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SOME APPLICATIONS OF BOCSAN'S FIXED POINT THEOREM

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1. Introduction

In this paper we shall prove some fixed point theorems in probabilistic metric spaces, using Bocsan's fixed point theorem from [3] and some of the results from [7].

In [27] A.N. Sherstnev introduced the notion of random normed space which is a special Menger probabilistic metric space. Some fixed point theorems in probabilistic metric and random normed spaces are proved in [6], [16], [17], [18].

A probabilistic metric space $(S, \mathcal{F})$ is formed by a nonempty set S together with a mapping $\mathcal{F}$ which assigns to each $(x, y) \in S \times S$ a distribution function $F_{x,y}$ such that the following conditions are satisfied:

(F1) $F_{x,y}(t)=1$ for all $t \geqslant 0$ if and only if $x=y$.

$(F2)$ For every $(x, y) \in S \times S$, $F_{x,y}(0)=0$.

$(F3)$ For every $(x,y) \in S \times S$, $F_{x,y}=F_{y,x}$.

(F4) If $F_{x,y}(r)=1$ and $F_{y,z}(s)=1$ then $F_{x,z}(r+s)=1$.

By a Menger space $(S, \mathcal{F}, t)$, we mean a probabilistic metric space $(S, \mathcal{F})$ with (F4) replaced by the condition:

(F4) For every $(x, y, z) \in S \times S \times S$ and every $r, s \geqslant 0$:

$$F_{x,z}(r+s) \geqslant t\,(F_{x,y}(r), \quad F_{y,z}(s))$$

where t is a T-norm [20].

The (ε, λ)-topology is introduced by the family $\{U_v(\varepsilon, \lambda)\}_{v \in S,\, \varepsilon>0,\, \lambda \in (0,1)}$ where:

$$U_v(\varepsilon, \lambda)=\{u \mid F_{u,v}(\varepsilon)>1-\lambda\}$$

and this topology is metrisable if $\sup t\,(x, x)=1$.

A *random normed space* $(S, \mathcal{F}, t)$ is a triplet, where S is a linear space over $\mathcal{K}$, $\mathcal{F}: S \to \Delta^+(\Delta^+$ is the set of all the distribution functions F such that $F(0)=0)$. and t is a T-norm such that the following conditions are satisfied:

(R1) $F_p(0)=0$, for all $p \in S$.

(R2) $F_p = H \Leftrightarrow p = 0 \in X$, where $H(u) = \begin{cases} 1 & u>0 \\ 0 & u \leqslant 0 \end{cases}$

(R3) If λ is a non-zero scalar then:

$$F_{\lambda p}(u) = F_p\left(\frac{u}{|\lambda|}\right), \quad \text{for all } p \in S, \; u \in R.$$

(R4) $F_{p+q}(u+v) \geqslant t(F_p(u), F_q(v))$, for all $p, q \in S$, $u \geqslant 0, v \geqslant 0$.

(R5) $t(u, v) \geqslant \max\{u+v-1, 0\}$, for all $u, v \in [0, 1]$.

A random normed space $(S, \mathcal{F}, t)$ is a Menger space under the mapping $\mathcal{F}$ defined by:

$$\mathcal{F}(p, q) = F_{p-q}, \quad \text{for all } p, q \in S.$$

If T-norm t is continuous then S is a Hausdorff topological vector space under the (ε, λ)-topology.

Let Δ be the set of all the distribution function F and $F \geqslant G$ $(F, G \in \Delta)$ iff $F(x) \geqslant G(x)$, for every $x \in R$. Furthermore, $F > G$ iff $F \geqslant G$ and $F \neq G$. If $F \in \Delta$ then:

$$S_F = \{G \mid G \in \Delta, \quad G \geqslant F\}.$$

DEFINITION 1. [3] *The topology in* Δ *for which is the family*

$$\{S_F \mid F \in \Delta\}$$

the subbase of closed subsets is τ*-topology.*

Bocsan and Constantin [8] introduced the notion of probabilistic bounded subset in a probabilistic metric space.

DEFINITION 2. [5] *Let* $(S, \mathcal{F})$ *be a probabilistic metric space and* $A \subseteq S$. *The function* D_A *on* R *defined by:*

$$D_A(u) = \sup_{v<u} \inf_{p, q \in A} F_{p,q}(v)$$

is called the probabilistic diameter of A *and* A *is probabilistic bounded if:*

$$\sup_{u \in R} D_A(u) = 1.$$

By $\mathfrak{B}(S)$ we shall denote the set of all probabilistic bounded subsets of a probabilistic metric space.

