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STRUCTURE OF GENERALIZED EQUIVALENCES CONTAINED IN $(2,n\bar{A}_1)$ - RT RELATIONS

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It is well-known that if ρ is a binary reflexive and transitive relation on S, then $\sigma = \rho \cap \rho^{-1}$ is an equivalence on S, and that an ordering χ can be defined on S/σ by: $(X,Y) \in \chi$ iff $(x,y) \in \rho$ for any $x \in X$, $y \in Y$. Binary relation σ is a maximal (in regard to the set inclusion) equivalence relation contained in ρ, and morever, the set of all equivalences in ρ is a complete lattice.

The class of binary reflexive and transitive relations is uniquely determined. In |3| it is shown that this is not the case with (n+1)-ary relations, when $n \geq 2$. Here we consider 2-reflexive, $n\bar{A}_1$-transitive, (n+1)-ary relations on the given set S, denoted as $(2,n\bar{A}_1)$-RT relations, induced among some other classes of (n+1)-ary relations in |3|. The structure of generalized equivalences (defined in |1|) included in such an generalized quasi-order is the subject of this article. We show that this poset always has the maximal elements, and we give the necessary and sufficent conditions under which it is a complete lattice. Finally, we describe two generalized orderings induced on the corresponding partition of type n (Hartmanis, see |1|) by one class of $(2,n\bar{A}_1)$-RT relations. We note that the considerations of some of these problems, we started in |2|.

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1. (n+1)-ary relation ρ on S is (i_1^t)-reflexive, $i_1,\ldots,i_t \in \{2,\ldots,n+1\}$, iff

$$(a_1^{i_1-1}, a, a_{i_1+1}^{i_2-1},\ldots,a_{i_{t-1}+1}^{i_t-1},a,a_{i_t+1}^{n+1}) \in \rho ,$$

for all $a_1,\ldots,a_{i_1-1},a_{i_1+1},\ldots,a_{i_t-1},a_{i_t+1},\ldots,a_{n+1},a \in S$[1].

ρ is t-reflexive, $t \in \{2,\ldots,n+1\}$, iff it is (i_1^t)-reflexive for all different $i_1,\ldots,i_t \in \{1,\ldots,n+1\}$[2]. An (i_1^{n+1})-reflexive relation ρ is (trivialy) (n+1)-reflexive, and it is described by the formula:

$$(\forall a \in S)((\overset{n+1}{a}) \in \rho) .$$

2. (n+1)-ary relation ρ on S is k-antisymmetric, $k \in \{2,\ldots,n+1)\}$, iff for all $a_1,\ldots,a_k \in S$ the following is satisfied:

If all permutations of $a_1,\ldots,a_k$ are included in (n+1) - tuples of ρ, then $a_1 =\ldots= a_k$.

3. (n+1)-ary relation ρ on S is $n\bar{A}_1$ -transitive iff from $(a_o^n) \in \rho$, $(a_1^{n+1}) \in \rho$, and $a_i \neq a_j$ for $i \neq j$, $i,j \in \{1,\ldots,n\}$, it follows that $(a_o^{n-1},a_{n+1}) \in \rho$, for all $a_o,\ldots,a_{n+1} \in S$.

REMARK:

Some other generalizations of the antisymmetric and transitive relations are given in |3|,|4| and |5|.

4. (n+1)-ary relation ρ on S is symmetric, iff for all $a_1,\ldots,a_{n+1} \in S$, the following is satisfied:
$(a_1^{n+1}) \in \rho$ implies $(a_{\pi(1)},\ldots,a_{\pi(n+1)}) \in \rho$, for each $\pi \in \{1,\ldots,n+1\}!$.

1) For t=2, this is (i,j)-reflexivity from |5|.

2) "Reflexive" in |5| is 2-reflexive here.

5. (n+1)-ary relation ρ on S is generalized equivalence, iff it is (1,n+1)-reflexive, symmetric, and $n\bar{A}_1$-transitive (|1|).

