\hat{y}\|^*, \text{ for every } 0 \leq s \leq 1$$

and every $\hat{x},\hat{y} \in K_\alpha$. Let $\{x(t)\} \in \hat{x}$ and $\{y(t)\} \in \hat{y}$ so that:

$$|x(t)| \leq \alpha, \quad |y(t)| \leq \alpha, \quad t \in [0,1].$$

Then $1 + |x(t)-y(t)| \leq (1+2\alpha) + (1+2\alpha)s|x(t)-y(t)| =$

$$= (1+2\alpha)(1+s|x(t)-y(t)|)$$

and so:

$$\|s(\hat{x}-\hat{y})\|^* = \int_0^1 \frac{s|x(t)-y(t)|}{1+s|x(t)-y(t)|}\mu(dt) \le (1+2\alpha)s\int_0^1 \frac{|x(t)-y(t)|}{1+|x(t)-y(t)|}\mu(dt)$$
$$= (1+2\alpha)s\,\|\hat{x}-\hat{y}\|^* .$$

Let E be a topological vector space, $K \subseteq E$ and $2^K$ be the family of all nomepty subsets of K. If $U \subseteq K$ then $cl_K(U)$ is the closure (in K) of U and $\partial_K U$ denotes the boundary of U in K. If C is a lattice with a minimal element $\theta$ then a map $\chi: 2^K \to C$ is called a measure of noncompactness of K provided that the following conditions are satisfied for any $X, Y \in 2^K$:

1. $\chi(X) = \theta$ if and only if X is relatively compact.
2. $\chi(\overline{co}(X)) = \chi(X)$, where $\overline{co}(X)$ denotes the convex closure of X.
3. $\chi(X \cup Y) = \max\{\chi(X), \chi(Y)\}$.

If $\chi$ is a measure of noncompactness of K, $D \subseteq K$ and $g: D \to 2^K$ then g is called $\chi$-condensing if and only if the following implication holds:

$$X \subseteq D,\ \chi(X) \le \chi(g(X)) \Rightarrow X \text{ is relatively compact .}$$

In |15| the following theorem is proved.

THEOREM B. *Let* $\chi$ *be a measure of noncompactness of* K, $\mathcal{M}$ *be a set of nonempty compact subsets of* K, $U \subseteq K$ *be an open neighborhood of zero in* E, $0 \in K$ *and* $H: [0,1] \times cl_K(U) \to \mathcal{M}$ *be an upper semicontinuous mapping. Suppose that the following conditions are satisfied:*

*(i)* $x \notin H(t,x)$ *for* $t \in [0,1]$ *and* $x \in \partial_K U$.

*(ii)* $X \subset cl_K(U)$ *and* $\chi(X) \le \chi(H([0,1] \times X))$ *imply that* X *is relatively compact.*

*(iii) For every* $t > 1$ *and* $x \in \partial_K U$, $tx \notin H(1,x)$.

*(iv)* $tM \in \mathcal{M}$ *for* $t \in [0,1]$ *and* $M \in \mathcal{M}$.

*Then there exists an upper semicontinuous mapping* $g: K \to \mathcal{M}$ *such that for* $x \in K$ *:*

$x \in g(x)$ *if and only if* $x \in cl_K(U)$ *and* $x \in H(0,x)$.

Using Theorems A and B we shall prove the following fixed point theorem.

THEOREM *Let* $K, U \in \mathcal{U}$ *and* $\chi$ *be as in Theorem B and* $\mathcal{M}'$ *the family of all nonempty, convex and compact subsets of* K *so that* $co(\{0\} \cup H([0,1] \times cl_K(U)))$ *is of Zima's type where* H *is an upper semicontinuous mapping from* $[0,1] \times cl_K(U)$ *into* $\mathcal{M}'$ *so that (i), (ii) and (iii) in Theorem* B *are satisfied. Then there exists* $x \in cl_K(U)$ *such that* $x \in H(0,x)$.

P r o o f. The family $\mathcal{M}'$ has the property (iv) from Theorem B. Namely $0 \in K$ and K is convex, so for every $M \subseteq K$ it follows that $tM \subseteq K$ for every $t \in [0,1]$ and so $M \in \mathcal{M}'$ implies that $tM \in \mathcal{M}'$. From Theorem B it follows that there exists an upper semicontinuous $\chi$-condensing mapping $g: K \to \mathcal{M}'$ so that for all $x \in K$:

(1) $x \in g(x) \iff x \in cl_K(U), \quad x \in H(0,x)$.