DEFINITION 3. [7] *A mapping* $\gamma: \mathfrak{B}(S) \to \Delta$ *is a random measure of noncompactness if the following implication holds:*

$$\gamma(A) = H \Leftrightarrow A \text{ is precompact in the } (\varepsilon, \lambda)\text{-topology}.$$

Kuratowski's function α_A for a probabilistic bounded subset $A \subseteq S$, defined by:

$$\alpha_A(u) = \sup\{\varepsilon \mid \varepsilon > 0, \text{ there exists a finite cover } \mathcal{A} \text{ of } A$$

$$\text{such that } D_S(u) \geqslant \varepsilon, \text{ for all } S \in \mathcal{A}\}$$

is an example of the random measure of noncompactness.

DEFINITION 4. *Let* $M: S \to S$ *be continuous and* γ *be a random measure of noncompactness. The mapping* M *is* γ *probabilistic densifying if and only if:*

For every $A \in \mathfrak{B}(S)$, $\gamma_A < H \Rightarrow \gamma_{M(A)} > \gamma_A$.

In the next theorem $\Phi: S \times S \to \Delta$ is a τ-continuous mapping. It is easy to see, similarly as in [3], that the following Theorem is valid.

THEOREM 1. *Let* $(S, \mathcal{F}, t)$ *be a complete Menger space with a T-norm* t *such that* $\sup_{a<1} t(a, a) = 1$ *and* $M: S \to S$ *be a* γ*-probabilistic densifying where the random measure of noncompactness satisfies the following condition:*

$$\gamma_{A \cup \{p\}} = \gamma_A, \quad \text{for every } A \in \mathfrak{B}(S),\ p \in S.$$

Furthermore suppose that the following conditions are satisfied:

1. *There exists* $p_0 \in S$ *such that* $\sup_{x \in R} G_{p_0}(x) = 1$, *where:*

$$G_{p_0}(x) = \inf\{F_{M^n p_0 - p_0}(x) \mid n \in N\}.$$

2. *For every* $p, q \in S, p \neq q$ *is:*

$$\Phi(Mp, Mq) > \Phi(p, q).$$

Then there exists one and only one fixed point of M.

Let X be a separable Banach space and V be the set of all random variables on the complete probability measure space $(\Omega, \mathcal{K}, P)$ with values in X. So $\xi \in V$ if and only if $\xi: \Omega \to X$ and:

$$\{\omega \mid \omega \in \Omega, \quad \xi(\omega) \in B\} \in \mathcal{K}$$

for every Borel's subset B of X. For every $\xi \in V$ let:

$$F_\xi(x) = P\{\omega \mid \omega \in \Omega, \quad \|\xi(\omega)\| < x\}, \text{ for every } x \in R.$$

The mapping $F: \xi \to F_\xi(\cdot)$ is a random seminorm on V if T-norm t is t_m and the (ε, λ)-topology on V induced by F is the convergence in probability [4].

Let $\mathcal{V}$ be the set of all classes of random variables from V which are P equal almost everywhere. Then the triplet $(\mathcal{V}, \mathcal{F}, t_m)$ is a random normed space where the mapping $\mathcal{F}: \mathcal{V} \to \Delta$ is defined by:

$$\mathcal{F}: \xi \to F_\xi, \quad \text{for every } \xi \in \mathcal{V}.$$

2. Fixed point theorems

Applying Theorem 1 we shall prove a fixed point theorem for mapping $M:\mathcal{U}\to\mathcal{U}$ i.e. the existence of an element $\xi\in\mathcal{U}$ such that $M\xi=\xi$.

In the next Theorem we shall suppose that γ is a random measure of noncompactness such that $\gamma_{A\cup\{p\}}=\gamma_A$ for every $A\in\mathfrak{B}(\mathcal{U})$ and every $p\in\mathcal{U}$.