6. We denote by d_2 the intersection of all 2-reflexive (n+1)-ary relations on S, i.e.

$$d_2 = \{(a_o^n) \mid a_o,\ldots,a_n \in S,\ a_i = a_j \text{ for some } i,j \in \{0,\ldots,n\}\}.$$

(d_2 is $n\bar{A}_1$-transitive too, see |3|).

In the following, we assume that $|S| \geq n$.

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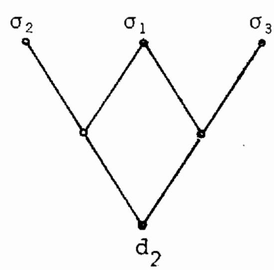
To illustrate the problems that arise in considering the structure of equvalences contained in generalized quasi-order , we start with one example.

EXAMPLE 1. $S = \{a,b,c,d,e\}$, n=2.

$$\rho = \pi(a,b,c)^{1)} \cup \pi(b,c,d) \cup \pi(b,c,e) \cup \pi(a,b,e) \cup \pi(a,c,e) \cup$$
$$\{(a,b,d),(d,b,a),(a,c,d),(d,c,a),(b,a,d),(b,d,a),$$
$$(c,a,d),(c,d,a),(e,a,d),(e,b,d),(e,c,d),(a,e,d),$$
$$(b,e,d),(c,e,d),(d,b,e),(d,c,e),(b,d,e),(c,d,e)\}.$$

ρ is $(2,2\bar{A}_1)$-RT relation on S. The following relations are maximal ternary equivalences contained in ρ .

$$\sigma_1 = d_2 \cup \pi(a,b,c) \cup \pi(b,c,e) \cup \pi(a,c,e) \cup \pi(a,b,e) ;$$
$$\sigma_2 = d_2 \cup \pi(b,c,d) \cup \pi(a,b,a);$$
$$\sigma_3 = d_2 \cup \pi(b,c,d) \cup \pi(a,c,e);$$



Hasse diagram of the partialy order set of equivalences in ρ illustrates the situation.

1) $\pi(a,b,c)$ denotes $\{(a,b,c),(a,c,b),(b,a,c),(b,c,a),(c,a,b),(c,b,a)\}$

THEOREM 1. *Let ρ be $(2,n\bar{A}_1)$ -RT relation on* S . *Now, if* E *denotes the set of all equivalence relations* σ *on* S *, such that* $\sigma \subseteq \rho$, *then the partially ordered set* $\langle E, \subseteq \rangle$ *contains at least one maximal equivalence relation.*

P r o o f. $E \neq \emptyset$, since $d_2 \subseteq E$ (see |3|). Let $\{\sigma_i; i \in I\}$ be a chain in $\langle E, \subseteq \rangle$. $\bar{\sigma} = \bigcup_{i \in I} \sigma_i$ is an upper bound for that chain. Realy, $\bar{\sigma}$ is 2-reflexive, since $d_2 \subseteq \bar{\sigma}$. $\bar{\sigma}$ is symmetric: if $(a_1^{n+1}) \in \bar{\sigma}$ then $(a_1^{n+1}) \in \sigma_i$, for some $i \in I$, and since σ_i is symmetric, $(a_{\pi(1)}, \ldots, a_{\pi(n+1)}) \in \sigma_i$, for every $\pi \in \{1,\ldots,n+1\}!$, and thus $(a_{\pi(1)}, \ldots, a_{\pi(n+1)}) \in \bar{\sigma}$, for every π. $\bar{\sigma}$ is $n\bar{A}_1$-transitive: suppose that $(a_0^n) \in \bar{\sigma}$ and $(a_1^{n+1}) \in \bar{\sigma}$, and $a_1, \ldots, a_n$ are different. Then $(a_0^n) \in \sigma_i$ and $(a_1^{n+1}) \in \sigma_j$, for some $i, j \in I$. Let $\sigma_i \subseteq \sigma_j$. Then both $(n+1)$-tuples belong to σ_j and by $n\bar{A}_1$-transitivity it follows that $(a_0^{n-1}, a_{n+1}) \in \sigma_j$, and thus $(a_0^{n-1}, a_{n+1}) \in \bar{\sigma}$. By Zorn's Lemma we conclude that $\langle E, \subseteq \rangle$ has a maximal element.