Now, we shall prove that the set of fixed points of the mapping g, Fix(g), is nonempty. Let us prove that there exists a closed and convex subset $T \subseteq K$ so that $g(T) \subseteq T$ and $\overline{g(T)}$ is compact. In order to do this we shall use Remark 6 from Hahn's paper |6|. The measure $\chi$ has the following properties:

(a) $X \subseteq Y (X,Y \subseteq K) \implies \chi(X) \leq \chi(Y)$.

(b) $\chi(X \cup \{y_0\}) = \chi(X)$, for every $X \subseteq K$, $y_0 \in K$.

Indeed $X \subseteq Y$ implies $\chi(X \cup Y) = \chi(Y) = \max\{\chi(X), \chi(Y)\}$ and so $\chi(Y) \geq \chi(X)$. Further $\chi(X \cup \{y_0\}) = \max\{\chi(X), \chi(\{y_0\})\} = \chi(X)$ since $\chi(\{y_0\}) = 0$. If $M = K$ in Remark 6 |6| as in |6| it follows that there exists a nonempty, closed and convex subset $T_0$ of E such that $K \cap T_0 \neq \emptyset$, $g(K \cap T_0) \subseteq K \cap T_0$ and $\chi(K \cap T_0) \leq \chi(g(K \cap T_0))$. Since the mapping g is $\chi$-condensing it follows that $K \cap T_0$ is compact and so from $\overline{g(K \cap T_0)} \subseteq K \cap T_0$, it follows that $\overline{g(K \cap T_0)}$ is compact. Further in the proof of Theorem B the mapping g is defined in the following way ($\lambda : K \to [0,1]$ is defined in |15|)

$$g(x) = \begin{cases} H(2\lambda(x),x) & \lambda(x) \le \frac{1}{2}, \ x \in cl_K(U) \\ 2(1-\lambda(x))H(1,x) & \lambda(x) \ge \frac{1}{2}, \ x \in cl_K(U) \\ 0 & x \notin cl_K(U) \end{cases}$$

From this it follows that $g(K) \subseteq co(\{0\} \cup H([0,1] \times cl_K(U)))$. Since the set $co(\{0\} \cup H([0,1] \times cl_K(U)))$ is of Zima's type we conclude that the set $g(K)$ is of Zima's type. So, on the set $T = K \cap T_o$ the mapping $g$ satisfies all the conditions of Theorem A and there exists $x \in T$ so that $x \in g(x)$. From (1) it follows that there exists $x \in cl_K(U)$ such that $x \in H(0,x)$.

From the Theorem we obtain the following Corollary.

COROLLARY 1. *Let* $E$ *be a complete topological vector space,* $0 \in K \subseteq E$ *and* $K$ *be a closed and convex of Zima type subset,* $U$ *be an open neighborhood of zero in* $E$ *and* $U \subset K$, $\mathcal{M}'$ *be as in the Theorem and* $H : [0,1] \times cl_K(U) \to \mathcal{M}'$ *so that the following conditions are satisfied:*

(A) *For every* $t \in [0,1]$ *and every* $x \in \partial_K U$, $x \notin H(t,x)$.

(B) $\overline{H([0,1] \times cl_K(U))}$ *is compact.*

(C) *For every* $t > 1$ *and every* $x \in \partial_K U$, $tx \notin H(1,x)$.

*Then there exists* $x \in cl_K(U)$ *such that* $x \in H(0,x)$.

P r o o f. Let $C = \{0,1\}$ and $\chi : 2^K \to C$ be defined in the following way:

$$\chi(X) = \begin{cases} 0 & \bar{X} \text{ is compact} \\ 1 & \bar{X} \text{ is not compact.} \end{cases}$$

Then $\chi$ is a measure of noncompactness. Indeed 1. follows from the definition of the mapping $\chi$. Let us prove 2.. Since $A \subset B$ implies $\chi(A) \le \chi(B)$ we have that $\chi(X) \le \chi(\overline{co}\, X)$ for every $X \in 2^K$. Further, if $\bar{X}$ is compact it follows that $\overline{co}\, X$ is compact as we have proved in |4|. So we have that $\chi(X) = 1$ implies that $\chi(\overline{co}\, X) = 1 = \chi(X)$. If $X$ is not relatively compact then $\chi(\overline{co} X) \le 1 = \chi(X)$ and so 2. is proved. 3. is obvious. Further, we have the following implication:

$$X \subseteq cl_K(U), \quad \chi(X) \le \chi(H([0,1] \times X)) \Rightarrow \bar{X} \text{ is compact.}$$