THEOREM 2. *Let $M:\mathcal{U}\to\mathcal{U}$ be a γ probabilistic densifying mapping such that the following conditions are satisfied:*

1. *There exists $C>O$ so that for every $U\in\mathcal{U}$:*

$$P\{\omega\mid\omega\in\Omega,\quad \|(MU)(\omega)\|\leqslant C\}=1.$$

2. a) *For every $U, V\in\mathcal{U}$:*

$$P\{\omega\mid\omega\in\Omega,\quad \|(MU)(\omega)-(MV)(\omega)\|\leqslant\|U(\omega)-V(\omega)\|\}=1.$$

b) *For every $U, V\in\mathcal{U}$ there exists $\varepsilon_{U,V}>0$ so that:*

$$P\{\omega\mid\omega\in\Omega,\quad \|(MU)(\omega)-(MV)(\omega)\|<\varepsilon_{U,V}\leqslant\|U(\omega)-V(\omega)\|\}>0.$$

Then there exists one and only one fixed point of the mapping M.

Proof: We shall prove that all the conditions of Theorem 1 are satisfied, where $(S, \mathcal{F}, t)$ is random normed space $(\mathcal{U}, \mathcal{G}, t_m)$ and the mapping $\Phi:\mathcal{U}\times\mathcal{U}\to\Delta$ is defined by:

$$\Phi(U, V)=F_{U-V},\quad \text{for every}\quad U, V\in\mathcal{U}.$$

The mapping Φ is τ continuous. Let us prove this fact. If $G\in\Delta$ and $S_G=\{F\mid F\in\Delta,\ F\geqslant G\}$ is a closed subset in Δ then $\Phi^{-1}(S_G)=\{(U, V)\mid(U, V)\in\mathcal{U}\times\mathcal{U}$ and $F_{U-V}\geqslant G\}$. Since $S\times S$ is metrisable, it is sufficient to prove that:

$$\{(U_n, V_n)\}_{n\in N}\subseteq\Phi^{-1}(S_G)\quad\text{and}\quad \lim_{n\to\infty}(U_n, V_n)=(U, V)\Rightarrow(U,V)\in\Phi^{-1}(S_G).$$

From $(U_n, V_n)\in\Phi^{-1}(S_G)$, it follows that $F_{U_n-V_n}\geqslant G$, and since:

$$\liminf_{n\in N} F_{U_n-V_n}=F_{U-V}$$

we have $F_{U-V}\geqslant G$ which implies that $(U, V)\in\Phi^{-1}(S_G)$. Furthermore:

$$(1)\qquad \Phi(MU, MV)>\Phi(U, V),\quad \text{for every}\quad U\neq V\ (U, V\in\mathcal{U})$$

which means that $F_{MU-MV}\geqslant F_{U-V}$ but $F_{MU-MV}\neq F_{U-V}$. Let us prove (1). In the subsequent text we shall use the following notations: If $U, V\in\mathcal{U}$ and $\varepsilon>0$ then $B_{U,V,\varepsilon}=\{\omega\mid\omega\in\Omega,\ \|U(\omega)-V(\omega)\|<\varepsilon\}$ and $C_{U,V}=\{\omega\mid\omega\in\Omega,\|(MU)(\omega)-(MV)(\omega)\|\leqslant\|U(\omega)\|-V(\omega)\}$. It is obvious that:

$$B_{U,V,\varepsilon}\cap C_{U,V,\varepsilon}\subseteq B_{MU,MV,\varepsilon}\quad \text{for every}\ U, V\in\mathcal{U}$$

and so from 2. a, it follows that:

$$F_{U,V}(\varepsilon)=P(B_{U,V,\varepsilon})\leqslant P(B_{MU,MV,\varepsilon})=F_{MU-MV}(\varepsilon).$$

Let us show that:

(2) $$F_{U-V}(\varepsilon_{U,V}) < F_{MU-MV}(\varepsilon_{U,V}).$$

For every $U, V \in \mathfrak{V}$ we shall denote by $A_{U,V}$ the set:

$$\{\omega \mid \omega \in \Omega, \| (MU)(\omega) - (MV)(\omega) \| < \varepsilon_{U,V} \leqslant \| U(\omega) - V(\omega) \| \}.$$