Generalizing binary case for $(2,n\bar{A}_1)$-RT relation ρ on S, we get the following definition of the relation σ_ρ:

(1) $(a_1^{n+1}) \in \sigma_\rho$ iff $(a_{\pi(1)}, \ldots, a_{\pi(n+1)}) \in \rho$, for every $\pi \in \{1,\ldots,n+1\}!$.

It is obvious that the following proposition holds.

Lemma 2. *If* $\sigma \in E$, *then*

a) $(a_1^{n+1}) \in \sigma$ *implies* $(a_{\pi(1)}, \ldots, a_{\pi(n+1)}) \in \rho$, *for every* $\pi \in \{1,\ldots,n+1\}!$

b) $d_2 \subseteq \sigma \subseteq \sigma_\rho$.

Up to now we have found that for $n > 1$

i) σ_ρ is not always transitive;

ii) $\langle E, \subset \rangle$ can have more than one maximal equivalence; and thus iii) $\langle E, \subset\rangle$ is not always a lattice.

If the following we discuss some of these problems.

THEOREM 3. $\sigma_\rho = \cup E$ *(union of all* (n+1)-*ary equivalences in* E *).*

P r o o f. 1) $\cup E \subset \sigma_\rho$. Realy, if $(a_1^{n+1}) \in \sigma$, $\sigma \in E$, then by a), Lemma 2, $(a_1^{n+1}) \in \sigma_\rho$.

2 $\sigma_\rho \subset \cup E$. Indeed, if $(a_1^{n+1}) \in \sigma_\rho$, then for every $\pi \in \{1, \dots, n+1\}!$ $(a_{\pi(1)}, \dots, a_{\pi(n+1)}) \in \sigma_\rho$, and (a_1^{n+1}) belongs at least to equivalence relation $\sigma = d_2 \cup \{(a_{\pi(1)}, \dots, a_{\pi(n+1)}); \pi \in \{1, \dots \dots, n+1\}!\}$. Thus, $(a_1^{n+1}) \in \cup E$.

It follows from 1) and 2) that $\sigma_\rho = \cup E$.

It is obvious that $\langle E, \subseteq\rangle$ is a meet semilattice with zero d_2. Now we can give the necessary and sufficent conditions under which it is a lattice.

We start with the following definition of a special (n+1)-ary quasiorder.

(n+1)-ary relation ρ on S is $(2,n\bar{A}_1)^*$ RT relation iff it is $(2,n\bar{A}_1)$-RT relation and the following is satisfied:

(*) If

(a) $(a_{\alpha(o)}, \dots, a_{\alpha(n)}) \in \rho$ and $(a_{\beta(1)}, \dots, a_{\beta(n+1)}) \in \rho$,

for each $\alpha \in \{0, \dots, n\}!$ and for each $\beta \in \{1, \dots, n+1\}!$, and $a_o, \dots \dots, a_{n+1}$ are different elements of S, then, with each cosequence

(b) $(b_1, \dots, b_{n+1}) \in \rho$, $(b_1, \dots, b_{n+1} \in \{a_o, \dots, a_{n+1}\})$,

for the corresponding premises of (a) by $n\bar{A}_1$-transitivity, in ρ is also

$(\bar{b})$ $(b_1, \dots, b_{n-1}, b_{n+1}, b_n)$,

for all $a_o, \dots, a_{n+1} \in S$.

THEOREM 4. *If* ρ *is* $(2,n\bar{A}_1)^{*}$ *RT ralation, then* $\cup E$ *is* (n+1)-*ary equivalence relation on* S.