Indeed, from $H([0,1]\times X)\subseteq \overline{H([0,1]\times cl_K(U))}$ and from the compactness of the set $H([0,1]\times cl_K(U))$ it follows that $\chi(H([0,1]\times X)=0$, for every $X\subseteq cl_K(U)$. From this we conclude that the inequality $\chi(X)\le\chi(H([0,1]\times X)$ implies $\chi(X)=0$ and so $\bar{X}$ is compact. From the Theorem it follows that there exists $x\in cl_K(U)$ so that $x\in H(0,x)$.

From Corollary 1 we obtain the following Corollary.

COROLLARY 2. *Let* $(E,\|\ \|^*)$ *be a complete paranormed space,* $0\in K\subseteq E$, $K$ *be a closed and convex subset of* $E$ *such that there exists* $C(K)>0$ *so that:* $\|tx\|^*\le t\,C(K)\,\|x\|^*$, *for every* $t\in[0,1]$ *and every* $x\in K-K$, *and* $U,H$ *and* $\chi$, $M'$ *be as in Corollary 1. Then there exists* $x\in cl_K(U)$ *such that* $x\in H(0,x)$.

Now, we shall give another corollary of the Theorem using some results from the theory of topological semifields. First, we shall recall some definitions from |9|.

Let $\mathbb{R}$ be the set of all real numbers, E be a vector space over $\mathbb{K}$ (real or complex number field), $\mathbb{R}_\Delta$ be the set of all mappings from $\Delta$ into $\mathbb{R}$ with the Tychonoff product topology and the operations addition and scalar multiplication as usual. If $f,g\in\mathbb{R}_\Delta$, we shall say that $f<g$ if and only if $f(t)\le g(t)$ for $t\in\Delta$ and $f\neq g$. By $\mathbb{P}_\Delta$ we shall denote the cone of non-negative elements in $\mathbb{R}_\Delta$. In |9| the notion of a paranormed space over a topological semifield is introduced. For the semifield $\mathbb{R}_\Delta$ we obtain the following definition.

DEFINITION 2. *The triplet* $(E,\|\ \|,\Phi)$ *is a paranormed space over a topological semifield* $\mathbb{R}_\Delta$ *if and only if* $\|\ \|:E\to\mathbb{P}_\Delta$ *and* $\Phi$ *is a linear, continuous, positive mapping from* $\mathbb{R}_\Delta$ *into* $\mathbb{R}_\Delta$ *such that the following conditions are satisfied:*

1. $\|x\| = 0 \iff x = 0.$
2. $\|\lambda x\| = |\lambda|\,\|x\|$ *, for every* $x\in E$ *and every* $\lambda\in\mathbb{K}$
3. $\|x+y\|\le\Phi(\|x\|)+\Phi(\|y\|)$, *for every* $x,y\in E$.

Let us denote by $\mathcal{U}$ the family of neighborhoods of zero in $\mathbb{R}_\Delta$ and for every $U\in\mathcal{U}$ we shall denote the set

$$\{x \mid x \in E,\ \|x\| \in U\}$$

by $V_U$. Then E is a topological vector space in which $V = \{V_U\}_{U \in \mathcal{U}}$ is the family of neighborhoods of zero in E. In |9| it is proved that every Hausdorff topological vector space E is a paranormed space $(E, \|\ \|, \Phi)$ over a topological semified $\mathbb{R}_\Delta$ (for some set $\Delta$) and we shall say that the triplet $(E, \|\ \|, \Phi)$ is the associated paranormed space. In |1| we have introduced the following definition.

DEFINITION 3. *Let* E *be a Hausdorff topological vector space and* $(E, \|\ \|, \Phi)$ *be the associated paranormed space. The set* $K \subseteq E$ *is of* $\Phi$*-type if and only if for every* $n \in N$:

$$\left\| \sum_{i=1}^{n} t_i x_i \right\| \leq \sum_{i=1}^{n} t_i\, \Phi(\|x_i\|)$$

*for every* $x_i \in K-K$ $(i=1,2,\ldots,n)$ *and every* $t_i \in [0,1]$ $\sum_{i=1}^{n} t_i = 1$.