We have that:

$$A_{U,V} = \overline{B}_{U,V,\varepsilon_{U,V}} \cap B_{MU,MV\varepsilon_{U,V}}$$

and:

$$B_{MU,MV,\varepsilon_{U,V}} = (B_{U,V,\varepsilon_{U,V}} \cap B_{MU,MV,\varepsilon_{U,V}}) \cup (\overline{B}_{U,V,\varepsilon_{U,V}} \cap B_{MU,MV,\varepsilon_{U,V}})$$

and so:

$$P(B_{MU,MV,\varepsilon_{U,V}}) = P(B_{U,V,\varepsilon_{U,V}} \cap B_{MU,MV,\varepsilon_{U,V}}) + P(A_{U,V}).$$

Since from 2. b), it follows that $P(A_{U,V}) > 0$, we conclude that:

$$P(B_{MU,MV,\varepsilon_{U,V}}) > P(B_{U,V,\varepsilon_{U,V}} \cap B_{MU,MV,\varepsilon_{U,V}}).$$

Furthermore:

(3) $$B_{U,V,\varepsilon_{U,V}} = (B_{U,V,\varepsilon_{U,V}} \cap C_{U,V}) \cup (B_{U,V,\varepsilon_{U,V}} \cap \overline{C}_{U,V})$$

and (3) implies that:

(4) $$B_{U,V,\varepsilon_{U,V}} \cap B_{MU,MV,\varepsilon_{U,V}} = (B_{U,V,\varepsilon_{U,V}} \cap B_{MU,MV,\varepsilon_{U,V}} \cap C_{U,V}) \cup$$
$$\cup (B_{U,V,\varepsilon_{U,V}} \cap B_{MU,MV,\varepsilon_{U,V}} \cap \overline{C}_{U,V}).$$

Since $B_{U,V,\varepsilon_{U,V}} \cap C_{U,V} \subseteq B_{MU,MV,\varepsilon_{U,V}}$ (4) implies that:

(5) $$P(B_{U,V,\varepsilon_{U,V}} \cap B_{MU,MV,\varepsilon_{U,V}}) = P(B_{U,V,\varepsilon_{U,V}})$$

because $P(C_{U,V}) = 1$ and $P(\overline{C}_{U,V}) = 0$. From (5) we have that:

$$P(B_{MU,MV,\varepsilon_{U,V}}) > P(B_{U,V,\varepsilon_{U,V}})$$

and so (2) is satisfied, which implies (1).

It remains to be proved that condition 1 of Theorem 1 is satisfied. Indeed we have that for every $U \in \mathfrak{V}$:

$$\sup_x G_U(x) = 1$$

where:

$$G_U(x) = \inf \{ F_{M^n U - U}(x) \mid n \in N \}.$$

Since $F_{U-V}(x) = P\{\omega \mid \omega \in \Omega, \| U(\omega) - V(\omega) \| < x\}$ we shall prove that:

(6) $$\sup_{t>0} \inf_{n \in N} P\{\omega \mid \omega \in \Omega, \| (M^n U)(\omega) - U(\omega) \| < t\} = 1$$

for every $U \in \mathfrak{V}$.

In order to prove (6) we shall show that for every $\lambda \in (0, 1)$ there exists $t(\lambda)>0$ so that:

$$\inf_{n \in N} P\{\omega \mid \omega \in \Omega, \|(M^n U)(\omega)-U(\omega)\|<t(\lambda)\}>1-\lambda.$$

Let $U \in \mathcal{U}$ and $\delta(\lambda')$ for $\lambda'<\lambda$ such that $F_U(\delta(\lambda'))>1-\lambda'$.

Furthermore let: $D=\{\omega \mid \omega \in \Omega, \|U(\omega)\|<\delta(\lambda')\}$

$$A_n=\{\omega \mid \omega \in \Omega, \|(M^n U)(\omega)-U(\omega)\| \leqslant C+\delta(\lambda')\}, \quad n \in N$$

$$B_n=\{\omega \mid \omega \in \Omega, \|(M^n U)(\omega)\| \leqslant C\}.$$

Since $B_n \cap D \subseteq A_n$ and $P(B_n)=1$, it follows that:

$$P(A_n)>1-\lambda', \text{ for every } n \in N$$

and so:

$$\inf_{n \in N} P(A_n) \geqslant 1-\lambda'>1-\lambda$$

which implies (6).