P r o o f.

a) $\cup E$ is (by definition) 2-reflexive and symmetric.

b) $\cup E$ ia $n\bar{A}_1$- transitive:

Let $(a_o^n) \in \cup E$ and $(a_1^{n+1}) \in \cup E$, $a_i \neq a_j$, for $i \neq j$, $i,j \in \{1, \ldots, n\}$. By $(\bar{I})$ this is equivalent to (a) in (*).

b_1) If $a_o, \ldots, a_{n+1}$ are not all different, and the conditions for the application of $n\bar{A}_1$-transitivity are satisfied, then $(a_{\gamma(o)}, \ldots, a_{\gamma(n-1)}, a_{\gamma(n+1)}) \in \rho$, for each $\gamma \in \{0, \ldots, n-1, n+1\}!$, because of

1) 2-reflexivity of ρ ; or

2) the consequence becomes one of the premises in (a).

Thus, $(a_o, \ldots, a_{n-1}, a_{n+1}) \in \cup E$.

b_2) Suppose now that $a_o, \ldots, a_{n+1}$ are all different. Then, starting with (a), we get that with $(a_o, \ldots, a_{n-1}, a_{n+1})$,all (n+1) tuples of the form

$$(a_o, a_{\eta(1)}, \ldots, a_{\eta(n-1)}, a_{n+1}), \quad \eta \in \{1, \ldots, n-1\}! \ .$$

also belong to ρ. Since 2-reflexive and $n\bar{A}_1$-transitive relation admits all cyclic permutations of first n coordinates of it´s elements, and by (*), it follows that for each

$$\gamma \in \{0, \ldots, n-1, n+1\}!, \ (a_{\gamma(o)}, \ldots, a_{\gamma(n-1)}, a_{\gamma(n+1)}) \in \rho \ .$$

Thus, $(a_o, \ldots, a_{n-1}, a_{n+1}) \in \cup E$, completing the proof of the proposition.

THEOREM 5. $\langle E, \subset \rangle$ *is a complete lattice iff* ρ *is* $(2,n\bar{A}_1)^{*}$ *RT relation.*

P r o o f.

a) Let ρ be $(2,n\bar{A}_1)^{\pm}$RT relation on S. Then by Theorem 4., $\cup E \in E$. That is why $\cup E$ is the only maximal element in $\langle E, \subset\rangle$. and clearly, the gratest one. E is closed under arbitrary intersections, and thus, it is a complete lattice.

b) Let now $\langle E,\subset\rangle$ be a complete lattice. Then it has a unit element $\cup E$. $\cup E$ is thus $(2,n\bar{A}_1)^{\pm}$ RT relation. Indeed, let

(o) $\quad (a_o^n) \in \cup E$ and $(a_1^{n+1}) \in \cup E$ imply $(a_o^{n-1}, a_{n+1}) \in \cup E$.

Then by $(\bar{1})$

$(a_o^n) \in \cup E$ iff $(a_{\alpha(o)},\dots,a_{\alpha(n)}) \in \rho$, for every $\alpha\in\{0,\dots,n\}!$;

$(a_1^{n+1}) \in \cup E$ iff $(a_{\beta(1)},\dots,a_{\beta(n+1)}) \in \rho$, for every $\beta \in \{1,\dots,n+1\}!$

$(a_o^{n-1}, a_{n+1}) \in \cup E$ iff $(a_{\gamma(o)},\dots,a_{\gamma(n-1)},a_{\gamma(n+1)}) \in \rho$, for every $\gamma \in \{0,\dots,n-1,n+1\}!$.

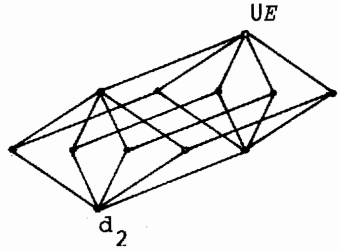
In this way it is shown that ρ satisfies (*), and thus it is $(2,n\bar{A}_1)^{\pm}$RT relation.

EXAMPLE 2. $S=\{a,b,c,d,e,f\}$, $n=2$.