In |1| it is proved that every subset of $\Phi$ is admissible in the sense of V.Klee. A fixed point theorem for such sets is also proved in |1|.

LEMMA *Let* E *be a Hausdorff topological vector space and* K *be a subset of* E *of* $\Phi$ *type. Then* K *is of Zima's type.*

P r o o f. We shall prove that for every $W \in V$ there exists $U \in V$ so that $co(U \cap (K-K)) \subset W$. Let $W \in V$ and $(E, \|\ \|, \Phi)$ be the associated paranormed space over the topological semifield $\mathbb{R}_\Delta$. For every $\varepsilon > 0$ and every finite subset $\Delta'$ of $\Delta$, let us denote by $U_{\Delta',\varepsilon}$ the set $\{\tilde{x} \mid \tilde{x} \in \mathbb{R}_\Delta,\ \tilde{x}(t) < \varepsilon, \text{ for every } t \in \Delta'\}$. From the definition of the family V it follows that there exists $\{t_1, t_2, \ldots, t_n\} = \Delta' \subset \Delta$ and $\varepsilon > 0$ so that:

$$\|u\| \in U_{\Delta',\varepsilon} \Rightarrow u \in W \quad .$$

The mapping $\Phi$ is linear and continuous and so let $U_1$ be such neighborhood of zero in $\mathbb{R}_\Delta$ that:

$$\|u\| \in U_1 \Rightarrow \Phi(\|u\|) \in U_{\Delta',\varepsilon} .$$

If $U_2$ is a symmetric neighborhood of zero in E such that $U_2 \subset V_{U_1}$ then:

$$co(U_2 \cap (K-K)) \subset W .$$

Let $u \in co(U_2 \cap (K-K))$. Then $u = \sum_{i=1}^{n} s_i u_i$, where $u_i \in U_2 \cap (K-K)$ for every $i \in \{1,2,\dots,n\}$ and $s_i \in [0,1]$ $(i \in \{1,2,\dots,n\})$ so that $\sum_{i=1}^{n} s_i = 1$. Since the set K is of $\Phi$ type it follows that:

$$\|u\|(t) = \|\sum_{i=1}^{n} s_i u_i\|(t) \le \sum_{i=1}^{n} s_i \phi(\|u_i\|)(t) < \varepsilon$$

for every $t \in \Delta'$ .

COROLLARY 3. *Let* E *be a complete topological vector space. Further, let* $0 \in K \subseteq E$, K *be a closed and convex subset of* $\Phi$ *type and* U ,H *and* $\chi$, $\mathcal{M}'$ *be as in Corollary 1. Then there exists* $x \in cl_K(U)$ *such that* $x \in H(0,x)$.

P r o o f. From the Lemma it follows that all the conditions of Corollary 1 are satisfied.

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**REZIME**

## LERAY-SCHAUDEROV PRINCIP ZA VIŠEZNAČNA PRESLIKAVANJA U VEKTORSKO TOPOLOŠKIM PROSTORIMA

Korišćenjem rezultata Schöneberga |15| i teoreme o nepokretnoj tački iz |2| u ovom radu je dokazana sledeća teorema.

TEOREMA. *Neka je* E *vektorsko topološki prostor*, $K \subseteq E$, $\chi$ *mera nekompaktnosti nad* K, $0 \in K$, $U \subseteq K$ *i* U *je okolina nule u* E, $M'$ *familija nepraznih, konveksnih i kompaktnih podskupova od* K, $H : [0,1] \times cl_K(U) \to M'$, $co(\{0\} \cup H([0,1] \times cl_K(U)))$ *zadovoljava Zimin uslov (Definicija 1) tako da su zadovoljeni sledeći uslovi:*

(i) $x \notin H(t,x)$ *za* $t \in [0,1]$ *i* $x \in \partial_K U$.

(ii) $X \subset cl_K(U)$ *i* $\chi(X) \leq \chi(H([0,1] \times X))$ *implicira da je skup* X *relativno kompaktan.*

(iii) *Za svako* $t > 1$ *i* $x \in \partial_K U$, $tx \notin H(1,x)$.

*Ako je preslikavanje* H *od gore poluneprekidno tada postoji* $x \in cl_K(U)$ *tako da je* $x \in H(0,x)$.