We can apply Theorem 2 in order to obtain a fixed point theorem for the random normed operator $T: \Omega \times X \to X$, where X is a separable Banach space. The mapping $T: \Omega \times X \to X$ is a random operator if and only if for every $x \in X$ the mapping $\omega \to T(\omega, x)$ is a random variable. Here we shall suppose that $(\Omega, \mathcal{K}, P)$ is an atomic probability measure space i. e. $\Omega = \bigcup_{n \in N} \Omega_n$, where Ω_n are different atoms.

Then every random variable $\xi: \Omega \to X$ is constant P a.e. on every atom.

In [7] it is proved that a continuous random operator $T: \Omega \times X \to X$ has a fixed point $\xi \in \mathcal{U}$ if and only if ξ is the fixed point of the operator $T: \mathcal{U} \to \mathcal{U}$ defined by:

$$(T\xi)(\omega)=T(\omega, \xi(\omega)), \ \omega \in \Omega.$$

A random operator $T: \Omega \times X \to X$ is a *random nonexpansive operator* if and only if:

$$P\{\omega \mid \omega \in \Omega, T(\omega, \cdot): X \to X \text{ is a nonexpansive mapping}\}=1$$

and a random densifying if and only if:

$$P\{\omega \mid \omega \in \Omega, \quad T(\omega, \cdot): X \to X \text{ is } \alpha \text{ densifying}\}=1.$$

In [7] is defined [on a random normed space $(\mathcal{U}, \mathcal{F}, t_m)$] the random measure of noncompactness $\tilde{\alpha}_A(\cdot)$ for $A \in \mathfrak{B}(\mathcal{U})$ in the following way:

$\tilde{\alpha}_A(x)=P\{\omega \mid \omega \in \Omega, \alpha(A(\omega))<x\}$, for every $x \in R$ where $A(\omega)=\{\xi(\omega) \mid \xi \in A\}$.

In [7] it is proved that the mapping $\tilde{T}:\mathcal{V}\to\mathcal{V}$ is $\tilde{\alpha}$-densifying if and only if the random operator $T:\Omega\times X\to X$ is α-densifying. From Theorem 2 we obtain the following Corollary.

COROLLARY. *Let $T:\Omega\times X\to X$ be a random nonexpansive α-densifying operator such that there exists a bounded subset $M\subseteq X$ such that:*

$$P\{\omega \mid \omega\in\Omega,\quad T(\omega, X)\subseteq M\}=1.$$

If for each two measurable mappings $\xi_1, \xi_2:\Omega\to X$ there exists $\varepsilon(\xi_1, \xi_2)>0$ such that:

$$P\{\omega \mid \omega\in\Omega, \|T(\omega, \xi_1(\omega))-T(\omega, \xi_2(\omega)\|<\varepsilon(\xi_1, \xi_2)\leqslant\|\xi_1(\omega)-\xi_2(\omega)\|\}>0$$

then there exists one and only one $\xi\in\mathcal{V}$ such that:

$$T(\omega, \xi(\omega))=\xi(\omega) \quad P \text{ a.e.}$$

3. Continuous dependance of fixed points on the parameter

We shall apply Theorem 1 on a continuous dependance of fixed points on the parameter.

In the following Theorem we shall suppose that γ is a random measure of noncompactness defined on $\mathfrak{B}(S)$, where $(S, \mathcal{F}, t)$ is a random normed space, such that for every $A, B\in\mathfrak{B}(S)$:

$$A\subseteq B\Rightarrow\gamma_A\geqslant\gamma_B. \tag{7}$$

It is known that the Kuratowski measure of noncompactness has this property.

As in [18] we shall prove the following Theorem.

THEOREM 3. *Let $(S, \mathcal{F}, t)$ be a complete random normed space with T--norm t such that t is continuous, let X be a probabilistic bounded and closed subset of S, Λ be a metrisable topological space and $M:X\times\Lambda\to X$ be a continuous mapping and $\Phi:X\times X\to\Delta$ be a τ continuous mapping such that the following conditions are satisfied:*

(a) *For every $x, y\in X, x\neq y$ and every $\lambda\in\Lambda$:*

$$\Phi(M(x, \lambda),\quad M(y, \lambda))>\Phi(x, y).$$

(b) *For every $X'\subseteq X$ such that $\gamma_{X'}<H$ and every $\lambda\in\Lambda$ there exists a neighbourhood $V(X', \lambda)$ of $\lambda\in\Lambda$ such that:*

$$L'\subseteq V, \overline{L'} \quad \textit{is compact}\Rightarrow\gamma_{M(X', L')}>\gamma_{X'}.$$