$\rho = d_2 \cup \pi(a,b,c) \cup \pi(a,b,d) \cup \pi(a,c,d) \cup \pi(b,c,d) \cup \pi(d,e,f)$ (i)

$\cup \{(a,b,e),(b,a,e),(a,c,e),(c,a,e),(a,b,e),(d,a,e),(b,c,e),$
$(c,b,e),(b,d,e),(d,b,e),(c,d,e),(d,c,e),(a,d,f),(b,a,f),$
$(a,c,f),(c,a,f),(a,d,f),(d,a,f),(b,c,f),(c,b,f),(b,d,f),$
$(d,b,f),(c,d,f),(d,c,f)\}.$

ρ is $(2,n\bar{A}_1)^{\pm}$RT relation. The lattice $\langle E,\subset\rangle$ is given by it's Hasse diagram, where zero is d_2, and unit is $\cup E$, described by (i) in ρ.



Since in binary case there is only one class of RT relations, and it satisfies (*), the fact that for n=1 $\langle E, \subseteq \rangle$ is a lattice is a direct consequence of Theorem 5.

* * *

Consider now the binary relation χ, defined at the beginning of the article, concerning the induced order on the partition. The following two theorems deal with the same problems for (n+1)-ary relations.

THEOREM 6. *Let* ρ *be* $(2,n\bar{A}_1)^{*}$-RT *relation on* S , *and denote* $\cup E$ *by* σ. *Let* S/σ *be the corresponding partition of type* n. *Now, if* χ *is* (n+1)-*ary relation on* S/σ, *defined by*

$$(x)\qquad (Q_1^{n+1}) \in \chi \text{ iff } (x_{i_1},\dots,x_{i_{n+1}}) \in \rho, \text{ for all}$$

$$(x_{i_1},\dots,x_{i_{n+1}}) \in Q_1 \times \dots \times Q_{n+1}, Q_1,\dots,Q_{n+1} \in S/\sigma ,$$

then in this way induced (by ρ*) relation* χ *is* (n+1)-*reflexive,* (n+1) -*antisymmetric, and* $n\bar{A}_1$-*transitive.* 1)

P r o o f.

a) $(\overset{n+1}{Q}) \in \chi$ iff $(x_{i_1},\dots,x_{i_{n+1}}) \in \rho$, for all

$x_{i_1},\dots,x_{i_{n+1}} \in Q$, and this is true since $(x_{i_1},\dots,x_{i_{n+1}}) \in$

$\in \sigma \subseteq \rho$.

Thus χ is (n+1)-reflexive.

b) χ is (n+1)-antisymmetric:

Let $(Q_{\pi(1)},\dots,Q_{\pi(n+1)}) \in \chi$, for each $\pi \in \{1,\dots,n+1\}!$. Then

$(x_{\pi(i_1)},\dots,x_{\pi(i_{n+1})}) \in \rho$, whenever this (n+1)-tuple belongs

to $Q_{\pi(1)} \times \dots \times Q_{\pi(n+1)}$, i.e. when

1) These properties are consistent as shown in |3|.

$(x_{i_1},\dots,x_{i_{n+1}}) \in \sigma$. But this means that $x_{i_1},\dots,x_{i_{n+1}}$ belong to the same class, i.e. $Q_1 = \dots = Q_{n+1}$.

c) χ is $n\bar{A}_1$-transitive:

Let $(Q_0^n) \in \chi$, $(Q_1^{n+1}) \in \chi$, $Q_i' \neq Q_j$, for $i\neq j$, $i,j \in \{1,\dots,n\}$. This holds if and only if

$(x_{i_0},\dots,x_{i_n}) \in \rho$, $(x_{i_1},\dots,x_{i_{n+1}}) \in \rho$, whenever $x_{i_j} \in Q_j$ $(j \in \{0,\dots,n+1\})$.