Then there exists one and only one continuous mapping $f:\Lambda\to X$ such that:

$$f(\lambda)=M[f(\lambda), \lambda] \quad \textit{for every} \quad \lambda\in\Lambda.$$

Proof: From Theorem 1 it follows that there exists, for every $\lambda\in\Lambda$, one and only one element $f(\lambda)\in X$ such that:

$$f(\lambda)=M(f(\lambda), \lambda).$$

That the mapping f is continuous we can prove, as in [18], since

$$\gamma_{\{M(f(\lambda_n),\lambda_n),|n\in N\}} \geqslant \gamma_{M(X',L')}$$

where X' and L' are defined by:

$$X'=\{f(\lambda_n)\mid n\in N\},\quad L'=\{\lambda_n \mid n\geqslant n_0\}\subseteq \Lambda\cap V(X',\lambda)$$

where $\lim_{n\to\infty}\lambda_n=\lambda$ $(\{\lambda_n\}_{n\in N}\subseteq X)$. The rest of the proof is as in [18].

From Theorem 3 we obtain the following Corollary.

COROLLARY 2. *Let X' be a probabilistic bounded and closed subset of the random normed space $(\mathcal{V}, \mathcal{F}, t_m)$, Λ be a metrisable topological space, γ be a random measure of noncompactness on $\mathfrak{B}(\mathcal{V})$ such that* (7) *holds, $M: X'\times\Lambda\to X'$ be a continuous mapping such that the following conditions are satisfied:*

(A) *For every $U, V\in X'$ and every $\lambda\in\Lambda$:*

$$P\{\omega\mid\omega\in\Omega, \|M(U,\lambda)(\omega)-M(V,\lambda)(\omega)\|\leqslant\|U(\omega)-V(\omega)\|\}=1.$$

(B) *For every $U, V\in X'$ there exists $\varepsilon_{U,V}>0$ so that:*

$$P\{\omega\mid\omega\in\Omega, \|M(U,\lambda)(\omega)-M(V,\lambda)(\omega)\|\leqslant\varepsilon_{U,V}\leqslant\|U(\omega)-V(\omega)\|\}>0.$$

(C) *For $X=X'$, condition* (b) *of Theorem* 3 *is satisfied. Then there exists one and only one continuous mapping $f:\Lambda\to X'$ so that:*

$$f(\lambda)(\omega)=M(f(\lambda),\lambda)(\omega),\ \textit{for every}\ \lambda\in\Lambda\ P\ \text{a.e.}$$

Let X be a separable Banach space, $CB(X)$ the family of all nonempty, closed and bounded subsets of X and $E:\omega\to E(\omega)$ be a mapping from Ω into $CB(X)$. The mapping E is measurable if and only if for every closed subset $C\subset X$:

$$\{\omega\mid\omega\in\Omega,\quad E(\omega)\cap C\neq\emptyset\}\in\mathcal{K}.$$

If $(\Omega, \mathcal{K}, P)$ is an atomic probability measure space, then E is constant a.s. on every atom. In the subsequent text we shall suppose that E is constant on every atom.

The mapping $U:\Omega\to X$ is a *measurable selector* of the mapping E if and only if $U(\omega)\in E(\omega)$, for every $\omega\in\Omega$.

Using Corollary 2 we can prove the following Corollary in which Λ is a metri sable topological space, $(\Omega, \mathcal{K}, P)$ is an atomic probability measure space and $E:\omega\to E(\omega)$ is a measurable mapping from Ω into $CB(X)$, $\mathcal{E}$ is the set of all measurable selectors with convergence in probability.

COROLLARY 3. *Let for every $\lambda\in\Lambda$, $T(\lambda,\cdot,\cdot)$ be a random operator on X such that for every $\lambda\in\Lambda$ and every $\omega\in\Omega$, $T(\lambda,\omega,E(\omega))\subseteq E(\omega)$ and $P\{\omega\mid\omega\in\Omega, (\lambda,x)\mapsto T(\lambda,\omega,x)$ is continuous on $\Lambda\times E(\omega)\}=1$. Suppose that the following conditions are satisfied:*

(I) *For every $\lambda\in\Lambda$:*

$$P\{\omega\mid\omega\in\Omega, \|T(\lambda,\omega,x_1)-T(\lambda,\omega,x_2)\|\leqslant\|x_1-x_2\|,\ \textit{for every}\ x_1,x_2\in E(\omega)\}=1.$$

(II) *For each two measurable selectors* U, V *of* E *there exists* $\varepsilon_{U,V}>0$ *so that for every* $\lambda\in\Lambda$:

$$P\{\omega\mid\omega\in\Omega, \|T(\lambda,\omega,U(\omega))-T(\lambda,\omega,V(\omega))\|<\varepsilon_{U,V}\leqslant\|U(\omega)-V(\omega)\|\}>0.$$

(III) *For every* $\omega\in\Omega$ *the following condition is satisfied: For everv* $B\subseteq E(\omega)$ *and every* $\lambda\in\Lambda$ *there exists a neighbourhood* $V(\lambda)$ *of* $\lambda\in\Lambda$ *so that for every compact* $L'\subseteq V$: $\alpha(T(L',\omega,B))\leqslant\alpha(B)$ *with a strict inequality if* $\alpha(B)>0$.

Then there exists one and only one continuous mapping $f:\Lambda\to\mathcal{E}$ *such that*:

$$f(\lambda)(\omega)=T(\lambda,\omega,f(\lambda)(\omega))\quad P \text{ a.e.}$$

Proof: We shall show that all the conditions of Corollary 2 are satisfied where:

$$X=\mathcal{E} \text{ and for every } U\in\mathcal{E}$$

$$M(U,\lambda)(\omega)=T(\lambda,\omega,U(\omega)), \text{ for every } \omega\in\Omega,\ \lambda\in\Lambda.$$

The random measure of noncompactness α, defined by:

$$\alpha_A(x)=P\{\omega\mid\omega\in\Omega,\ \alpha(A(\omega))<x\} \text{ for every } x\in R\ (A\in\mathfrak{W}(\mathcal{U}))$$

is such that:

$$A\subseteq B\ (A,B\in\mathfrak{W}(\mathcal{U}))\Rightarrow\alpha_B\leqslant\alpha_A.$$

Indeed from $A\subseteq B$ we have that:

$$A(\omega)\subseteq B(\omega), \text{ for every } \omega\in\Omega.$$

Using the property of Kuratowski's measure of noncompactness α, we conclude that $\alpha(A(\omega))\leqslant\alpha(B(\omega))$ and so:

$$\{\omega\mid\omega\in\Omega,\ \alpha(B(\omega))<x\}\subseteq\{\omega\mid\omega\in\Omega,\ \alpha(A(\omega))<x\}$$

which implies that:

$$\alpha_B(x)\leqslant\alpha_A(x), \text{ for every } x\in R.$$

In [7] it is proved that the set $\mathcal{E}$ is closed in the (ε,λ)-topology and probabilistic ally bounded. From (I) and (II), it follows (A) and (B) and it remains to be proved that condition (C) of Corollary 2 is satisfied. Let $X'\subseteq\mathcal{E}$ be such that $\alpha_{X'}<H$. Since $X'\subseteq\mathcal{E}$, for every $\omega\in\Omega$ we have that $X'(\omega)\subseteq E(\omega)$ and there exists $n_0\in N$ such that $X'(\omega)$ is not procompact for every $\omega\in\Omega_{n_0}$ Furthermore, $P(\Omega_{n_0})>0$ and for every $\lambda\in\Lambda$ there exists a neighbourhood $V(\lambda)$ of λ such that for every $L'\subseteq V(\lambda)$ where $\overline{L'}$ is compact:

$$\alpha(T(L',\omega,X'(\omega))\leqslant\alpha(X'(\omega)), \text{ for every } \omega\in\Omega. \tag{8}$$

From (8) it follows that:

$$\alpha_{M(X',L')}\geqslant\alpha_{X'},$$

and it remains to be proved that we have strict inequality. Suppose, or the contrary, that:

$$\alpha_{M(X',L')}=\alpha_{X'}. \tag{9}$$

From (9) we have that:

$$\alpha(T(L', \omega, X'(\omega)) = \alpha(X'(\omega)) \quad P \text{ a.e.}$$

But, for every $\omega \in \Omega_{n_0}$ we have:

$$\alpha(X'(\omega)) > 0$$

and since $P(\Omega_{n_0}) > 0$, we obtain a contradiction. Hence, all the conditions of Corollary 2 are satisfied.