Then $(x_{i_0},\dots,x_{i_{n-1}},x_{i_{n+1}}) \in \rho$, $x_{i_j} \in Q_j$, $j \in \{0,\dots,n-1,n+1\}$,

since : a) ρ is 2-reflexive (if $x_{i_1},\dots,x_{i_n}$ are not all different) or b) ρ is $n\bar{A}_1$-transitive (otherwise).

By a), b) and c), the proof is complete.

Generalized ordering relation χ , defined in the preceding proposition, in binary case reduces to the usual one. The same is with the relation ψ , given in the following proposition. This one has already been defined in |3|, but with some unpreciseness included. That is why we repeat it here, together with one example, illustrating both, χ and ψ.

THEOREM 7. *Let* $|S| > n$, $n\neq 2$ *and* ρ, σ , *and* S/σ *be as in Theorem 6. Define* $(n+1)$*-ary relation* ψ *on* S/σ *in the following way:*

For $Q_1,\dots,Q_{n+1} \in S/\sigma$, *if*

a) $|\{Q_1,\dots,Q_{n+1}\}| \neq 2$, *then*

$(Q_1^{n+1}) \in \psi$ *if and only if there are* $x_1,\dots,x_{n+1} \in S$, $x_i \neq x_j$, *for* $i\neq j$, $i,j \in \{1,\dots,n+1\}$, *such that*

I $A_i = \{x_1,\dots,x_{n+1}\} \setminus \{x_i\} \subseteq Q_i$, $i=1,\dots,n+1$, *and that*

II $(x_{i_1},\dots,x_{i_{n+1}}) \in \rho$ *when* $(x_{i_1},\dots,x_{i_{n+1}}) \in A_1 \times\dots\times A_{n+1}$;

and if

b) $|\{Q_1,\dots,Q_{n+1}\}| = 2$, *then*

$(Q_1^{n+1}) \in \psi$ *iff there is exactly one set with* n+1 *element*

$\{x_1,\dots,x_{n+1}\} \in S$, $x_i \neq x_j$, *for* $i \neq j$, $i,j \in \{1,\dots,n+1\}$, *such that*

I *and* II *hold.*

Then ψ *is* (n+1)*-reflexive,* (n+1)*-antisymmetric, and* $n\bar{A}_1$*-transitive relation on* S/σ.

P r o o f.

a) $(Q_1^{n+1}) \in \psi$ if and only if there are $x_1,\dots,x_{n+1}$, $x_i \neq x_j$, such that A_1 defined in I is a subset of Q_1, and that II is satisfied for $A_i = A_1$, i=1,...,n+1. Since each class contains at least n elements, A_1 always exists, and II is a consequence of the definition of S/σ. Thus, ψ is (n+1)-reflexive.

b) Let $(Q_{\pi(1)},\dots,Q_{\pi(n+1)}) \in \psi$, for each $\pi \in \{1,\dots,n+1\}!$.

Then for each such π , there is exactly n+1 element $x_1,\dots,x_{n+1}$ such that I and II are satisfied, provided that $Q_1,\dots,Q_{n+1}$ are not all equal. Realy, if $\{Q_1,\dots,Q_{n+1}\}$ consists of only two different classes, then this uniqueness is postulated. Otherwise, suppose that for some $\alpha,\beta \in \{1,\dots,n+1\}!$ $Q_{\alpha(1)},\dots,Q_{\alpha(n+1)}$ determines $x_1,\dots,x_{i-1},x_i,\dots,x_{n+1}$, and $Q_{\beta(1)},\dots,Q_{\beta(n+1)}$ determines $x_1,\dots,x_{i-1},x_i',\dots,x_{n+1}$. Now, for Q_r and Q_s, $r, s \in \{1,\dots,n+1\} \setminus \{i\}$, $Q_r \neq Q_s$, we have

$|Q_r \cap Q_s| = |\{x_1,\dots,x_i,x_i',\dots,x_{n+1}\} \setminus \{x_r,x_s\}| = n$ which

means thet $Q_r = Q_s$, contrary to our assumption. So we can consider $x_1,\dots,x_{n+1}$. Each of this elements is in at least one class and thus all permutations of (x_1^{n+1}) are in ρ , i.e. all those classes are equal, proving (n+1)-antisymmetry for ψ .