REFERENCES

[1] A T. Bharucha-Reid, *Random Integral Equations*, Academic Press, New York, 1972.

[2] A. T. Bharucha-Reid, *Fixed point theorems in probabilistic analysis*, Bull. Amer. Math. Soc., 82, 1976, 641–657.

[3] G. Bocsan, *On some fixed point theorems in probabilistic metric spaces*, Mathematica Balkanica 4, (1974), 67–70.

[4] G. Bocsan, *Sur certains seminormes aleatories et applications*, C. R. Acad. Sci. Paris, 282, 22, (1976), 1319–1321.

[5] G. Bocsan, *On the Kuratowski's function in random normed spaces*, Sem. de teoria funct. si matematici aplicate, Timisoara, No. 8, 1974.

[6] G. Bocsan, *On some fixed point theorems in random normed spaces*, Sem. de teoria funct. si matematici aplicate, Timisoara, No. 13, 1974.

[7] G. Bocsan, *On random operators on separable Banach spaces*, Sem. de teoria funct. si matematici aplicate, Timisoara, No. 38, 1978.

[8] G. Bocsan and G. Constantin, *The Kuratowski function and some applications to the probabilistic metric spaces*, Atti Acad. Naz. Lincei, Rend. Cl. Sci. Fis. Math. Nat. LV (1973), Fasc. 3–4, 236–240.

[9] H. W. Engl, *Random fixed point theorems for multivalued mappings*, Pacific Journal of Mathematics, Vol. 76, No 2, 1978.

[10] H. W. Engl, *Some random fixed point theorems for strict contraction and nonexpansive mappings*, Nonlinear Analysis, Theory, Methods & Applications, Vol. 2, No 5, (1978), 619–626.

[11] H. W. Engl, *Random Fixed Point Theorems*, Nonlinear Equations in Abstract Spaces, Ed. V. Lakschmikantham, (1978), 67–81.

[12] H. W. Engl, *A general stochastic fixed point theorem for continuous random operators on stochastic domains*, J. Math. Anal. Appl, *in print*.

[13] H. W. Engl, *Weak convergence of Mann iteration for nonexpansive mappings without convexitity assumptions*, Boll. Un. Mat. Ital., 14 (1977), 471–475.

[14] H. W. Engl, *Fixed point theorems for random operators on stochastic domains*, Dissertation, Johannes Kepler Universität, Linz, 1977.

[15] O. Hadžić, *Osnovi teorije nepokretne tačke*, Institut za matematiku, Novi Sad, 1978.

[16] O. Hadžić, *Fixed point theorems for multivalued mappings in probabilistic metric spaces*, Matematički vesnik, 3(16)(31), 1979, 125-133.

[17] O. Hadžić and D. Despotović, *Some fixed point theorems in random normed spaces*, Ann. Univ. Timisoara, Stiinte Matematice, vol. XVII, Fasc. 1, 1979, 39-47.

[18] O. Hadžić and M. Stojaković, *On the existance of a solution of the system $x=H(x, y), y=K(x, y)$ in random normed spaces*, Zbornik radova Prirodno-matematičkog fakulteta u Novom Sadu, knjiga 9, (1979), 43–48.

[19] O. Hadžić and M. Stojaković, *Two random fixed point theorems*, Zbornik radova Prirodno-matematičkog fakulteta u Novom Sadu, knjiga 9, (1979), 49–52.

[20] V. Istratescu, *Introducere in teoria spatiilor metrice probabiliste cu aplicatii*, Editura Technica, Bucuresti, 1974.

[21] M. Stojaković, *Neke klase slučajnih operatorskih jednačina*, Magistarski rad, Beograd, 1979.

[22] A. N. Sherstnev, *The notion of a random normed spaces*, DAN SSSR, (1973), (Russian).

NEKE PRIMENE TEOREME BOCSANA O NEPOKRETNOJ TAČKI

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REZIME

U ovom radu su date neke primene uopštenja teoreme Boscana iz rada [3] u prostoru $(\mathcal{X}, \mathcal{F}, t_m)$, gde je $\mathcal{X}$ skup klasa slučajnih promenljivih sa vrednostima u separabilnom Banahovom prostoru.