c) ψ is $n\bar{A}_1$ -transitive: Let $(Q_0^n) \in \psi$, $(Q_1^{n+1}) \in \psi$ satisfy the conditions of $n\bar{A}_1$-transitivity. It means that there are $x_0,\dots,x_n$

and $y_1,\dots,y_{n+1}$, satisfying I and II. By the definition of the sets A_i, $\{x_o,\dots,x_n\}=\{y_1,\dots,y_{n+1}\}$, and we can deduce that $x_i=y_i$ for $i=1,\dots,n$, and $x_o=y_{n+1}$[1]. Now, $n\bar{A}_1$-transitivity for ψ follows directly from the same property of ρ .

EXAMPLE 3. $S = \{1,2,3,4\}$, $n=2$.

$$\rho = d_2 \cup \pi(1,2,3) \cup \{(1,2,4),(2,1,4),(1,3,4),(3,1,4),(3,4,1),(4,3,1),(2,3,4),(3,2,4),(3,4,2),(4,3,2)\}.$$

ρ is $(2,2\bar{A}_1)^{*}$-RT relation on S.

$$\sigma = d_2 \cup \pi(1,2,3) .$$

S/σ: $Q_1 = \{1,2,3\}$, $Q_2 = \{1,4\}$, $Q_3 = \{2,4\}$, $Q_4 = \{3,4\}$.

3-reflexive, 3-antisymmetric and $2\bar{A}_1$- transitive relation χ , defined in Theorem 6, is given by:

$$\chi = \{(Q_1,Q_1,Q_1),(Q_2,Q_2,Q_2),(Q_3,Q_3,Q_3),(Q_4,Q_4,Q_4),(Q_1,Q_1,Q_2),(Q_1,Q_1,Q_3),(Q_1,Q_1,Q_4),(Q_3Q_4Q_3),(Q_4,Q_3,Q_3),(Q_4,Q_4,Q_2),(Q_4,Q_2,Q_2),(Q_2,Q_4.Q_2),(Q_4,Q_4,Q_3)\}.$$

3-reflexive, 3-antisymmetric and $2\bar{A}_1$-transitive relation ψ, defined in Theorem 7., is given by:

$$\psi = \{\{(Q_i^3)_{i=1,2,3,4}\} \cup \{(Q_1,Q_2,Q_2),(Q_2,Q_1,Q_2),(Q_1,Q_3,Q_3),(Q_3,Q_1,Q_3),(Q_4,Q_4,Q_1),(Q_1,Q_1,Q_2),(Q_1,Q_1,Q_3),(Q_1,Q_1,Q_4),(Q_4,Q_4,Q_2),(Q_4,Q_2,Q_2),(Q_2,Q_4,Q_2),(Q_4,Q_4,Q_3),(Q_4,Q_3,Q_3),(Q_3,Q_4,Q_3),(Q_4,Q_1,Q_3),(Q_1,Q_4,Q_3),(Q_1,Q_4,Q_2),(Q_4,Q_1,Q_2)\}.$$

1) The statement holds in ternary case also if we require that $x_1=y_1$ and $x_2=y_2$.

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REZIME

STRUKTURA UOPŠTENIH EKVIVALENCIJA SADRŽANIH U $(2,n\bar{A}_1)$-RT RELACIJAMA

U radu se razmatra jedna klasa generalisanih relacija pretporetka ($(2,n\bar{A}_1)$-RT relacija) i ispituje se struktura u njima sadržanih ekvivalencija. Daju se potrebni i dovoljni uslovi pod kojima je taj parcijalno uredjen skup kompletna mreža. Takodje se pokazuje da se na odgovarajućim particijama tipa n može posmatrati uopšteni poredak, indukovan spomenutim generalisanim pretporetkom.